Problem 1:
A wooden ball with mass 250 g swings back and forth on a string, pendulum style, reaching a maximum speed of 3.00 m/s when it passes through its equilibrium position. Use g = 10 m/s².

(a) At what maximum height above the equilibrium position was the ball released from rest?

Energy is conserved, so kinetic energy is being converted entirely into potential energy and back. The maximum height is reached when the ball stops, at the peak of its motion, i.e. all the mechanical energy is converted to potential energy, and there is no kinetic energy. For simplicity, we can say that the ball’s potential energy is zero at the equilibrium position. Our energy conservation thus is written as:

\[ K_0 = U_f \]

so

\[ \frac{1}{2} m v_0^2 = m g h_f \]

which leads to a maximum height of:

\[ h_f = \frac{v_0^2}{2g} = \frac{(3.0 \text{ m/s})^2}{2 \cdot 10 \text{ m/s}^2} = 0.45 \text{ m} \]

(b) At one instant, when the ball is at its equilibrium position and moving left at 3.00 m/s, it is struck by a bullet with a mass of 12.0 g. This is shown in the figure. Before the collision, the bullet has a velocity of 312 m/s to the right. The bullet passes through the ball and emerges with a velocity of 100 m/s to the right. What is the magnitude and direction of the ball’s velocity immediately after the collision? To indicate the direction, use a positive sign if the ball’s velocity is to the right, and a negative sign if the ball’s velocity is to the left. Neglect any change in mass for the ball.

First, it is useful to draw a before/after picture of the collision:

Momentum is conserved here, so the initial momentum of the the bullet and the pendulum summed together should be the same as the momenta of the bullet and pendulum summed together directly after the collision.

Here, we have:

\[ m_b v_b - m_p v_p = m_b v_{bf} + m_p v_{pf} \]

where \( m_b \) and \( m_p \) are the masses of the bullet and pendulum, \( v_b \) and \( v_p \) are the initial velocities of the bullet and pendulum, and \( v_{bf} \) and \( v_{pf} \) are the final velocities of the bullet.
and pendulum, respectively. (Note that since the problem says that a positive velocity is to the right, we are choosing the pendulum’s final velocity to be to the right. If we get a negative number for its velocity, we know it is directed to the left.)

Then we just need to solve for $v_{pf}$:

$$v_{pf} = \frac{m_b}{m_p} v_b - v_p - \frac{m_b}{m_p} v_{bf}$$

$$= \frac{12 \text{ g}}{250 \text{ g}} \cdot 312 \text{ m/s} - 3.0 \text{ m/s} - \frac{12 \text{ g}}{250} \cdot 100 \text{ m/s}$$

$$= 7.18 \text{ m/s}$$

Problem 2:

A block of mass $m$ is released from rest, at a height $h = 5R$ above the base of a frictionless loop-the-loop track, as shown in the figure. The loop has a radius $R = 30.0 \text{ cm}$. Use $g = 10.0 \text{ m/s}^2$, to make the calculations easier.

(a) What is the speed of the block when it reaches position $b$, shown in the figure?

We can use conservation of energy here as there are no non-conservative forces (e.g. friction) along the track. This is, therefore, simply conversion between potential and kinetic energy. To make our lives easier, we can set the potential energy at position $b$ to be zero. Since we are releasing the object at rest, the kinetic energy at the beginning is also zero. Our energy conservation equation between the start and point $b$ will thus be:

$$U_0 = K_b$$

or

$$mg(h - 2R) = \frac{1}{2}mv^2$$

Note that since we set the potential energy to be zero at $b$, a point that is $2R$ above the ground, the potential energy at the initial point is related to its height subtracted by the zero-point height, or $h - 2R$. Now we can solve for $v$:

$$v = \sqrt{2g(h - 2R)} = \sqrt{2 \cdot 10 \text{ m/s}^2 (3 \cdot .3 \text{ m})}$$

$$= 4.24 \text{ m/s}$$

(b) If the block has a mass of 100 g, calculate the magnitude of the normal force acting on the block when the block is at point $b$.

