Advanced Analysis Methods

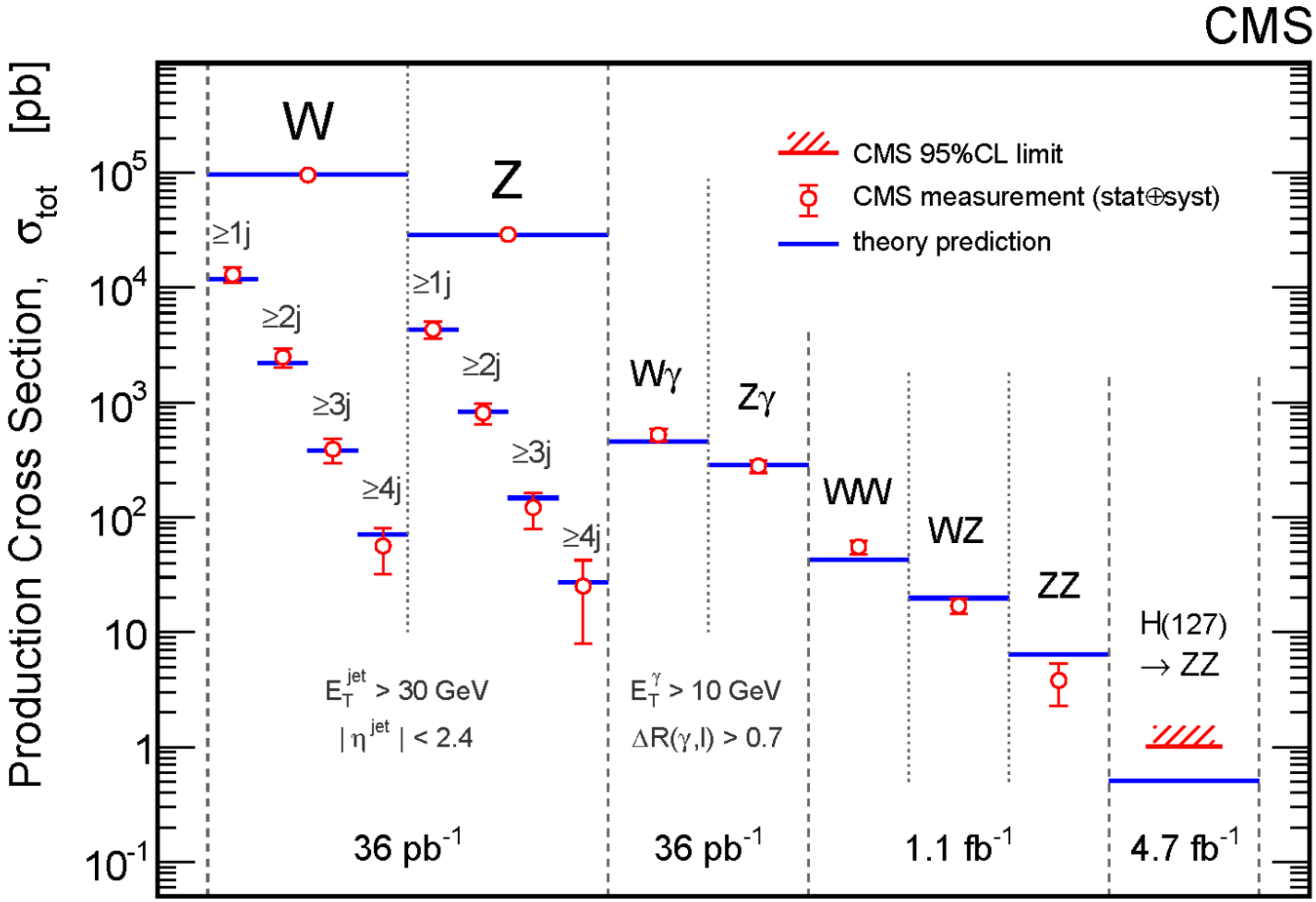
Tulika Bose

April 6th, 2016

Searching for rare signals

circa 2011

Higgs and new physics cross-sections are small...



JHEP10(2011)132
CMS-PAS-EWK-10-012

PLB701(2011)535

CMS-PAS-EWK-11-010

CMS-PAS-HIG-11-025

Examples of background to H→ZZ searches

5 orders of magnitude!

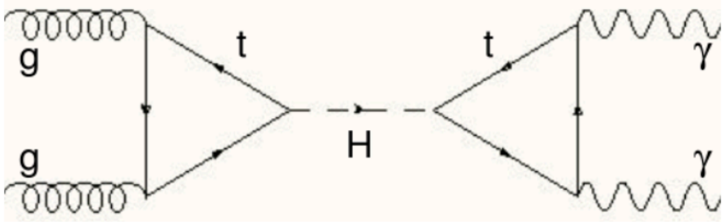
Over huge backgrounds

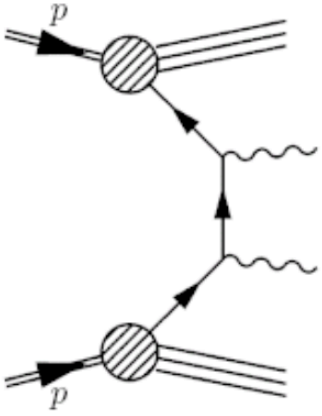
To achieve a discovery, huge background reduction rate needed

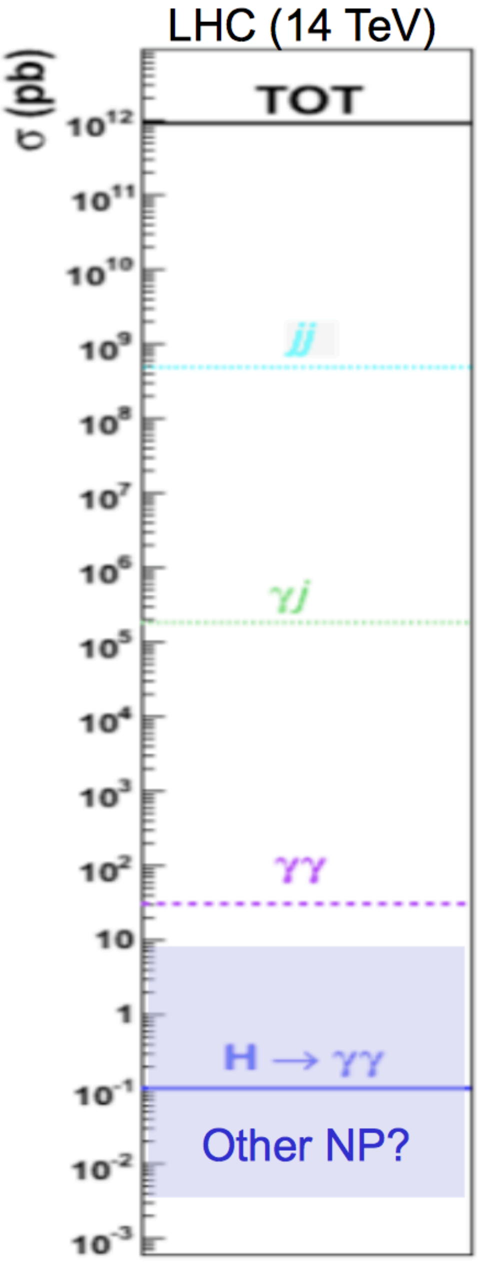
- Example of **H→γγ** : typically 9 orders of magnitude under the QCD jets background
- **Reducible background** : jet-jet, photon-jet
  - Jets can be mis-identified as photons

