2.3. (a) Using the thermodynamic relation
\[ C_p - C_v = T(\partial P / \partial T)_v(\partial V / \partial T)_p = -T(\partial P / \partial T)_v^2 / (\partial P / \partial V)_T \]
and the equation of state (9.3.9), we get
\[ \frac{C_p - C_v}{Nk} = \frac{T(\partial P / \partial T)_v^2}{k(\partial P / \partial V)_T} = \frac{T(k / (v - b))^2}{k(-kT / (v - b)^2 + 2a / v^3)} = \frac{1}{1 - 2a(v - b)^2 / kTv^3}. \]

(b) In view of the thermodynamic relation
\[ TdS = C_v dT + T(\partial P / \partial T)_v dV \]
and the equation of state (9.3.9), an adiabatic process is characterized by the fact that
\[ C_v dT + NkT(v - b)^{-1} dv = 0. \]
Integrating this result, under the assumption that \( C_v = \text{const.} \), we get
\[ T^{C_v/Nk} (v - b) = \text{const.}. \]

(c) For this process we evaluate the Joule coefficient
\[ \left( \frac{\partial T}{\partial V} \right)_u = \left( \frac{\partial U / \partial V}_T \right)_F = -\frac{T(\partial P / \partial T)_v - P}{C_v} = \frac{-a}{C_v} = -\frac{N^2a}{C_vV^2}. \]
Now integrating from state 1 to state 2, we readily obtain the desired result.
\[ P/P_{\text{sea}} = \exp\left(-\frac{Mg}{R\gamma T}\right) = \exp\left(-\frac{1}{2}\right) \]

\[ \frac{dP}{dT} = P \frac{L}{kT^2} \Rightarrow \int \frac{dP}{P} = \int \frac{L}{kT^2} \, dT \]

\[ \ln P - \ln P_{\text{sea}} = -\frac{L}{kT_{\text{sea}}} + \frac{L}{kT} \]

\[ \Rightarrow T = \left[\frac{L}{kT_{\text{sea}}} + -\frac{L}{2} \ln \left(\frac{P}{P_{\text{sea}}}\right)\right]^{-1} \]

\[ T_{\text{sea}} = 373 K \quad \Rightarrow \quad T = 359 K = 86^\circ C \]
3. (a) Let \( z_i = z_{i+1}^{\pm 1} \)

\[
q_i = -J, \quad z_i z_{i+1} - J \quad z_i z_{i+1}^{2} \quad z_{i+2} = -J, \quad z_{i+1} - J \quad z_{i+1}^{2} \quad c = \text{const.}
\]

\[
z_i = z_i e^{-\beta H} = \text{Tr}(T^\mu) = \lambda^+ + \lambda^-
\]

\[
T = \left( \begin{array}{cc}
\sigma^+ & 0 \\
0 & \sigma^-
\end{array} \right)
\]

\[\lambda^+ \text{ are eigenvalues of } T\]

\[
0 = \begin{vmatrix}
e^{\sigma^+} & e^{\sigma^+} \\
-\sigma^+ & -\sigma^-
\end{vmatrix}
= \lambda^2 - 2\lambda e^{\beta J_2} \cosh(\beta J_1) + 2 \sinh(2 \beta J_2)
\]

\[
= \lambda^2 - 2\lambda e^{\beta J_2} \cosh(\beta J_1) + 2 \sinh(2 \beta J_2)
\]

\[
\lambda^+ = e^{\beta J_2} \cosh(\beta J_1) \pm \sqrt{e^{2\beta J_2} \cosh^2(\beta J_1) - 2 \sinh(2 \beta J_2)}
\]

\[
\lambda^+ > \lambda^-, \quad \lambda^+ \to \infty \quad \lambda^+ \gg \lambda^-
\]

\[
z^\ast \approx \frac{\lambda^+}{\sqrt{e^{2\beta J_2} \cosh^2(\beta J_1) - 2 \sinh(2 \beta J_2)}}
\]
(b) For $T=0$, $\beta \to \infty$

\[ Z_N = \left( \frac{1}{2} e^{\beta J_1} e^{\beta J_2} + \sqrt{\frac{1}{4} (e^{\beta J_1})^2 (e^{\beta J_2})^2 - e^{2\beta J_1} + e^{-2\beta J_2}} \right) \]