The block is experiencing circular motion, so we can draw the force diagram of the block and write Newton’s 2nd Law, keeping in mind that any forces that go towards or away from the center of the circle lead to centripetal acceleration which is related to the velocity of the object. Here we only have forces
in that direction, normal force and gravitational force, so we can write:

\[ N + mg = ma = \frac{mv^2}{R} \]

Solving for \( N \) gives us:

\[ N = \frac{mv^2}{R} - mg = \frac{1 \text{ kg} \cdot (4.24 \text{ m/s})^2}{0.3 \text{ m}} - 0.1 \text{ kg} \cdot 10 \text{ m/s}^2 = 5.99 \text{ m/s} \]

**Problem 3:**

A wooden block with a mass of 190 g rests on two supports. A piece of sticky chewing gum with a mass of 60.0 g is fired straight up at the block, colliding with the block when the gum’s speed is 10.0 m/s. The gum sticks to the block. Use \( g = 10.0 \text{ m/s}^2 \).

(a) What is the speed of the gum-block system immediately after the collision?

We can draw a picture again of the pre- and post-collision, and then use conservation of momentum to find the speed of the entire system:

So if \( m_g \) is the mass of the gum, \( m_b \) is the mass of the block, \( v_g \) is the initial speed of the gum, and \( v \) is the speed of the gum-block system right after collision, our conservation of momentum equation looks like this:

\[ m_g v_g = (m_g + m_b) v \]

Solving for \( v \) gives us:

\[ v = \frac{m_g v_g}{m_g + m_b} = \frac{60 \text{ g} \cdot 10.0 \text{ m/s}}{250 \text{ g}} = 2.4 \text{ m/s} \]

(b) What is the maximum height reached by the gum-block system, measured from the point of the collision?

We can use energy conservation to find this, plus the knowledge that the gum+block system will reach maximum height when its velocity becomes zero. Thus, its initial kinetic energy is converted entirely to potential energy (here we’re setting \( U = 0 \) at the point where the collision occurs).

\[ \frac{1}{2} (m_g + m_b) v^2 = (m_g + m_b) gh \]

Solving for \( h \) gives us:

\[ h = \frac{v^2}{2g} = \frac{(2.4 \text{ m/s})^2}{2 \cdot 10 \text{ m/s}^2} = 0.288 \text{ m} \]
Problem 4:
An electric drill accelerates a drill bit, which has a radius of 3.00 mm, from rest to a maximum angular speed of 250 rpm (revolutions per minute) in 2.20 seconds. Assuming that the drill bit’s angular acceleration is constant, what is the magnitude of that angular acceleration?

The angular acceleration is defined as:

$$\alpha = \frac{\Delta \omega}{t}$$

So to find \(\alpha\) we need to find the change in angular velocity in our given time interval. The problem gives us units of revolutions per minute, which is a frequency. To convert between frequency and angular velocity, we remember that one revolution is \(2\pi\) radians, so:

$$\omega = 2\pi f$$

Thus, our initial angular velocity is \(\omega_0 = 0\) rad/s, while our final angular velocity is:

$$\omega_f = \frac{2\pi}{1 \text{ revolution}} \cdot \frac{250 \text{ revolution}}{1 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ s}} = \frac{25\pi}{3} \text{ rad/s}$$

And our angular acceleration is:

$$\alpha = \frac{\omega_f - \omega_0}{t} = \frac{\frac{25\pi}{3} \text{ rad/s}}{2.2 \text{ s}} = 11.9 \text{ rad/s}^2$$

Problem 5:
You release a ball from rest at the top of a ramp, and it experiences a constant angular acceleration of 1.40 rad/s^2. At the bottom of the ramp, the ball is rotating at 5.00 revolutions per second. How long did it take the ball to travel down the ramp?

We can use our constant angular acceleration equations. Here we are given \(\alpha\), \(\omega_0\), and \(\omega_f\), and we are looking for \(t\), so we can use:

$$\omega_f = \omega_0 + \alpha t$$

We have to be careful and remember that revolutions per second is a frequency, so to convert to radians per second, we must multiply it by \(2\pi\).

Solving for \(t\) and plugging in our numbers yields:

$$t = \frac{\omega_f - \omega_0}{\alpha} = \frac{2\pi \cdot 5.0 \text{ rad/s} - 0}{1.40 \text{ rad/s}^2} = 22.4 \text{ s}$$

Problem 6:
A hollow sphere has a mass of \(M = 6.00\) kg and a radius of \(R = 50.0\) cm.
(a) Determine the sphere’s rotational inertia about an axis passing through the center of the sphere.

