1. (Jackson 6.11)
A transverse plane wave is incident normally in vacuum on a perfectly absorbing flat screen.
(a) From the law of conservation of linear momentum, show that the pressure (called radiation pressure) exerted on the screen is equal to the field energy per unit volume in the wave.

Let our wave travel in the $x$ direction, with $E$ in the $y$ direction and $B$ in the $z$ direction

$$E = E_2 \hat{y}$$
$$B = B_3 \hat{z}$$

From Jackson 6.121 we have

$$\frac{d\left(P_{\text{mech}}\right)_{\alpha}}{dt} = \sum_{\beta \nu} \int d^3r \partial_{\beta} T_{\alpha\beta} - \frac{d\left(P_{\text{field}}\right)_{\alpha}}{dt}$$

$$= \sum_{\beta \nu} \int d^3r \partial_{\beta} T_{\alpha\beta} - \varepsilon_0 \frac{d}{dt} \int d^3r (E \times B)_{\alpha}$$

The left-hand-side is the force on the screen and we want the pressure so we will take the force per volume and then integrate in the $x$ direction

$$\sum_{\alpha} \frac{d}{dt} \frac{d\left(P_{\text{mech}}\right)_{\alpha}}{dV} = \sum_{\alpha, \beta} \partial_{\beta} T_{\alpha\beta} - \varepsilon_0 \sum_{\alpha} \frac{d\left(E \times B\right)_{\alpha}}{dt}$$

Now evaluate the stress tensor:

$$T_{\alpha\beta} = \varepsilon_0 \left[ E_{\alpha} E_{\beta} + c^2 B_{\alpha} B_{\beta} - \frac{1}{2} \left(E^2 + c^2 B^2\right) \delta_{\alpha\beta}\right]$$

$T$ has only diagonal components

$$T_{11} = -\frac{\varepsilon_0}{2} \left[ E_2^2 + c^2 B_3^2 \right]$$

$$T_{22} = \frac{\varepsilon_0}{2} \left[ E_2^2 - c^2 B_3^2 \right]$$

$$T_{33} = \frac{\varepsilon_0}{2} \left[ -E_2^2 + c^2 B_3^2 \right]$$
The force is only in the $x$ direction because $E$ and $B$ do not depend on $y$ or $z$ and the cross product (Poynting vector) is also in the $x$ direction.

\[
\frac{d(P_{\text{mech}})}{dt} = \int_v \partial_1 T_{11} - \varepsilon_0 \frac{d}{dt} \left[ \int_v (E \times B) \right]
\]

\[
= -\int_v d^3 r \left[ \partial_1 \left( \frac{\varepsilon_0}{2} \left( E_2^2 + c^2 B_3^2 \right) \right) + \varepsilon_0 d(E_2 B_3) \right]
\]

The pressure is

\[
P = \int_0 dx \frac{d(P_{\text{mech}})}{dt} = -\frac{\varepsilon_0}{2} \left[ E_2^2 + c^2 B_3^2 \right] + \varepsilon_0 \int_0^\infty dx \frac{d(E_2 B_3)}{dt}
\]

Now we must take the time average. The last term contributes zero because its time average is zero.

\[
\bar{P} = \frac{\varepsilon_0}{2} \left[ \bar{E}_2^2 + c^2 \bar{B}_3^2 \right] = \frac{\varepsilon_0}{2} \bar{E}_2^2 + \frac{1}{2 \mu_0} \bar{B}_3^2 = \bar{u}
\]

(b) In the neighborhood of the earth the flux of electromagnetic energy from the sun is approximately 1.4 kW/m$^2$. If an interplanetary "sailplane" had a sail of mass 1 g/m$^2$ of area and negligible other weight, what would be its maximum acceleration in meters per second squared due to the solar radiation pressure? How does this answer compare with the acceleration due to the solar "wind" (corpuscular radiation)?