  => can be suppressed by tight photon identification criteria
- **Irreducible background :** photon-photon
  - Non-resonant diphoton continuum

  => Can be discriminated using kinematic properties





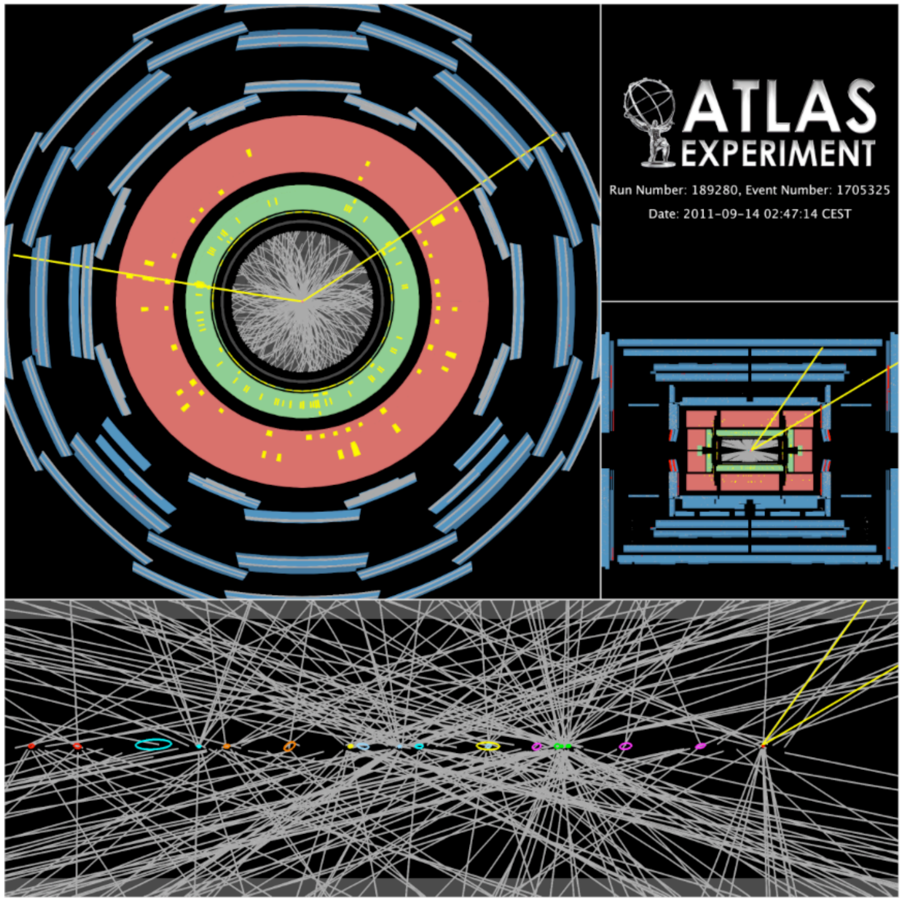


With experimental challenges

Experimental challenges :

- Detector calibration
- Identification of the tracks / energy deposits in the sub-detectors
- Particle reconstruction
- Particle identification
- Finding the vertex of hard interaction among all pile-up vertices
- Discriminate the signal process against all other background processes
- ...

- **Multivariate methods can help for that**


ATLAS EXPERIMENT
Run Number: 189280, Event Number: 1705325
Date: 2011-09-14 02:47:14 CEST

Collision with 20 pile-up events recorded with the ATLAS detector

Multi-variate methods: definitions

MultiVariate Analysis :

- Set of statistical analysis methods that simultaneously analyze multiple measurements (variables) on the object studied
- Variables can be dependent or correlated in various ways

Classification / regression :

- **Classification** : discriminant analysis to separate classes of events, given already known results on a training sample
- **Regression** : analysis which provides an output variable taking into account the correlations of the input variables

Statistical learning :

- **Supervised learning :** the multivariate method is trained over a sample where the result is known (e.g. Monte Carlo simulation of signal and background)
- **Unsupervised learning :** no prior knowledge is required. The algorithm will cluster events in an optimal way

Multivariate analyses in HEP

- **Signal/background discrimination :**
  - **Object reconstruction :** discriminate against instrumental background (electronic noise…)
  - **Object identification :** e.g. electron, bottom quark identification, to improve the rejection other objects resembling (e.g. jets)
  - **Discriminating physics process against physics backgrounds**. Many examples, e.g. single top against W+jets, H->WW against WW background…
- **Improving the energy measurement**, via regression. Allows to narrow the reconstructed mass peak, improve the resolution.
- **Estimate the sensitivity of the analysis :**
  - **Sensitivity to signal exclusion or discoveries :** Likelihood of the data to be consistent with background only or signal+background hypothesis
  - **Combination** of many channels