For $J_2 > -J_1$, $Z_N = e^{\beta J_2 (N+1)}$ \(\Rightarrow U = -N (J_1 + J_2)\)

For $J_2 \leq -J_1$, $Z_N = e^{\beta J_2 \beta J_1}$ \(\Rightarrow U = N J_2\)

For $J_2 > -J_1$ case

\[ U = -N (J_1 + J_2) = \left\langle \sum_{i=1}^{\infty} \sigma_i \sigma_{i+1} \right\rangle \]

\(\Rightarrow \sigma_1 \sigma_{i+1} = 1\) and \(\sigma_{i+1} \sigma_{i+2} = 1\) \(\Rightarrow\) ferromagnetic state.

For $J_2 \leq -J_1$, we can get \(\sigma_1 \sigma_{i+1} = \pm 1\) \(\Rightarrow\) paramagnetic state.
A plot of the internal energy vs. inverse temperature is shown in Figure 3.3. The internal energy is given by:

\[
\left(\frac{\frac{x^2 + \frac{2}{3}x^2 - \frac{2}{3}x^3 + \frac{2}{3}x^4 - 1/4 + \frac{2}{3}x + x + 1}{x^2 + \frac{2}{3}x^2 - \frac{2}{3}x^3 + \frac{2}{3}x^4 - 1/4 + \frac{2}{3}x + x^2 + x}} - 1\right) f N = \frac{x\theta - x}{\theta} f N = \mathcal{A}
\]

where \( x = x \). The largest eigenvalue is \( \lambda \). The Internal energy is thus

\[
\left(\frac{x^2 + \frac{2}{3}x^2 - \frac{2}{3}x^3 + \frac{2}{3}x^4 - 1/4 + \frac{2}{3}x + x + 1}{x^2 + \frac{2}{3}x^2 - \frac{2}{3}x^3 + \frac{2}{3}x^4 - 1/4 + \frac{2}{3}x + x^2 + x}\right) = \gamma
\]

The eigenvalues of this matrix can be obtained by solving the characteristic equation

The eigenvalues of the transfer matrix for the spin-1 Ising ferromagnet are given by

\[
\exp(\omega\beta) = \rho
\]

where \( \beta \) and \( \rho \) are constant. Therefore,

\[
\rho = \exp(\omega\beta)
\]

The elements of the transfer matrix for the spin-1 Ising ferromagnet are:

\[
\begin{pmatrix}
\rho_a & \rho_b \\
\rho_c & \rho_d
\end{pmatrix}
\]

which shows the expected increase of the length of the chain as a function of \( L \).

\[
\rho^{-\beta} N^{\beta} = \frac{a^2 + 1}{a - 1} N^\beta = l
\]

Relating only terms proportional to \( N \) we arrive at the result

\[
\begin{pmatrix}
\sum_{i=1}^{L} z_i + \rho & N = \gamma l
\end{pmatrix}
\]

where \( L \) is the length with \( \gamma \). Therefore

\[
\rho^{-\beta} = \langle f | \omega \rangle
\]

as for the Ising ferromagnet with the result

The correlation function for this model can be worked out in the same fashion

\[
\begin{pmatrix}
\sum_{i=1}^{L} f_i + \rho & N = \gamma l
\end{pmatrix}
\]

The possible model is the Ising antiferromagnet with

\[
\begin{pmatrix}
\sum_{i=1}^{L} f_i + \rho & N = \gamma l
\end{pmatrix}
\]

Following way

Length of a polymer with the end-to-end distance of a random walk is

\[
\begin{pmatrix}
\sum_{i=1}^{L} f_i + \rho & N = \gamma l
\end{pmatrix}
\]

Following way