Electric Potential Energy (Cont'd)

Example: where is the potential zero?
Two charges, +3Q and −Q, are separated by 4 cm. Is there a point along the line passing through them (and a finite distance from the charges) where the net electric potential is zero? If so, where?

First, think qualitatively.
Is there such a point in Region I?
Region II?
Region III?

Example: where is the potential zero?
Unlike electric field, where we had to worry about two vectors being equal and opposite, we just have to worry about two numbers having the same magnitude but opposite sign.

One charge has three times the magnitude of the other. Thus, we're looking for points that are three times farther from the +3Q charge than the −Q charge. (Recall that \( V = \frac{kq}{r} \))

In which region(s) can we find such points?
Ans. Region II and Region III.

Let's also set up and solve the equations …

Case I: Location lying in Region II
For this case, we have the following relations:
\[ r_1 + r_2 = 4 \text{ cm} \quad (1) \]
\[ r_1 = 3r_2 \quad (2) \]
where \( r_1 \) is the distance of the location from the +3Q charge and \( r_2 \) is the distance of the location from the −Q charge.

Substitute (2) into (1), one finds: \( 3r_2 + r_2 = 4 \text{ cm} \Rightarrow r_2 = 1 \text{ cm} \)

Hence the location with zero potential is 1 cm to the left of the −Q charge.

Case II: Location lying in Region III
For this case, we have the following relations:
\[ r_1 - r_2 = 4 \text{ cm} \quad (1) \]
\[ r_1 = 3r_2 \quad (2) \]

Substitute (2) into (1), one finds: \( 3r_2 - r_2 = 4 \text{ cm} \Rightarrow r_2 = 2 \text{ cm} \)

Hence, the location with zero potential is 2 cm to the right of the −Q charge.
Let's look at \( V(r) \) ...

Plotting the potentials due to the +3Q and –Q charges may also help one visualize where \( V(r) = 0 \).

![Potential plot](image)

-6 -4 -2 0 2 4 6 8 10 \( r \)/cm

At these points, \( V_{+3Q}(r) = V_{-Q}(r) \)

<table>
<thead>
<tr>
<th>( r )/cm</th>
<th>Potential of the +3Q charge</th>
<th>Potential of the -Q charge</th>
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<tr>
<td>(-1 \times 10^9)</td>
<td>(1 \times 10^9)</td>
<td>0</td>
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<tr>
<td>(-2 \times 10^9)</td>
<td>(2 \times 10^9)</td>
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<td>(-3 \times 10^9)</td>
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<td>(-10 \times 10^9)</td>
<td>(10 \times 10^9)</td>
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</tbody>
</table>

Capacitors

A capacitor is a device for storing charge. The simplest type of capacitor is made up of two parallel conducting plates separated by either empty space or by an insulating material known as the dielectric. For a capacitor storing charge \( Q \), one conductor has a charge of +\( Q \) and the other has a charge of -\( Q \).

The amount of charge a capacitor can store per applied potential difference is given by its capacitance, \( C \). It is determined by the capacitor geometry and the dielectric material used.

The unit of capacitance is farad (F).

For a capacitor with a charge of +\( Q \) on one plate, -\( Q \) on the other and voltage \( \Delta V \) across the two plates:

\[
Q = C \Delta V
\]

A parallel-plate capacitor

A parallel-plate capacitor is a pair of identical conducting plates, each of area \( A \), placed parallel to one another and separated by a distance \( d \). With nothing between the plates, the capacitance is:

\[
C = \frac{\varepsilon_0 A}{d}
\]

\( \varepsilon_0 \) is known as the permittivity of free space.

\[
\varepsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12} \text{ C}^2/(\text{N m}^2)
\]

Playing with a capacitor I – While the capacitor is connected to the power supply, vary the gap between the plates

Take a parallel-plate capacitor and connect it to a power supply. The power supply sets the potential difference between the plates of the capacitor.

While the capacitor is still connected to the power supply, the distance between the plates is increased. When this occurs, what happens to \( C \), \( Q \), and \( \Delta V \)?

1. \( C \) decreases, \( Q \) decreases, and \( \Delta V \) stays the same
2. \( C \) decreases, \( Q \) increases, and \( \Delta V \) increases
3. \( C \) decreases, \( Q \) stays the same, and \( \Delta V \) increases
4. All three decrease
5. None of the above

Playing with a capacitor II – with the power supply disconnected, vary the gap

Take a parallel-plate capacitor and connect it to a power supply. Then disconnect the capacitor from the power supply. After this, the distance between the plates is increased. When this occurs, what happens to \( C \), \( Q \), and \( \Delta V \)?

1. \( C \) decreases, \( Q \) decreases, and \( \Delta V \) stays the same
2. \( C \) decreases, \( Q \) increases, and \( \Delta V \) increases
3. \( C \) decreases, \( Q \) stays the same, and \( \Delta V \) increases
4. All three decrease
5. None of the above

Playing with a capacitor I – While the capacitor is connected to the power supply, vary the gap between the plates

Does anything stay the same?

