New Scaling in an Old Earthquake Model

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$N_{GR} (M_0 > m) = A m^{-b}$
Empirical Scaling

• Gutenberg & Richter observe the now-famous relation [Gutenberg and Richter, Ann. Geophys. 9, 1 (1956)]:

\[ N_{GR} (M_0 > m) = A m^{-b} \]

where \( M_0 \) is the magnitude of the event and the exponent is referred to as the “b-value”.

• Scaling occurs over fault systems, not single faults and \( b \) is determined from a cumulative distribution.
Empirical Scaling


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• Scaling occurs over *fault systems*, not single faults and \( b \) is determined from a *cumulative* distribution.

• Typical b-values lie in \( 0.8 \leq b \leq 1.2 \) but can vary outside this region [Frohlich and Davis, *J. Geophys. Res.* 98, 631-644 (1993)].
OFC Model

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- These dynamics yield “cluster scaling”

\[ N(s) \propto s^{-\tau} \] with \( \tau = 3/2 \) where \( b = \tau - 1 \).
Introduce Damage

- We can “freeze in” a certain fraction of damaged sites, $1 - \phi$, that cannot receive stress from their neighbors.
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Reduced Scaling Regime

\[ N(s) = L \]

\( \phi = 1 \)

\( \phi = 0.9 \)

\( \phi = 0.8 \)

\( \phi = 0.5 \)
\( \phi = 0.1 \)

\( \phi = 0.2 \)

\( \phi = 0.3 \)

\( \phi = 0.4 \)

\( \phi = 0.5 \)

\( \phi = 0.6 \)

\( \phi = 0.7 \)

\( \phi = 0.8 \)

\( \phi = 0.9 \)

\( \phi = 1 \)

\( N(s) \)

\( s \)
\[ \phi = 0.1 \]

\[ \phi = 0.2 \]

\[ \phi = 0.3 \]

\[ \phi = 0.4 \]

\[ \phi = 0.5 \]

\[ \phi = 0.6 \]

\[ \phi = 0.7 \]

\[ N(s) \]

\[ s \]
“New” Scaling

![Graph](image)

- $N(s)$ vs. $s$
- Axes: $10^0$ to $10^7$ for $N(s)$ and $10^0$ to $10^4$ for $s$
"New" Scaling

\[ \tau = 2.073 \pm 0.001 \]

\[ b = 1.1 \]

\[ \chi^2/d.o.f = 5.3 \]
Summary

- Seismologists have observed that the frequency of earthquakes scales as a power-law with respect to the size or magnitude of the event.

- Empirical evidence suggests that most exponents, $b$, lie in a small window about $b = 1$.

- The OFC model gives rise to scaling with $b = 0.5$.

- The modified OFC model with damage produces new scaling with exponent $b = 1$. 
Thank You

Further information:

C. A. Serino et. al. arXiv:0905.3860 [cond-mat.stat-mech]