Assignment #3  PY 541  Week of Sept. 18–22, 2006

Reading: This week we will complete the theoretical discussion of the canonical ensemble and the connection between thermodynamics and statistical mechanics. Several illustrative examples of basic concepts will then be given. After the canonical ensemble, we will turn to the grand canonical ensemble as well as several examples of its usage. These topics comprise the crucial conceptual core of the course. Please complete reading in chapter 3 and then continue to the end of chapter 4 in Pathria.

Problems: Due Tuesday September 26.

1. Text 3.15.

2. Text 3.18.

3. Text 3.29.

4. Text 3.35.

5. Study the Einstein model for a crystal lattice. Assume that each atom in a solid is an independent quantum-mechanical harmonic oscillator, with energy levels \( \epsilon_n = (n + \frac{1}{2}) \hbar \omega \)
   
   (a) Compute the partition function of a single oscillator. Then compute the partition function of a solid of \( N \) atoms. Note that there is no factor of \( \frac{1}{N!} \) because the atoms are distinguishable.
   
   (b) Compute the mean energy of the system as a function of temperature and plot your answer. Determine the low- and high-temperature limiting behaviors and give a physical explanation for your results.
   
   (c) Compute the heat capacity at constant volume and sketch the result as a function of temperature. Show that the heat capacity is monotonically increasing with temperature. Discuss the low- and high-temperature limiting behaviors.