While the capacitor is still connected to the power supply, the potential difference \( \Delta V \) cannot change.

Moving the plates further apart decreases the capacitance, \( C \) because:

\[
C = \frac{\varepsilon_0 A}{d}
\]

To see what happens to the charge, use \( Q = C \Delta V \), which shows that decreasing \( C \) decreases the charge, \( Q \) stored on the capacitor.
When the power supply is disconnected, the charge is stranded on the capacitor plates. So, the charge cannot change.

Moving the plates further apart decreases the capacitance, because:

\[ C = \frac{\varepsilon_0 A}{d} \]

To see what happens to the potential difference, look at \( Q = C \Delta V \), which shows that decreasing \( C \) while keeping the charge the same, the potential difference increases.

Change in capacitance – effect of plate area

Capacitance is proportional to area, so increasing area increases capacitance.

\[ C = \frac{\varepsilon_0 A}{d} \]

Change in capacitance – effect of varying the charge

Recall: \( C = \frac{\varepsilon_0 A}{d} \)

Increasing \( Q \) does not change the capacitance at all. The capacitance is a constant because it is determined by how the capacitor looks like. With this, \( Q = C \Delta V \) tells us that the potential difference across the capacitor doubles when the charge on each plate doubles.

Change in capacitance – effect of varying the charge

Double the charge on each plate. The capacitance ...

1. Increases
2. Decreases
3. Stays the same

Energy in a capacitor

When we move a single charge \( q \) through a potential difference \( \Delta V \), its potential energy changes by \( q \Delta V \).

Charging a capacitor involves moving a large number of charges from one capacitor plate to another. If \( \Delta V \) is the final potential difference on the capacitor, and \( Q \) is the magnitude of the final charge on each plate, the energy stored in the capacitor is:

\[ U = \frac{1}{2} Q \Delta V \]

The factor of 1/2 is because, on average, the charges were moved through a potential difference of \( \frac{1}{2} \Delta V \). (Note that the potential difference changed from zero to \( \Delta V \) in the process. So the average value is \( \frac{1}{2} \Delta V \).

Using \( Q = C \Delta V \), the energy stored in a capacitor can be written as:

\[ U = \frac{1}{2} Q \Delta V = \frac{1}{2} C (\Delta V)^2 = \frac{Q^2}{2C} \]
Example: Energy stored in a capacitor

How much energy is stored in a 8 μF capacitor that has a potential difference of 4000 V?

Solution:

\[ U = \frac{1}{2} C (\Delta V)^2 = \frac{1}{2} (8 \times 10^{-6} \, \text{F})(4000 \, \text{V})^2 \]

The factor of $10^{-6}$ in the capacitance cancels the factor of $1000^2$, so we get:

\[ U = \frac{1}{2} (8 \, \text{F})(4 \, \text{V})^2 = 64 \, \text{J} \]

Dielectrics

When a material (generally an insulator) is inserted into a capacitor, we call the material a dielectric. Adding a dielectric allows the capacitor to store more charge for a given potential difference.

Dielectrics (cont’d)

Thus, when a dielectric is inserted in a charged capacitor (not connected to a power supply), the electric field would be decreased and so would the voltage ($\approx ED_0$). Since $C = Q/V$, this means that $C$ must be bigger when a dielectric is inserted.

For a parallel-plate capacitor containing a dielectric, the capacitance is:

\[ C = \frac{\kappa \varepsilon_0 A}{d} \]

where $\kappa$ is the dielectric constant.

In general, adding a dielectric to a capacitor increases the capacitance by a factor of $\kappa$.

The dielectric constant of a conductor

\[ \kappa = \frac{E_0}{E_{\text{ref}}} \]

What is the dielectric constant of a conductor?

1. Zero
2. Infinity
3. This question makes no sense – a dielectric is an insulator, so a conductor does not have a dielectric constant.

The dielectric constant

Every material has a dielectric constant $\kappa$, which as discussed above tells you how effective the dielectric is at increasing the amount of charge a capacitor can store for given voltage difference applied across it. Alternatively, we can think of the dielectric constant as telling you how effectively the dielectric is in reducing the electric field of a charged capacitor not connected to a power supply.

\[ \kappa = \frac{E_0}{E_{\text{ref}}} > 1 \]

$E_0$ is the field without the dielectric.
$E_{\text{ref}}$ is the field with the dielectric.

Dielectrics

When a dielectric is inserted into a charged capacitor, the dielectric is polarized by the field. The electric field from the dielectric will partially cancel the electric field from the charge on the capacitor plates. If the capacitor is connected to a battery at the time, the battery is able to store more charge in the capacitor, bringing the field back to its original value.