Homework #0. S. Y., June 10, 2004.

1. www.amazon.com.

2. $f: A \to B$ and $g: A \to B$ are equal if $f(a) = g(a)$ for all $a \in A$.

3. There are $|B|^{|A|}$ functions from finite sets A, B with $0^0 \equiv 1$.

4. It's not necessarily a function on equivalence classes.

5. No + No.

6. If $X \underset{\alpha'}{\overset{\alpha}{\rightrightarrows}} A \xrightarrow{\varphi} B$ commutes, so does $X \underset{\alpha'}{\overset{\alpha}{\rightrightarrows}} A \underset{\varphi^{-1}}{\overset{\varphi}{\rightleftarrows}} B$

$\Rightarrow \alpha = \alpha'$. Similar for epi.

7. If $A \xrightarrow{\varphi} B$ has two inverses $i_A \circlearrowright A \underset{\varphi_1}{\overset{\varphi}{\rightleftarrows}} B \circlearrowleft i_B$ and

$i_A \circlearrowright A \underset{\varphi_2}{\overset{\varphi}{\rightleftarrows}} B \circlearrowleft i_B \Rightarrow \varphi_1 \circ \varphi = \varphi_2 \circ \varphi \Rightarrow \varphi_1 \circ (\varphi \circ \varphi_2) = \varphi_2 \circ (\varphi \circ \varphi_2) \Rightarrow \varphi_1 = \varphi_2.$

8. Done in class.

9. Not true in any category, but if there is always a morphism $A \xrightarrow{x} B$ between any two objects,

$$\begin{tikzcd} A & A \times B \arrow[l, "\alpha"'] \arrow[r, "\beta"] & B \\ & A \arrow[ul, "i_A"] \arrow[u, "\gamma"] \arrow[ur, "x"'] & \end{tikzcd}$$

commutes for some γ. Since i_A is epic, so is α (by #8).

Similarly for $A \oplus B$, the insertions are monic.

10. Objects are pairs of objects (A, B) A from category C, B from C', morphisms are $(A, B) \xrightarrow{(\varphi, \psi)} (A', B')$ $\varphi: A \to A'$, $\psi: B \to B'$.

11. In "17" any epic is an isomorphism. Let $A \xrightarrow{\varphi} B$, then non-iso.

$$\begin{tikzcd} A & A \times B \arrow[l, "\alpha"'] \arrow[r, "\beta"] & B \\ & A \arrow[ul, "i_A"] \arrow[u, "\gamma"] \arrow[ur, "\varphi"'] & \end{tikzcd}$$

$\Rightarrow \alpha, \beta, \gamma$ are iso $\Rightarrow \varphi$ is iso $\Rightarrow \Leftarrow$.

Similar for $A \oplus B$.