\[
\bar{P} = \bar{u} = \frac{S}{c} = \frac{1.4 \times 10^3 \text{ W/m}^2}{3 \times 10^8 \text{ m/s}} \approx 5 \times 10^{-6} \text{ N/m}^2
\]

\[
a = \frac{\bar{P} A}{m} = \frac{5 \times 10^{-6} \text{ N/m}^2}{10^{-3} \text{ kg/m}^2} \approx 5 \times 10^{-3} \text{ m/s}^2
\]

Compare to solar wind:
At [http://sec.noaa.gov/ace/MAG_SWEPAM_7d.html](http://sec.noaa.gov/ace/MAG_SWEPAM_7d.html)
I found the following plot of the solar wind for the last 7 days beginning 9/12/01:
From this plot we see that the solar wind (in the vicinity of the earth) is made of protons traveling at
\[ v \sim (4 - 6) \times 10^5 \text{ m/s} \]
with a density of
\[ \rho \sim (10^6 - 10^7) \text{ m}^{-3} \]

Thus, the solar wind flux is
\[ N \approx \left(5 \times 10^5 \text{ m/s}\right)\left(3 \times 10^7 \text{ m}^{-3}\right) \approx 2 \times 10^{12} \text{ m}^{-2}\text{s}^{-1} \]

The corresponding acceleration on our sail plane is
\[ a \approx \frac{(2 \times 10^{12} \text{ m}^{-2}\text{s}^{-1}) m_p v}{10^{-3} \text{ kg/m}^2} = \frac{(2 \times 10^{12} \text{ m}^{-2}\text{s}^{-1})(1.6 \times 10^{-27} \text{ kg})(7 \times 10^5 \text{ m/s})}{10^{-3} \text{ kg/m}^2} \approx 2 \times 10^{-6} \text{ m/s}^2 \]

significantly smaller than the acceleration due to radiation.
2. (Jackson 6.16)
(a) Calculate the force in newtons acting on a Dirac monopole of the minimum magnetic charge located a distance 0.05 nm from and in the median plane of a magnetic dipole with dipole moment equal to one nuclear magneton \((\epsilon h/2m_p)\).

The force on monopole in dipole field

\[ F = gH = \frac{gB}{\mu_0} \]

Dipole field,

\[ B = \frac{m\mu_0}{2\pi r^3} \]

where

\[ g = \frac{2\pi\hbar}{\epsilon n} \]
\[ m = \frac{\epsilon h}{2m_p} \]

This gives

\[ F = \left( \frac{2\pi\hbar}{\epsilon n} \right) \left( \frac{\epsilon h}{2m_p} \right) \left( \frac{1}{2\pi r^3} \right) = \frac{\hbar}{r^3} \cdot n \cdot \frac{\epsilon h}{2m_p} \]

Numerically (for \(n = 1\))

\[ F = \left( \frac{1.05 \times 10^{-34} \text{ J} \cdot \text{s}}{\epsilon} \right) \left( \frac{5.0 \times 10^{-27} \text{ J/T}}{\left( 5 \times 10^{-11} \text{ m} \right)^3} \right) \approx 2.6 \times 10^{-11} \text{ N} \]

(b) Compare the force in part a with atomic forces such as the direct electrostatic force between charges (at the same separation), the spin-orbit force, the hyperfine interaction. Comment on the question of binding of magnetic monopoles to nuclei with magnetic moments. Assume the monopole mass is at least that of a proton.


For the direct electrostatic force

\[ F = \frac{ke^2}{r^2} = \left( 1.44 \text{ eV} \cdot \text{nm} \right) \left( \frac{10^{-9} \text{ m}}{\text{nm}} \right) \left( 1.6 \times 10^{-19} \text{ J/eV} \right) \approx 9 \times 10^{-8} \text{ N} \]

The force on the monopole is about 3000 times smaller than the direct electrostatic force.
The spin orbit force is $\alpha^2$ times smaller than the direct electrostatic force, putting it about the same order-of-magnitude as the monopole force. The hyperfine interaction (electron spin with proton spin) is about 6 orders of magnitude smaller than direct electrostatic or about 100 times smaller than monopole.