Temperature and Heat

Temperature & Internal Energy

Temperature is a measure of the average internal energy of an object or a system. Internal energy is the energy associated with the motion of atoms and/or molecules. Temperature is thus a measure of the average kinetic energy of the atoms and molecules making up an object or a system.

Temperature scales

A change by 1°C is the same as a change by 1K. The Celsius and Kelvin scales are just offset by about 273.

A change by 1°C is the same as a change by 1.8°F. To convert between Celsius and Fahrenheit we use:

\[ T_F = \left(\frac{5}{9}\right) (T_C - 32) \]

Equations involving temperature

If the equation involves \( T \), use an absolute temperature (we generally use a Kelvin temperature).

If the equation involves \( \Delta T \), we can use Celsius or Kelvin.

Thermometer

A device used to measure temperature is called a thermometer. All thermometers exploit the physical properties of a material that depend on temperature. Examples of temperature-dependent properties include:

- the pressure in a sealed container of gas
- the volume occupied by a liquid
- the electrical resistance of a conductor

All these effects, and plenty of others, can be used in designing thermometers.

Thermal expansion

Linear expansion

Most materials expand when heated because the average distance between the atoms increases. As long as the temperature change isn't too large, each dimension, \( L \), of an object experiences a change in length, \( \Delta L \), that is proportional to the change in temperature, \( \Delta T \).

\[ \Delta L = L_0 \alpha \Delta T \]

or, equivalently,

\[ L = L_0 (1 + \alpha \Delta T) \]

where \( L_0 \) is the original length, and \( \alpha \) is the coefficient of linear expansion, which depends on the material.

<table>
<thead>
<tr>
<th>Material</th>
<th>( \alpha \times 10^{-6}/\degree C )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>23</td>
</tr>
<tr>
<td>Glass</td>
<td>8.5</td>
</tr>
<tr>
<td>Copper</td>
<td>17</td>
</tr>
<tr>
<td>Iron</td>
<td>12</td>
</tr>
</tbody>
</table>
Thermal expansion

Volume expansion
For small temperature changes, we can find the new volume using:

\[ \Delta V = V_0 (3\alpha \Delta T) \]

or, equivalently,

\[ V = V_0 (1 + 3\alpha \Delta T) \]

where \( V_0 \) is the original volume.

An application making use of thermal expansion of materials -- Bimetallic strip

A bimetallic strip is made from two different metals that are bonded together side-by-side. The strip is straight at room temperature, but it curves when it is heated. This is because the metals have equal lengths at room temperature but different thermal expansion coefficients, so they have different lengths when heated.

A common application of a bimetallic strip is found in switches controlled by a thermostat. When the room is too cold, the strip completes a circuit and turns on the furnace. The furnace goes off when the room (and the strip) warms up.

Thermal Stress

If an object is heated or cooled and it is not free to expand or contract, the thermal stresses can be large enough to cause damage. This is why bridges have expansion joints (check this out where the BU bridge meets Comm. Ave.). Even sidewalks are built accounting for thermal expansion.

Materials that are subjected to thermal stress can age prematurely. For instance, over the life of an airplane the metal is subjected to thousands of hot/cold cycles that weaken the airplane's structure.

Another common example occurs with water, which expands by 10% when it freezes. If the water is in a container when it freezes, the ice can exert a lot of pressure on the container.

Specific heat

The specific heat of a material is the amount of heat required to raise the temperature of 1 kg of the material by 1°C.

The symbol for specific heat is \( c \).

Heat lost or gained by an object is given by:

\[ Q = mc\Delta T \]

<table>
<thead>
<tr>
<th>Material</th>
<th>( c ) (J/(kg °C))</th>
<th>Material</th>
<th>( c ) (J/(kg °C))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>900</td>
<td>Water (gas)</td>
<td>1850</td>
</tr>
<tr>
<td>Copper</td>
<td>385</td>
<td>Water (liquid)</td>
<td>4186</td>
</tr>
<tr>
<td>Gold</td>
<td>128</td>
<td>Water (ice)</td>
<td>2060</td>
</tr>
</tbody>
</table>

Heat

Heat is the energy transferred between a system and its surrounding because of a temperature difference between them.

A change of state

Changes of state occur at particular temperatures (involving no temperature changes), so the heat associated with the process is given by:

Freezing or melting: \( Q = mL_f \)

where \( L_f \) is the latent heat of fusion

Boiling or condensing: \( Q = mL_v \)

where \( L_v \) is the latent heat of vaporization

For water the values are:

\[ L_f = 333 \text{ kJ/kg} \]
\[ L_v = 2256 \text{ kJ/kg} \]

(c.f. \( c = 4.186 \text{ kJ/(kg °C)} \))
Example 1: Heat Exchange Between Ice & Water

Question: 100 grams of ice, with a temperature of -10°C, is added to a styrofoam cup of water. The water is initially at +10°C, and has an unknown mass \( m \). If the final temperature of the mixture is 0°C, what is the possible range of \( m \)?

Assume that no heat is exchanged with the cup or with the surroundings.

Use the following approximate values in determining your answer:
Specific heat of liquid water: 4000 J/(kg \( °C \))
Specific heat of ice: 2000 J/(kg \( °C \))
Latent heat of fusion of water: 3 \( \times \) 10^5 J/kg

Example 1: Heat Exchange Between Ice & Water

Solution:

First, all the ice at -10°C must be warmed up to 0°C and all the water at 10°C must be cooled to 0°C. At the same time, there can be change of state occurring to either the water or the ice. Three extreme cases are possible:

1. All the water is transformed into ice at 0°C.
2. There is no change of state to the water nor the 100 grams of ice.
3. All the 100 grams of ice is transformed into water at 0°C.

In the following, we will find the value of \( m \) corresponding to each case.

Case (1): All the mass \( m \) of water transforms into ice at 0°C.

Since the water and ice are mixed inside a styrofoam cup, which is a good thermal insulator, there is no heat exchange. So: \[ \sum Q = 0 \]

\[ m_{\text{ice}}c_{\text{ice}}\Delta T_{\text{ice}} + m_{\text{water}}c_{\text{water}}\Delta T_{\text{water}} = 0 \]

Plugging in numbers gives:

\[ 100 \text{ g} \times 2 = 34m, \text{ so } m = 6 \text{ g}. \]

Case (2): No change in the state of the water nor the ice.

\[ \sum Q = 0 \]

\[ m_{\text{ice}}c_{\text{ice}}\Delta T_{\text{ice}} + m_{\text{water}}c_{\text{water}}\Delta T_{\text{water}} = 0 \]

Plugging in numbers gives:

\[ 100 \text{ g} = 2m, \text{ so } m = 50 \text{ g}. \]

Case (3): All the 100 grams of ice transforms into water.

\[ \sum Q = 0 \]

\[ m_{\text{ice}}c_{\text{ice}}\Delta T_{\text{ice}} + m_{\text{water}}c_{\text{water}}\Delta T_{\text{water}} = 0 \]

Plugging in numbers gives:

\[ 100 \text{ g} = 32 = 4m, \text{ so } m = 800 \text{ g}. \]

From the above results,

1. If 6 g < \( m \) < 50 g, some but not all of the water transforms into ice at 0°C.
2. If 50 g < \( m \) < 800 g, some but not all the 100 g of ice transforms into water.

Amazingly, we can add anywhere from 6 g to 800 g of water at +10°C to the 100 g of ice at -10°C and get a mixture with an equilibrium temperature of 0°C. This is because the equilibrium temperature we were trying to achieve is the temperature at which liquid and solid water can co-exist, and we get such a wide range of possible masses because of the large amount of energy associated with a phase change.