3. No, because $0$ has no inverse.

\[
\begin{align*}
\exp & \quad (\mathbb{R}, +) \quad \exp(a + b) = \exp(a) \exp(b) \\
\log & \quad (\mathbb{R}^{>0}, \cdot) \quad \log(ab) = \log(a) + \log(b)
\end{align*}
\]

$\Rightarrow (\mathbb{R}, +) \cong (\mathbb{R}^{>0}, \cdot)$.

4. $\mathbb{Z} / n$ mod \( \equiv \) has \( n \) elements.

5. No, e.g., the union of the $x$ and $y$ axes is \( (\mathbb{R}^2, +) \).

6. Not likely since $\text{Perm}(G)$ is so huge. To prove this, let \( f : G \rightarrow \text{Perm} G \) be

\[ f : g \mapsto \text{the permutation that swaps } g \text{ and } e \]

\( f \) is clearly 1-1. Suppose that \( G \) has two or more non-identity elements \( g, g' \). But then \( \text{Im} f \) does not contain the swap \((g, g') \in \text{Perm} G \Rightarrow G \neq \text{Perm} G \). If \( |G| = 1 \), or 2, however, you can check that \( G \cong \text{Perm} G \).  

7. No, it's a "semigroup" only since inverses are lacking.

8. Let \( \mathbb{G} \rightarrow G \rightarrow G / [G, G] \) be \( \{gg', gg'': g, g' \in G \} \) the (normal) commutator subgroup of \( \mathbb{G} \). Let \( A \) be an abelian group.

\[ S \xrightarrow{\phi} G \xrightarrow{\pi} G / [G, G] \xrightarrow{\pi} G / [G, G] \]

\[ S \xrightarrow{\phi} G / [G, G] \xrightarrow{\psi} A \xrightarrow{\phi} S \]

is the free abelian group on \( S \).
9. \( R \) is abelian, so all subgroups like \( \mathbb{Z} \) are normal.

Consider
\[
\mathbb{R} \xrightarrow{\alpha} \mathbb{R}/\mathbb{Z}, \quad \alpha(r) = r + \mathbb{Z}
\]

\[
\text{fraction} \xrightarrow{} (-1, 1)
\]

\[
\text{reals mod 1 addition}
\]

\[
\text{fraction } (r + s) = \text{fraction } (r) + \text{fraction } (s) \mod 1
\]

is a group homomorphism with \( \mathbb{Z} = \ker (\text{fraction}) \).

\( \Rightarrow \) there is a unique morphism \( \gamma \) causing the diagram to commute. fraction is onto \( \Rightarrow \gamma \) is epi as well.

\( \Rightarrow \mathbb{R}/\mathbb{Z} \cong (-1, 1) \).

10. Let \( H \) be a normal subgroup of \( G \). Notice that the set of left cosets \( \{ gH : g \in G \} \) and right cosets \( \{ Hg : g \in G \} \) are isomorphic as sets (why? ). Thus, if \( H \) has 2 left cosets, it also has 2 right cosets \( \{ e, G-H \} \) in both cases. \( \Rightarrow \) \( H \) is normal.

11. For \( g \in \text{Aut} \ G \), we must show that \( g \circ C_g \circ g^{-1} \) is also a conjugation. \( g \circ C_g \circ g^{-1}(x) = g( g^{-1} g(x) g ) = g( g^{-1} x g ) = C_{g(g)}(x) \).

13. If \( \ker g \) contains non-identity \( g \), then \( g(1) = g(e) = e \) and \( g \) is not 1-1. Conversely, if \( \ker g = \{ e \} \), and \( g(1) = g(1') \), then \( g(1) g^{-1} = e \Rightarrow g = 1' \Rightarrow g \) is 1-1.
14. Let $\mathbb{R}^\times$ be the multiplicative group of positive reals. Note that $\mathbb{R}^\times \to \mathbb{R}^\times$ is a group homomorphism with $\ker(\text{abs}) = \{-1, 1\}$, so

$$\mathbb{R}^\times \to \mathbb{R}^\times / \ker(\text{abs})$$

abs

$$\Rightarrow \mathbb{R}^\times / \{-1, 1\} \cong \mathbb{R}^\times$$

15. Suppose $G, H$ are groups. Let $G \sqcup H \xrightarrow{\lambda} F$ be the free group in $G \sqcup H$ as a set.

Given any $g, y, z$, the set theoretic direct sum means that the outer rim commutes with some function $\gamma$.

The freeness of $F$ then guarantees a unique group homomorphism $\gamma'$ such that $\gamma' \circ \lambda: G \sqcup H \to F$ commutes. Thus

$$\gamma' \circ \lambda = \gamma,$$ $\gamma' \circ \alpha = \gamma,$ $\gamma \circ \beta = \gamma$

$\Rightarrow \gamma' \circ (\lambda \circ \alpha) = \gamma,$ $\gamma' \circ (\lambda \circ \beta) = \gamma$

$\Rightarrow (F, \lambda \circ \alpha, \lambda \circ \beta)$ is the direct sum of $G$ and $H$. 