

1. Get a copy of *Mathematical Physics* by Robert Geroch, U.Chicago Press, 1985, and read chapter 1.
2. What's the definition of a function? When are two functions equal? Two sets? What's an "equivalence relation?" What are "equivalence classes?"
3. How many functions are there between  $\{1, 2, 3\}$  and  $\{a, b\}$ ? How many between  $\{1\}$  and  $\{a, b\}$ ? Between  $\{\}$  and  $\{a, b\}$ ? Is there a function from  $\{\}$  to  $\{\}$ ? What's the general formula?
4. Consider an equivalence relation  $E$  on set  $X$ . Let  $f$  be the function mapping  $[x]$  to  $x$ . What's wrong with this?
5. Define a category. What's an "object"? What's a morphism? Is a morphism always a function? Is an object always a set?
6. (Geroch's Exercise 1): Prove that every isomorphism is both a monomorphism and an epimorphism.
7. (Geroch's Exercise 5): Prove the the inverse of an isomorphism is unique.
8. (Geroch's Exercise 2): Let  $A \xrightarrow{\phi} B \xrightarrow{\psi} C$ . Prove that if  $\psi \circ \phi$  is a monomorphism, so is  $\phi$  and if  $\psi \circ \phi$  is an epimorphism, so is  $\psi$ . Find examples in the category of sets showing that the converses of these two statements are false.
9. (Geroch's Exercise 7): In the category of sets, the two projections in the definition of a direct product are epimorphisms and the two insertions in the direct sum are monomorphisms [Geroch's statement of Exercise 7 has epi and mono incorrectly switched.]. Is this true in every category?
10. (Geroch's Exercise 11): Fix two categories. Introduce a new category that can be thought of as the "product" of these. (Hint: choose, for objects, pairs consisting of one object from each of the given categories).
11. (Geroch's Exercise 4): Let the objects be sets with exactly 17 elements, the morphisms mappings of such sets, and composition composition. Verify that this is a category. Prove that in this category, no two objects have either a direct product or a direct sum.