Homework 3 (Due February 7): Vector Spaces
Mathematical Physics
Saul Youssef, February 2000

1. (Geroch's Exercise 58): Introduce the category of partially ordered sets, and discuss direct products and direct sums (both of which exist).

2. What is the free vector space on the empty set?

3. (Geroch's Exercise 63): Prove that, in the category of vector spaces, a linear mapping is one-to-one if and only if it is a monomorphism (see Geroch for hints).

4. Let W be a subspace of vector space V. Prove that any basis for W is the subset of some basis for V.

5. (Geroch's Exercise 83): In the Figure 3. (Geroch, page 61), let α and β be linear mappings. Prove that there exists a linear mapping γ which makes the diagram commute if and only if $\mathrm{Ker}(\alpha)$ is a subset of $\mathrm{Ker}(\beta)$. When is γ unique?

6. (Geroch's Exercise 67): Let V and V' be free vector spaces on sets S and S' respectively. Prove that there is a monomorphism from V to V' (category of vector spaces) if and only if there is a monomorphism from S to S' (category of sets). Similarly for epimorphism. What does this mean for finite–dimensional vector spaces?