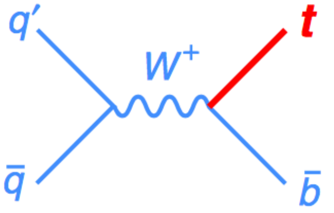
  => exclusion limits or discoveries

MVA example from Tevatron

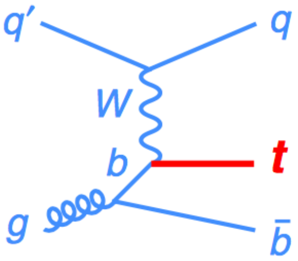
Single top discovery

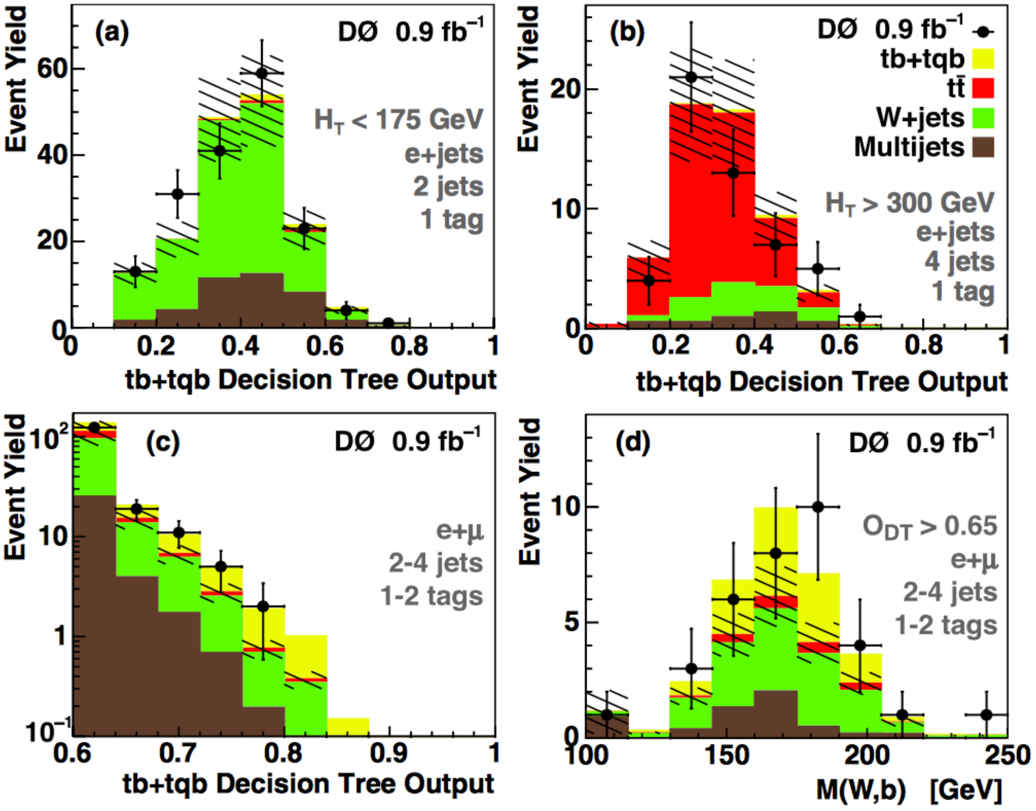
PhysRevLett.98.181802

(a)



(b)





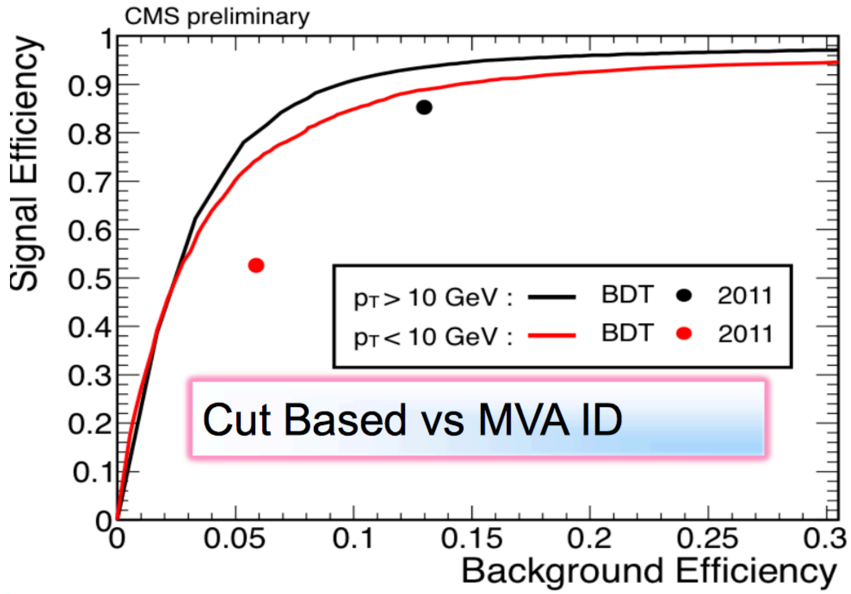
- When published, very controversial
- 36 boosted decision trees used to discriminate signal from background
- First measurement of the single top cross-section, today well established

No turning back!

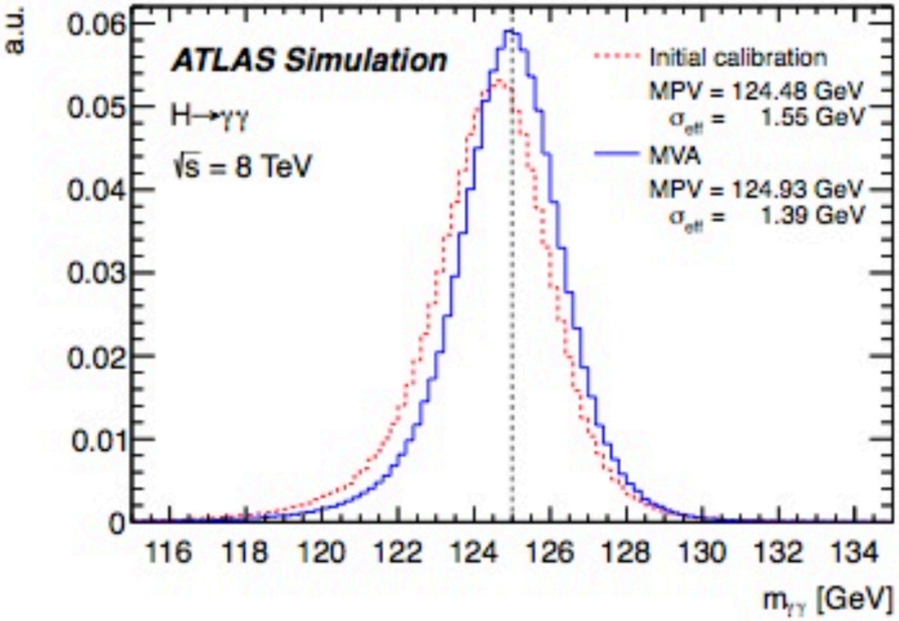
- Over the past ten years, Multivariate analysis (MVA) methods gained gradual acceptance in HEP.
- In fact, they are now “state of the art”
- Some of the most important physics results in HEP have come from the use MVA methods.

