Assignment #4    PY 541    Week of Sept. 25–29, 2006

Reading: This week, we will treat the grand canonical ensemble and then give a short general discussion of the applications of thermodynamics. Please finish reading chapter 4 of the text. Reif, chapter 5 is a good and more comprehensive reference for this material.

Notes: I will be absent next Monday October 2 because of the Jewish holiday of Yom Kippur. Please feel free to schedule a time to see me either on the previous Friday or on Tuesday October 3.

Problems: Due Tuesday October 3.

1. Prove the equipartition theorem within the framework of classical statistical mechanics. Namely, for each quadratic term in either the generalized coordinates or momenta which appears in the Hamiltonian, there is a contribution of $\frac{1}{2}kT$ to the average energy of the system.


3. (Adapted from Plischke and Bergersen 2.6) Show that the grand canonical partition function of the ideal gas in a box of volume $V$ is

$$Z = \exp \left[ e^{\beta \mu} \frac{V}{V_Q} \right]$$

From this expression for $Z$ compute the mean values of $N$, $P$, and $S$. Show that $PV = NkT$. Use the Gibbs-Duhem relation to derive $E = \frac{3}{2}NkT$. Finally show that $\left\langle (\Delta N)^2 \right\rangle = N$ in the grand canonical ensemble.

4. (Reif 5.23) Two identical bodies with the same temperature-independent heat capacity $C$ at constant pressure are used as heat reservoirs for a heat engine. The bodies remain at constant pressure and undergo no phase change. Initially, their temperatures are $T_1$ and $T_2 < T_1$. As a result of the heat engine operation, the bodies attain a common final temperature $T_f$.

(a) What is the total work $W$ done by the engine? Express your answer in terms of $C$, $T_1$, $T_2$, and $T_f$.
(b) From entropic considerations, derive an inequality that relates $T_f$ with $T_1$ and $T_2$.
(c) For given initial temperatures $T_1$ and $T_2$, find the maximum work obtainable from the engine.