1. Derive the expression for the electric dipole field (Jackson 9.18)

\[
E = \frac{1}{4\pi\varepsilon_0} \left\{ k^2 \left( \hat{\mathbf{n}} \times \mathbf{p} \right) \times \hat{\mathbf{n}} \frac{e^{ikr}}{r} + \left[ 3\hat{\mathbf{n}} (\hat{\mathbf{n}} \cdot \mathbf{p}) - \mathbf{p} \right] \left( \frac{e^{ikr}}{r^3} - \frac{i k e^{ikr}}{r^2} \right) \right\}
\]

by direct calculation of

\[
E = \frac{i}{k} \sqrt{\frac{\mu_0}{\varepsilon_0}} \nabla \times \left[ \frac{ck^2}{4\pi} \hat{\mathbf{n}} \times \mathbf{p} \frac{e^{ikr}}{r} \left( 1 - \frac{1}{ikr} \right) \right].
\]

2. Jackson 9.5
(a) Show that for harmonic time variation at frequency \( \omega \) the electric dipole scalar potential is

\[
\Phi = \frac{e^{ikr}}{4\pi\varepsilon_0 r^2} \hat{\mathbf{n}} \cdot \mathbf{p} (1 - ikr)
\]

where \( k = \omega/c \), \( \hat{\mathbf{n}} \) is a unit vector in the radial direction, \( \mathbf{p} \) is the dipole moment, and the time-dependence \( \exp(-i\omega t) \) is understood. (The vector potential \( \mathbf{A} \) is given by Jackson 9.16, you do not need to derive this.)

\[
\mathbf{A} \left( \mathbf{r} \right) = -i\omega \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} \mathbf{p}
\]

(b) Calculate the electric field \( \text{from the potentials} \) and show that it is given by Jackson 9.18,

\[
E = \frac{1}{4\pi\varepsilon_0} \left\{ k^2 \left( \hat{\mathbf{n}} \times \mathbf{p} \right) \times \hat{\mathbf{n}} \frac{e^{ikr}}{r} + \left[ 3\hat{\mathbf{n}} (\hat{\mathbf{n}} \cdot \mathbf{p}) - \mathbf{p} \right] \left( \frac{e^{ikr}}{r^3} - \frac{i k e^{ikr}}{r^2} \right) \right\}
\]

3. Jackson 9.3
Two halves of a spherical metallic shell of radius \( R \) and infinite conductivity are separated by a very small insulating gap. An alternating potential is applied between the two halves of the sphere so that the potentials are

\[
\Phi = \pm V \cos(\omega t).
\]

In the long-wavelength limit, find the radiation fields, the angular distribution of radiated power, and the total radiated power from the sphere.