MVA use in Higgs Discovery

- MVA used in every possible analysis aspect
  - Electrons/photons ID
  - MVA for EM cluster energy corrections
  - Vertex identification (diphotons)
  - b-tagging
  - S/B discrimination in all channels
    - γγ, ZZ→4l, (WW, bb, ττ)

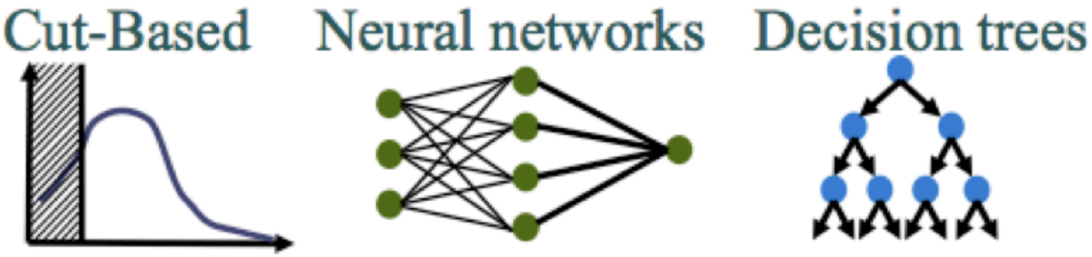


Cut Based vs MVA ID



Event Analysis Techniques

- Examples:



- **Characteristics :**
  - Level of complexity and transparency
  - Performance in term of background rejection
  - Way of dealing with non-linear correlations
  - Robustness while increasing the number of input variables

    …

“Rectangular” cuts

- Simplest multivariate method, very intuitive
  - All HEP analyses are using rectangular cuts, not always completely optimized

- **Rectangular cuts optimization :**
  - e.g. Grid search,

- **Characteristics :**
  - Difficult to discriminate signal from background if non-linear correlations
  - Optimization difficult to handle with high number of variables

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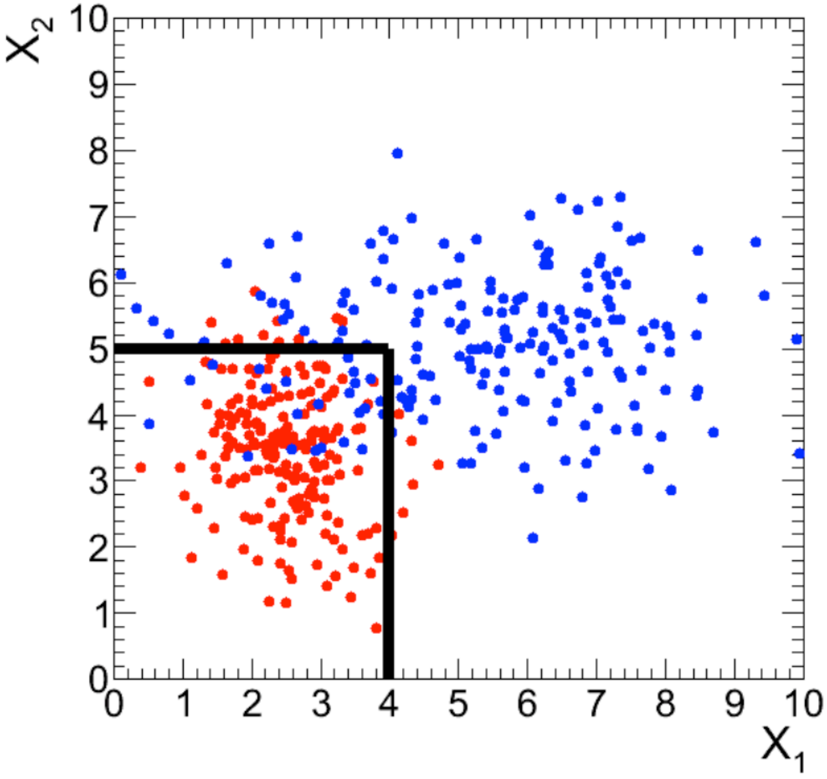
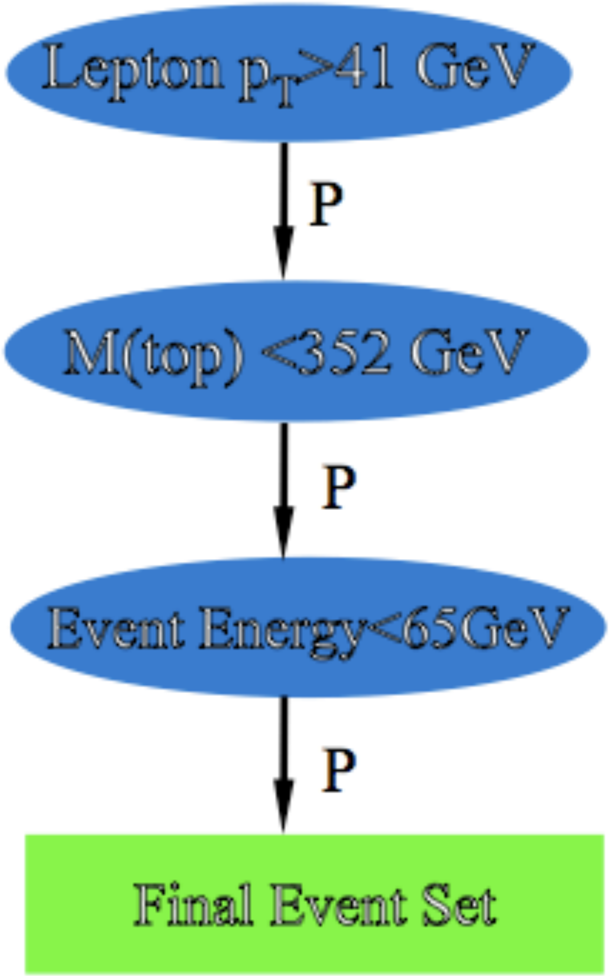


Define the signal region :
a1 < x1 < a2,
b1 < x2 < b2

Cut-based analysis



Lepton p_T>41 GeV

P

M(top) <352 GeV

P

Event Energy<65GeV

P

Final Event Set

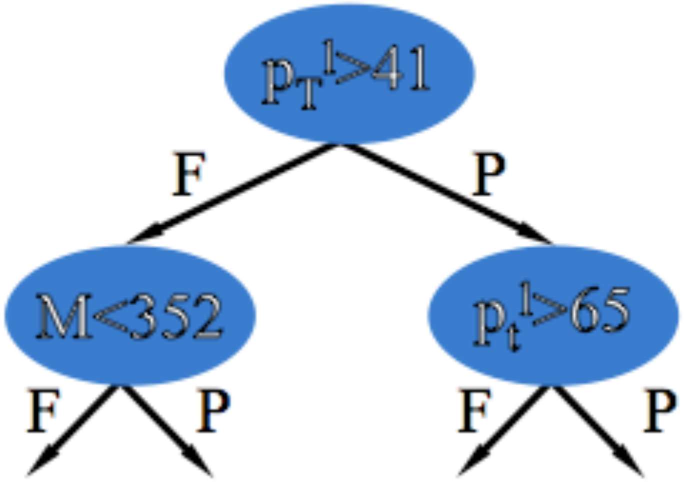
In the final event set

- Estimate background yield
- Compare to data
  $N_{\mathrm{obs}} = N_{\mathrm{data}} - N_{\mathrm{B}}$
- Calculate signal acceptance
  $\sigma = N_{\mathrm{obs}} / (A*L)$

