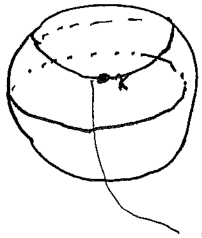
Group Actions



X = the sphere $\qquad G \xrightarrow{\psi} \mathrm{Aut}(X)$.

$G = SO(2)$

O_x : the orbit of x under the group action

G_x : the stabilizer subgroup of G fixing x,

ψ is transitive if, for any $x, y \in X$, $\psi(g)(x) = y$ for some $g \in G$.

$O_x \cong G/G_x$ because $gG_x \mapsto gx$ is an onto function and if $gx = g'x \Rightarrow g'^{-1}g \in G_x \Rightarrow g'G_x = g'g'^{-1}g\,G_x = g\,G_x$ so this function is a bijection.

The orbits are also equivalence classes of xEy iff $gx = y$ for some $g \in G$.

Example: Given subgroup H of G, define $G \longrightarrow \mathrm{Aut}(G/H)$ by $g' : gH \mapsto g'gH$. G thus acts on G/H transitively.