From a chart we can see that the rotational inertia of a hollow sphere about an axis passing through its center is:

$$I_0 = \frac{2}{3}mr^2$$

where \(m\) is the mass of the sphere and \(r\) is its radius. We can just plug-and-chug to get our answer:
\[ I_0 = \frac{2}{3} \cdot 6.0 \text{ kg} \cdot (0.5 \text{ m})^2 = 1 \text{ kg} \cdot \text{m}^2 \]

(b) Determine the sphere's rotational inertia about an axis that is tangent to the outer surface of the sphere.

Here we can use the parallel axis theorem, keeping in mind that the new axis is a distance \( r \) away from the old one. In the figure to the left, the gray axis is the original one, and the black one is the new tangential one.

The parallel axis theorem tells us:

\[ I = I_0 + mx^2 \]

where \( x \) is the distance between the new axis and the one that gave \( I_0 \). Here \( x = r \), and \( I_0 = \frac{2}{3}mr^2 \), so:

\[ I = \frac{2}{3}mr^2 + mr^2 = \frac{5}{3}mr^2 \]

\[ = \frac{5}{3} \cdot 6.0 \text{ kg} \cdot (0.5 \text{ m})^2 \]

\[ = 2.5 \text{ kg} \cdot \text{m}^2 \]

Problem 7:

In the “old days”, long before CD’s and MP3’s, people listened to music using vinyl records. Long-playing vinyl records spin at a constant rate of 33-and-one-third rpm (revolutions per minute). The music is encoded onto a continuous spiral track on the record that starts at a radius of 30 cm from the center and ends at a radius of 10 cm from the center. If a record plays for 24 minutes, how far apart are the grooves?

We are given a frequency and a time interval, so we can figure out the number of revolutions the record undergoes in the time interval given. We also know that the record goes through a certain “thickness” of the record in that same time interval. The number of grooves in that “thickness” is simply the number of circles it goes through, so the distance between grooves is:

\[ \frac{\Delta r}{n} = \frac{\Delta r}{\text{number of revolutions}} = \frac{\Delta r}{f \cdot \Delta t} \]

\[ = \frac{30 \text{ cm} - 10 \text{ cm}}{33.3 \text{ rpm} \cdot 24 \text{ min}} \]

\[ = 0.025 \text{ cm} \]

Problem 8:

You construct a mobile out of four objects, a sphere, a pyramid, an ellipsoid, and a cube. The mobile is in equilibrium in the configuration shown, where the vertical dashed lines are 20 cm apart. The mass of the strings (in blue) and rods (in red) can be neglected. If the pyramid has a mass of 800 g, what is the mass of each of the other objects? See if you can do this problem without a calculator.
(a) **What is the mass of the ellipsoid?**

Since this system is in equilibrium, we know two important things: the sum of the torques is zero, and the sum of the forces on the system is zero.

Since the sum of the torques is zero, we can relate the mass of the ellipse, \( m_e \) with the mass of the square, \( m_c \) as follows:

\[
m_e g \cdot 3r - m_c g \cdot r = 0
\]

This tells us that \( m_c = 3m_e \)

As the ellipse is \( 3r \) away from the turning point and the square is a distance \( r \) away.

Since the sum of the forces is zero, we know that the tension force of the string holding up the ellipse/square system is:

\[
T_{es} = m_e g + m_c g
\]

We can then relate \( m_e \) and \( m_c \) to the mass of the pyramid, \( m_p \) by again using the fact that the sum of the torques is zero:

\[
m_p g \cdot 3r - (m_e + m_c)g \cdot 3r
\]

This tells us that \( m_p = m_e + m_c \). Since we found above that \( m_c = 3m_e \), we can find the mass of the ellipse:

\[
m_p = m_e + 3m_e = 4m_e
\]

So:

\[
m_e = \frac{m_p}{4} = \frac{800 \text{ g}}{4} = 200 \text{ g}
\]

