Mathematical Physics Homework 3: Vector spaces II.

1. Prove that an n'th degree polynomial has at most n complex roots.
2. Prove that if operators $\phi_1, \ldots, \phi_n$ commute, then the kernel of the composition is the direct sum of the kernels.
3. Prove the four facts at the top of page 14.
4. Show that if a bilinear function area(v,w) is zero if either of it's arguments are zero, then it is anti-symmetric.
5. Prove that the eigenvalues of an isometry in a complex finite dimensional vector space are of the form $e^{i\theta}$.
6. Find an example of an operator on a complex vector space that has no eigenvector.
7. Prove that the eigenvalues of a non-negative operator is a non-negative real number.
8. Prove that the sum of two non-negative operators is a non-negative operator.
9. Given our treatment of determinants, guess a similar definition for the trace of an operator.
10. Prove that the category of all categories is a category.
11. State precisely and prove: "Free objects are unique."
12. The power set of a set is the set of all subsets of the set. Find a covariant and a contravariant functor that contain the power set as part of their definitions.
13. A group action of group G on set X is a mapping $. : G \times X \to X$ satisfying $g.h.x = (gh).x$ and $e.x = x$. A *representation* of a group G is a group homomorphism from G to the automorphism group of some vector space. Consider G to be a category with a single object and with all morphisms isomorphisms. Show that a group action is a functor from G to the category of sets and a group representation is a functor from G to the category of (real or complex) vector spaces.