Decision Trees

- Machine-learning technique, widely used in the social sciences
- Idea: recover events that fail criteria in cut-based analysis

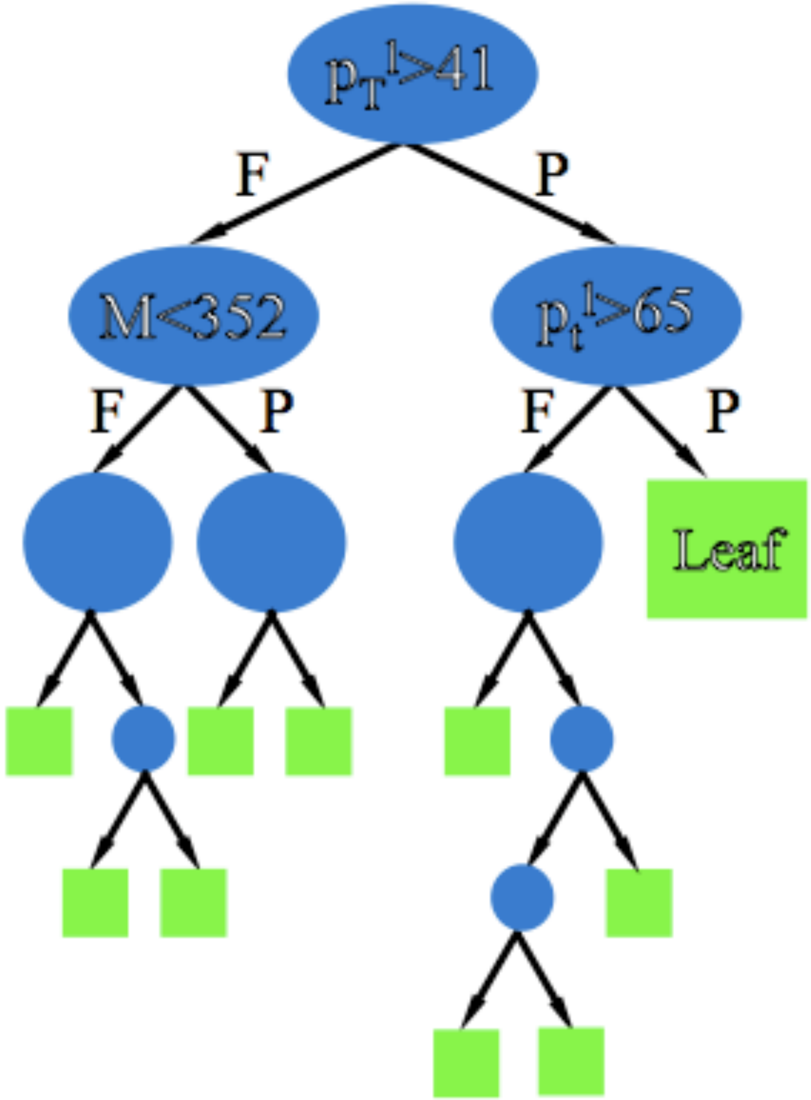
Including events that fail a cut



- Create a tree of cuts
- Divide sample into "pass" and "fail" sets
- Each node ● corresponds to a cut (branch)

- Start at first "node ● " with "training sample" of 1/3 of all signal and background events
  - For each variable, find splitting value with best separation between two children (mostly signal in one, mostly background in the other)
  - Select variable and splitting value with best separation to produce two "branches → " with corresponding events, (**F**)ailed and (**P**)assed cut

Trees and leafs

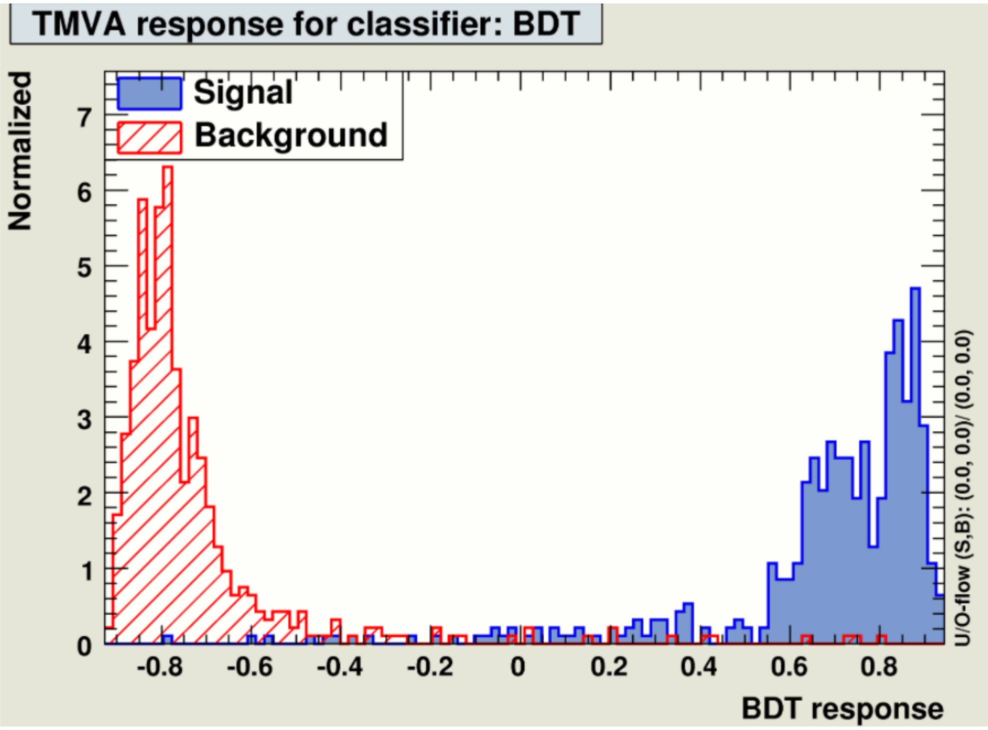


- Create a tree of cuts
- Divide sample into "pass" and "fail" sets
- Each node ● corresponds to a cut (branch)
- A leaf 🟩 corresponds to an end-point
- For each leaf, calculate purity (from MC):
  purity = $N_S/(N_S+N_B)$

Repeat recursively on each node
Stop (terminate at leaf) when improvement stops or when too few events left

Decision tree output

- Train on signal and background models (MC)
  - – Stop and create leaf when N_{MC}<100
- Compute purity value for each leaf
- Send data events through tree
  - – Assign purity value corresponding to the leaf to the event
- Result approximates a probability density distribution