(b) **What is the mass of the cube?**

As we found above,

\[
m_c = 3m_e = 3 \cdot 200 \text{ g} = 600 \text{ g}
\]

(c) **What is the mass of the sphere?**

Again, we use the fact that the sum of the forces is zero to find the force of tension on the string that pulls against the same bar as the sphere:

\[
T = (m_p + m_e + m_c)g = (4m_e + m_e + 3m_e)g = 8m_e g
\]

Now we use the sum of torques is zero, and say \( m_s \) is the mass of the sphere:

\[
m_s g \cdot 2r - 8m_e g \cdot 3r = 0
\]

Solving for \( m_s \) gives us:

\[
m_s = 12m_e = 12 \cdot 200 \text{ g} = 2400 \text{ g}
\]
Problem 9:
You are using a wheelbarrow to move a heavy rock, as shown in the figure below. The diagram shows the location of the upward force you exert \( (r_F = 1.2 \text{ m}) \) and the location of the force of gravity acting on the rock-wheelbarrow system. \( (r_g = 0.2 \text{ m}) \)

\[
\text{(a) If the force of gravity on the rock-wheelbarrow system has a magnitude of 360 N, solve for the magnitude of the force, } F, \text{ that you exert on the wheelbarrow.}
\]

Since the system has no angular acceleration, we know that the sum of the torques is zero. Thus:

\[
F \cdot r_F - mg \cdot r_g = 0
\]

Solving this for \( F \) yields:

\[
F = \frac{mg \cdot r_g}{r_F} = \frac{360 \text{ N} \cdot 0.2 \text{ m}}{1.2 \text{ m}} = 60 \text{ N}
\]

\[
\text{(b) What is the magnitude of the normal force exerted on the wheel of the wheelbarrow by the ground? The wheel is the only part of the wheelbarrow in contact with the ground in this situation.}
\]

Since the wheelbarrow has no acceleration whatsoever (i.e. it is in equilibrium), the sum of the forces is zero. So we can write:

\[
F_N + F - mg = 0
\]

Solving for \( F_N \) yields:

\[
F_N = mg - F = 360 \text{ N} - 60 \text{ N} = 300 \text{ N}
\]

Problem 11:
It is often useful to treat the lower arm as a uniform rod of length \( L \) and mass \( M = 1.30 \text{ kg} \) that can rotate about the elbow. Let’s say you are holding your arm so your upper arm is vertical (with your elbow below your shoulder), with a 90-degree bend at the elbow so the lower arm is horizontal, as shown above. In this position, we can say that three forces act on your lower arm: the force of gravity \( (Mg) \), the force exerted by the biceps, and the force exerted at the elbow joint by the humerus (the bone in the upper arm). Let’s say the biceps muscle is attached to the lower arm at a distance of \( L/10 \) from the elbow, moving away from the elbow toward the hand. Use \( g = 10 \text{ m/s}^2 \).

\[
\text{(a) Assuming that the force the biceps muscle exerts on the lower arm is vertical, what is the magnitude of the biceps force?}
\]
Because the system is in equilibrium, we know that the sum of the torques on the system is zero. We also know that the force $Mg$ will be exerted at $L/2$, as the center of mass is right in the middle of the uniform arm.

If we call the force the biceps muscle exerts on the arm $F_b$, that tells us:

$$\Sigma \tau = F_b \cdot \frac{L}{10} - Mg \cdot \frac{L}{2} = 0$$

We can now solve for $F_b$:

$$F_b = \frac{Mg \cdot 0.5L}{0.1L} = \frac{1.3 \text{ kg} \cdot 10 \text{ m/s}^2 \cdot 0.5}{0.1} = 65 \text{ N}$$

(b) Determine the force that the humerus exerts on the lower arm. If this force is directed up, use a positive sign. If this force is directed down, use a negative sign.

Here we use the fact that the sum of the forces is zero as the system is in equilibrium. We will assume that the humerus force is upwards, and if we get a negative number, we can tell it is downwards. Let’s write Newton’s 2nd Law:

$$\Sigma F = F_h + F_b - Mg = 0$$

From this we can solve for $F_h$:

$$F_h = Mg - F_b = 1.3 \text{ kg} \cdot 10 \text{ m/s}^2 - 65 \text{ N} = -52 \text{ N}$$