

Mathematical Physics Homework 2: Vector spaces.

1. Prove that $0 \cdot v = 0$ in any vector space.
2. What is the free vector space on the empty set?
3. Let objects be vector spaces over a fixed field F . Let morphisms be linear mappings with composition composition of functions. Prove that this is a category.
4. Prove the claims on page 2 of the notes.
5. Prove the three facts claimed on page 5 of the notes.
6. Let $\varphi : V \rightarrow W$ be a linear mapping. Prove that $\dim(\text{Ker } \varphi) + \dim(\text{Im } \varphi) = \dim V$.
7. If V and W are finite dimensional vector spaces, is $V \otimes W$ finite dimensional? If so, what is it's dimension?
8. Find a vector space which is isomorphic to one of it's proper subspaces.
9. Answer the question on page 13 of the notes.
10. If V is a finite dimensional vector space, show that $\text{Mor}(V, V) \cong V^* \otimes V$.
11. Show that $V \oplus (U \oplus W) \cong (V \oplus U) \oplus W$.