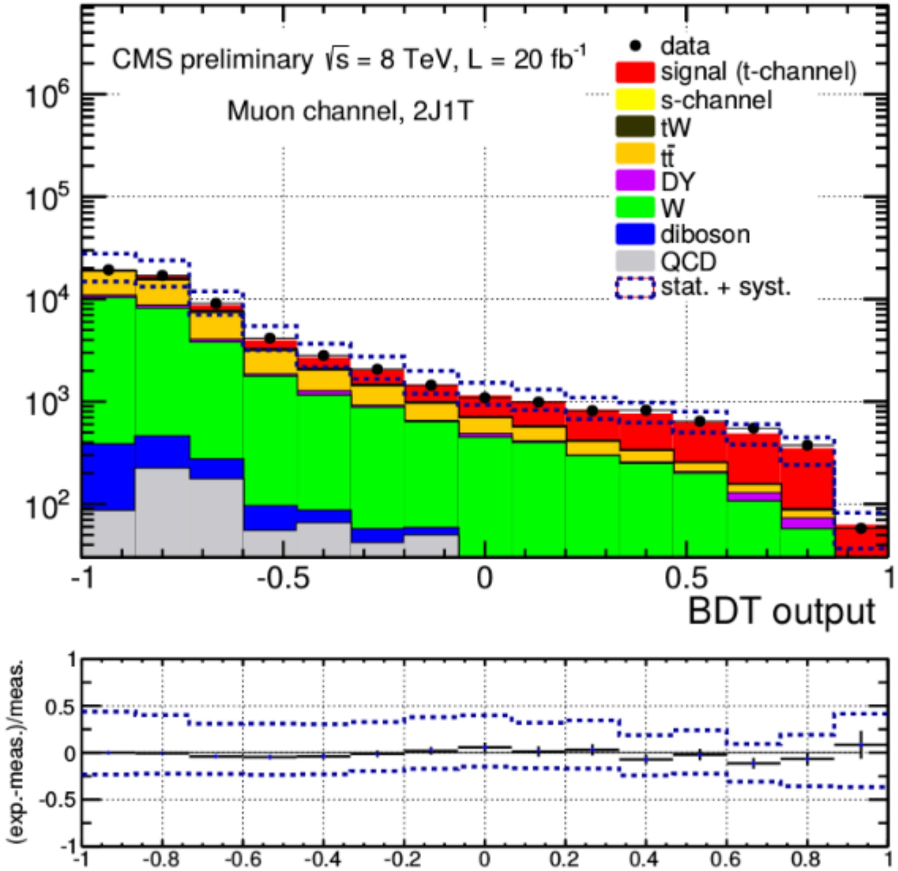


Decision tree output for each event = leaf purity
Closer to 1 for signal and closer to 0 for background

Measure and Apply

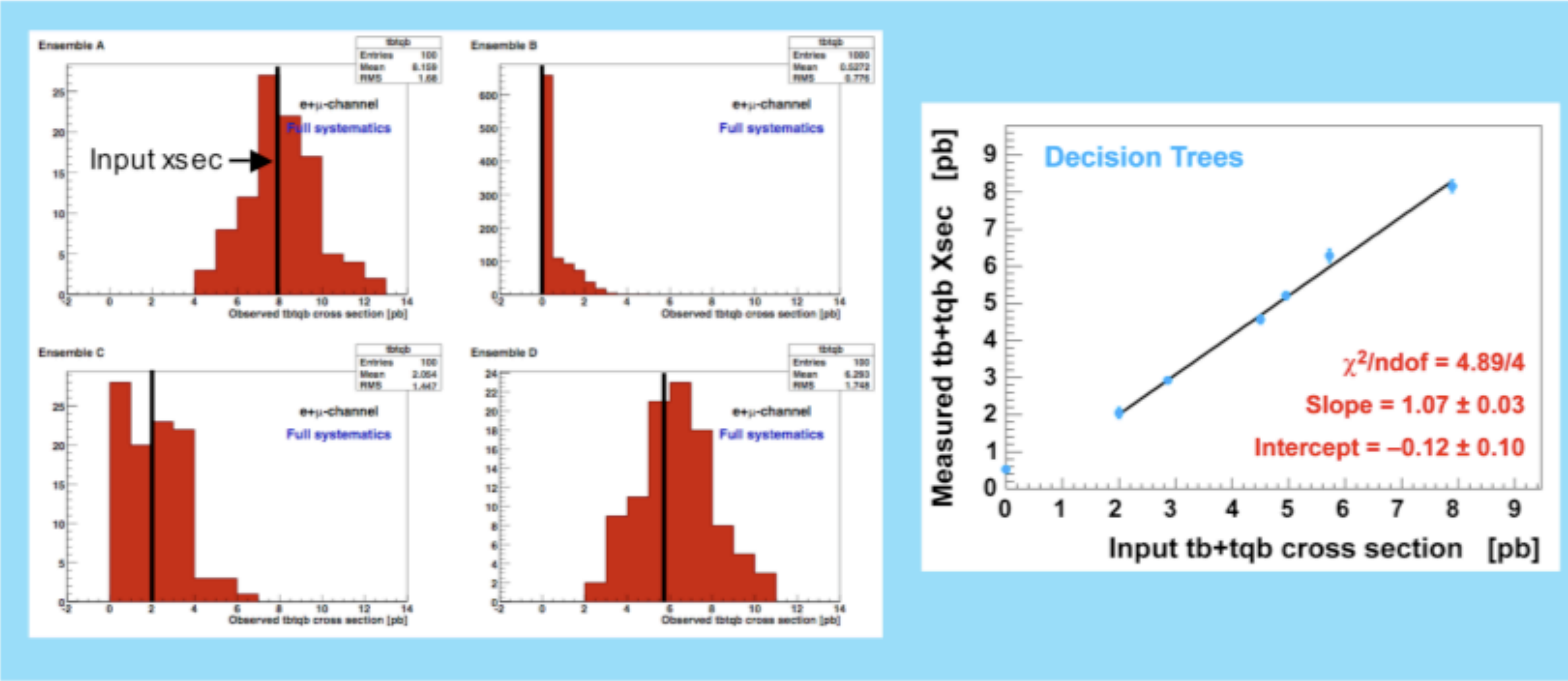
- Take trained tree and run on independent simulated sample, determine purities.
- Apply to Data
- Should see enhanced separation (signal right, background left)
- Could cut on output and measure, or use whole distribution to measure.

