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Homework hints / solutions

2. $f: A \to B$ and $g: A \to B$ are equal if they are equal as subsets of $A \times B$.

3. There are $|B|^{|A|}$ functions from finite set A to finite set B with $0^0 \equiv 1$.

4. This defines a relation but not necessarily a function.

5. No and No, as discussed in class.

6. If $A \xrightarrow{f} B$ is an isomorphism, then if $f \circ \alpha = f \circ \beta$, then $f^{-1} \circ f \circ \alpha = f^{-1} \circ f \circ \beta \Rightarrow \alpha = \beta \Rightarrow f$ is a monomorphism. Similarly for epi.

7. Easy

8. Let $A \xrightarrow{\varphi} B \xrightarrow{\psi} C$. If φ is not mono, then $\varphi \circ \alpha = \varphi \circ \beta$ for some $\alpha \neq \beta \Rightarrow (\psi \circ \varphi) \circ \alpha = (\psi \circ \varphi) \circ \beta$ for some $\alpha \neq \beta \Rightarrow \psi \circ \varphi$ is not a monomorphism.

9. It's not true in every category, but if the category is such that there is always a morphism $A \xrightarrow{x} B$ between any two objects A and B, and if the category has a product, then

$$\begin{tikzcd} A & A \times B \arrow[l, "\alpha"'] \arrow[r, "\beta"] & B \\ & A \arrow[ul, "i_A"] \arrow[u, "\gamma"] \arrow[ur, "x"'] & \end{tikzcd}$$

commutes for some γ. Since i_A is epi, so is α by problem 8.

Similarly, insertions for $A \oplus B$ are mono.

11. In "17", any epi is an isomorphism. Let $A \xrightarrow{\varphi} B$ be any non-isomorphism. Then

$$\begin{tikzcd} A & A \times B \arrow[l, "\alpha"'] \arrow[r, "\beta"] & B \\ & A \arrow[ul, "i_A"] \arrow[u, "\gamma"] \arrow[ur, "\varphi"'] & \end{tikzcd}$$

$\Rightarrow\Leftarrow$ since γ must also be iso.