Assignment #10   PY 541   Week of Nov. 27 – Dec. 1, 2006

Reading: We discuss the Landau theory of phase transitions (Sec. 11.9 in the text) and then continue with the two-dimensional Ising model (Sec. 12.3). I will then (probably next week) discuss disordered spin systems, such as spin glasses and random-field models. For additional readings on these topics, I suggest “Equilibrium Statistical Mechanics” by M. Plischke and B. Berenssen, (Prentice Hall), and “Statistical Mechanics” by S.-K. Ma (World Scientific).

Notes: The final examination is scheduled for Wednesday Dec. 20 at 12:30pm in SCI 113. Please plan for a 3-hour exam. The exam be closed book, with no notes, calculators, or other ancillary material allowed. Considerable algebra should not be needed in solving any problem.

Problems: Due Friday Dec. 8 by 5pm. I assigned problem 3 last year in PY 541, so please don’t consult with 2nd-year students.

1. Consider an Ising-like spin system on a hypercubic lattice in which the magnitude of the spin takes the values 1 and $\epsilon$ on alternating sites. Within the mean-field approximation, determine the critical temperature and the low temperature magnetization for both the “even” and “odd” sites. Comment on the behavior for small $\epsilon$.

2. Consider the Bethe approximation for the Ising model, which represents a next approximation beyond the Curie-Weiss theory. For a lattice of coordination number $z$, consider a cluster of $z+1$ spins (in zero external magnetic field $H$) which consists of a central spin and its $z$ surrounding nearest neighbors. Consider the ferromagnetic interaction between the central and boundary spins exactly, but assume that the interaction of the boundary spins with their nearest-neighbors can be replaced by an effective field of magnitude $H'$. Under these assumptions, compute the magnetization of the central and the boundary spins. Since these two magnetizations should be equal, this gives a self-consistency condition for $H'$ which determines the critical temperature. Give the numerical value of the critical temperature (in units of $J$) for a lattice with $z = 4$.

3. Consider a ferromagnetic Ising model on the complete graph in which $N$ sites are all interconnected by bonds. (For example, a 4-site complete graph is a tetrahedron with 6 sides. A general $N$-site complete graph has $\binom{N}{2}$ bonds). Further, suppose that the interaction strength of the Ising model on such a complete graph is $j = J/N$, with $J$ independent of $N$. (This is chosen so that the system has a finite energy per spin in the thermodynamic limit). (a) Without detailed computation, determine whether there is a spontaneous magnetization in this system at sufficiently low, but non-zero temperature. (b) Compute the partition function of this Ising model, in zero and finite magnetic field. (c) Compute the spontaneous magnetization.