- Cut on BDT yields signal enriched sample
- Allows to study top quark properties



Decision Tree Verification

- Use “mystery” ensembles with many different signal assumptions
- Measure signal cross section using decision tree outputs
- Compare measured cross sections to input ones
- **Observe linear relation close to unit slope**



χ^2/ndof = 4.89/4

Slope = 1.07 ± 0.03

Intercept = –0.12 ± 0.10

Object Kinematics

p_T(jet1)
p_T(jet2)
p_T(jet3)
p_T(jet4)
p_T(best1)
p_T(notbest1)
p_T(notbest2)
p_T(tag1)
p_T(untag1)
p_T(untag2)

Angular Correlations

ΔR(jet1,jet2)
cos(best1,lepton)$_{\mathrm{besttop}}$
cos(best1,notbest1)$_{\mathrm{besttop}}$
cos(tag1,alljets)$_{\mathrm{alljets}}$
cos(tag1,lepton)$_{\mathrm{btaggedtop}}$
cos(jet1,alljets)$_{\mathrm{alljets}}$
cos(jet1,lepton)$_{\mathrm{btaggedtop}}$
cos(jet2,alljets)$_{\mathrm{alljets}}$
cos(jet2,lepton)$_{\mathrm{btaggedtop}}$
cos(lepton, Q(lepton) $\times z$)$_{\mathrm{besttop}}$
cos(lepton,besttopframe)$_{\mathrm{besttopCMframe}}$
cos(lepton,btaggedtopframe)$_{\mathrm{btaggedtopCMframe}}$
cos(notbest,alljets)$_{\mathrm{alljets}}$
cos(notbest,lepton)$_{\mathrm{besttop}}$
cos(untag1,alljets)$_{\mathrm{alljets}}$
cos(untag1,lepton)$_{\mathrm{btaggedtop}}$

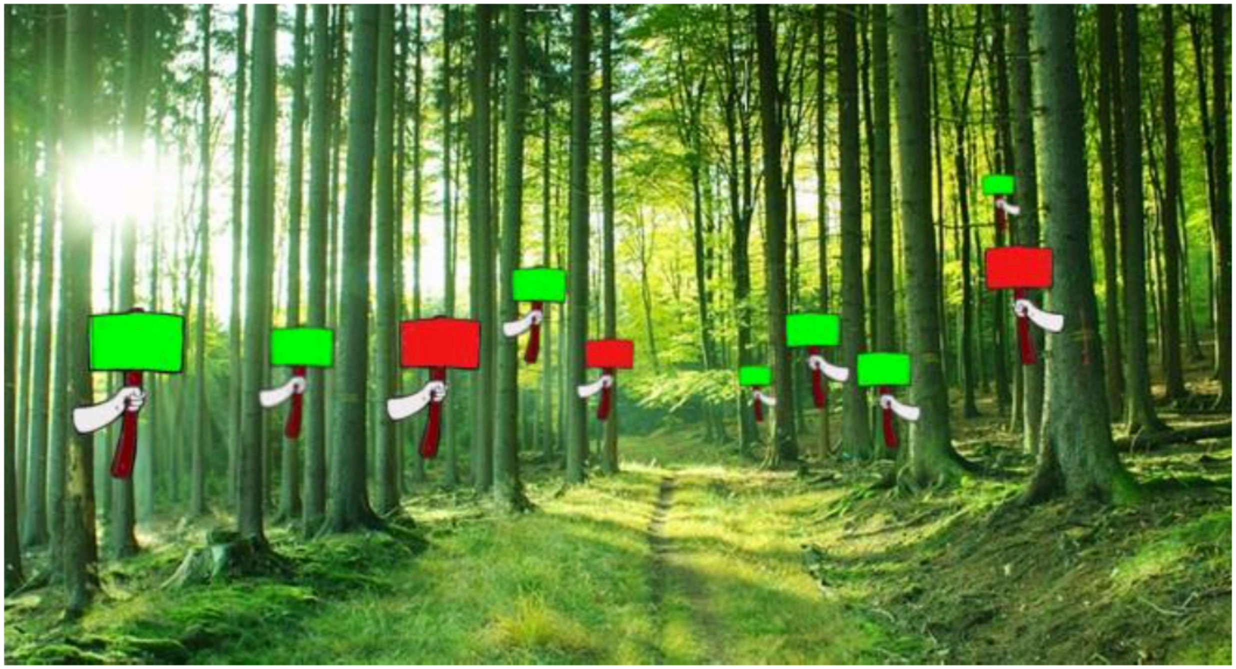
Event Kinematics

Aplanarity(alljets, W)
M(W,best1) ("best" top mass)
M(W,tag1) ("b-tagged" top mass)
H_T(alljets)
H_T(alljets$-$best1)
H_T(alljets$-$tag1)
H_T(alljets, W)
H_T(jet1,jet2)
H_T(jet1,jet2, W)
M(alljets)
M(alljets$-$best1)
M(alljets$-$tag1)
M(jet1,jet2)
M(jet1,jet2, W)
M_T(jet1,jet2)
M_T(W)
Missing E_T
p_T(alljets$-$best1)
p_T(alljets$-$tag1)
p_T(jet1,jet2)
Q(lepton) $\times \eta$(untag1)
$\sqrt{\hat{s}}$
Sphericity(alljets, W)

- Adding variables does not degrade performance
- Tested shorter lists, lose some sensitivity
- Same list used for all channels

Random Forest

- Random Forests is an ensemble method that combines different trees
- Final output is determined by the majority vote of all trees



A Random Forest combines the votes of all trees

Random forest

- Average over many decision trees
  - Typically O(100)
- Each tree is grown using m variables
  - For N total variables, m<<N
- Very fast algorithm
  - Even with large number of variables
- Very few parameters to adjust
  - Typically only m

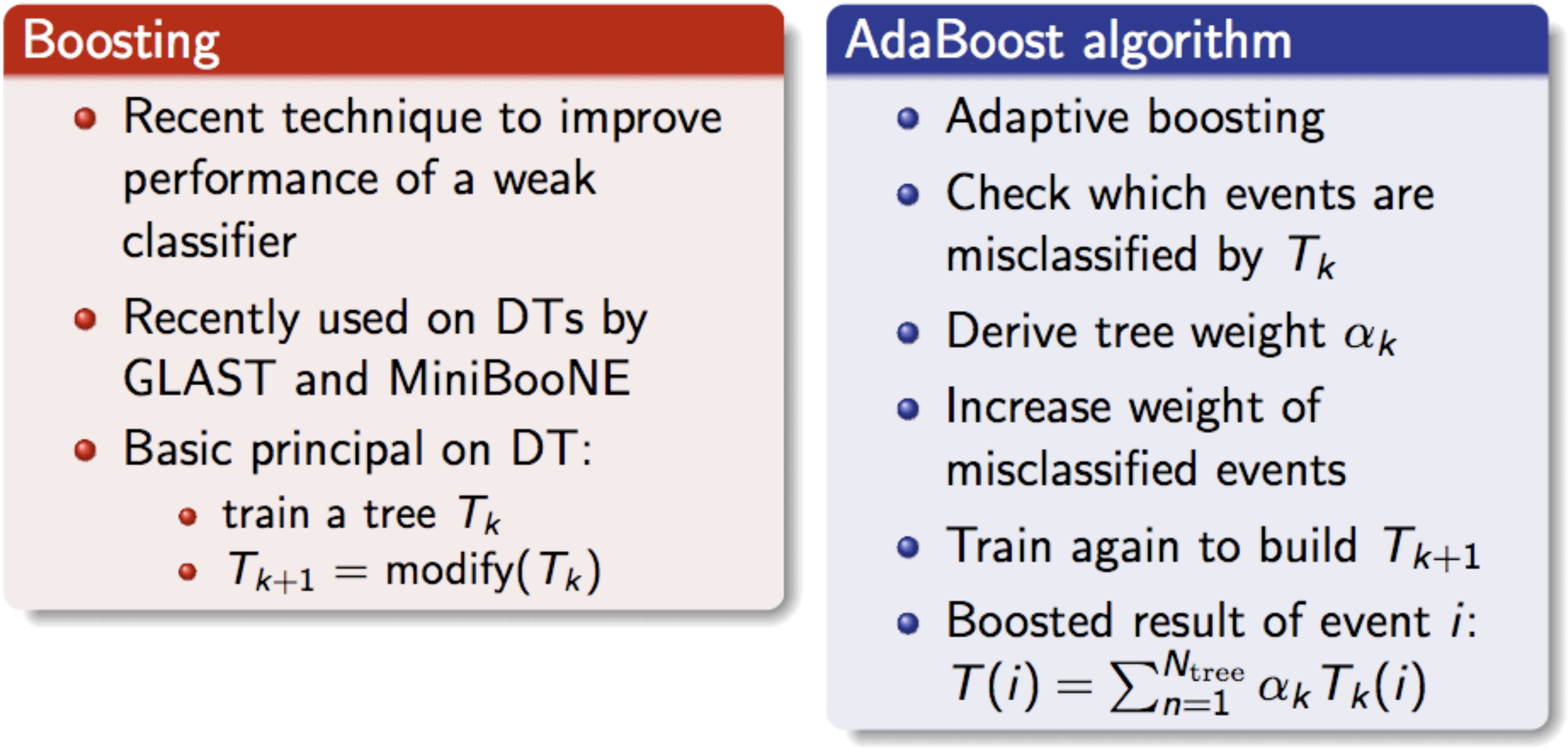
Random Forest

- Random Forests is an ensemble method that combines different trees
- Final output is determined by the majority vote of all trees
- The idea is, that a sum of weak learners results in a stronger learner

Simple example:

- 3 different trees which are uncorrelated and are correct in 60% of cases
- In order to correctly classify an event, only ⅔ trees have to be correct. That means, the misclassification probability is either 3 wrong or ⅔ wrong:
    - $P = ({}^{3}{}_{2}) *0.4^{2} * 0.6 + ({}^{3}{}_{3}) * 0.4^{3} * 0.6^{0} = 0.352$
- Therefore the ensemble of trees is better than only one tree even though their separation power is the same (if uncorrelated)

Boosted Decision Trees



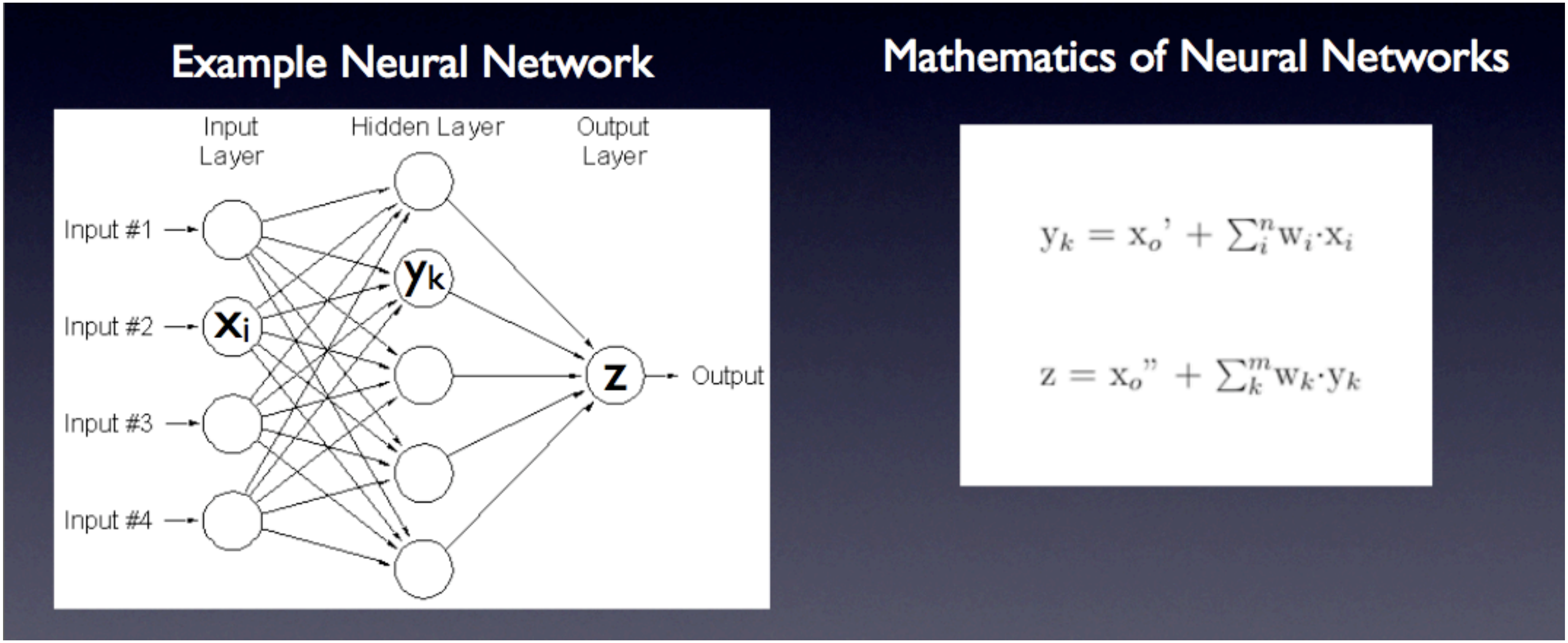
Boosting

- Recent technique to improve performance of a weak classifier
- Recently used on DTs by GLAST and MiniBooNE
- Basic principal on DT:
  - train a tree T_k
  - $T_{k+1} = \text{modify}(T_k)$

AdaBoost algorithm

- Adaptive boosting
- Check which events are misclassified by T_k
- Derive tree weight α_k
- Increase weight of misclassified events
- Train again to build T_{k+1}
- Boosted result of event i: $T(i) = \sum_{n=1}^{N_{\text{tree}}} \alpha_k T_k(i)$

Neural Networks



Example Neural Network

Mathematics of Neural Networks

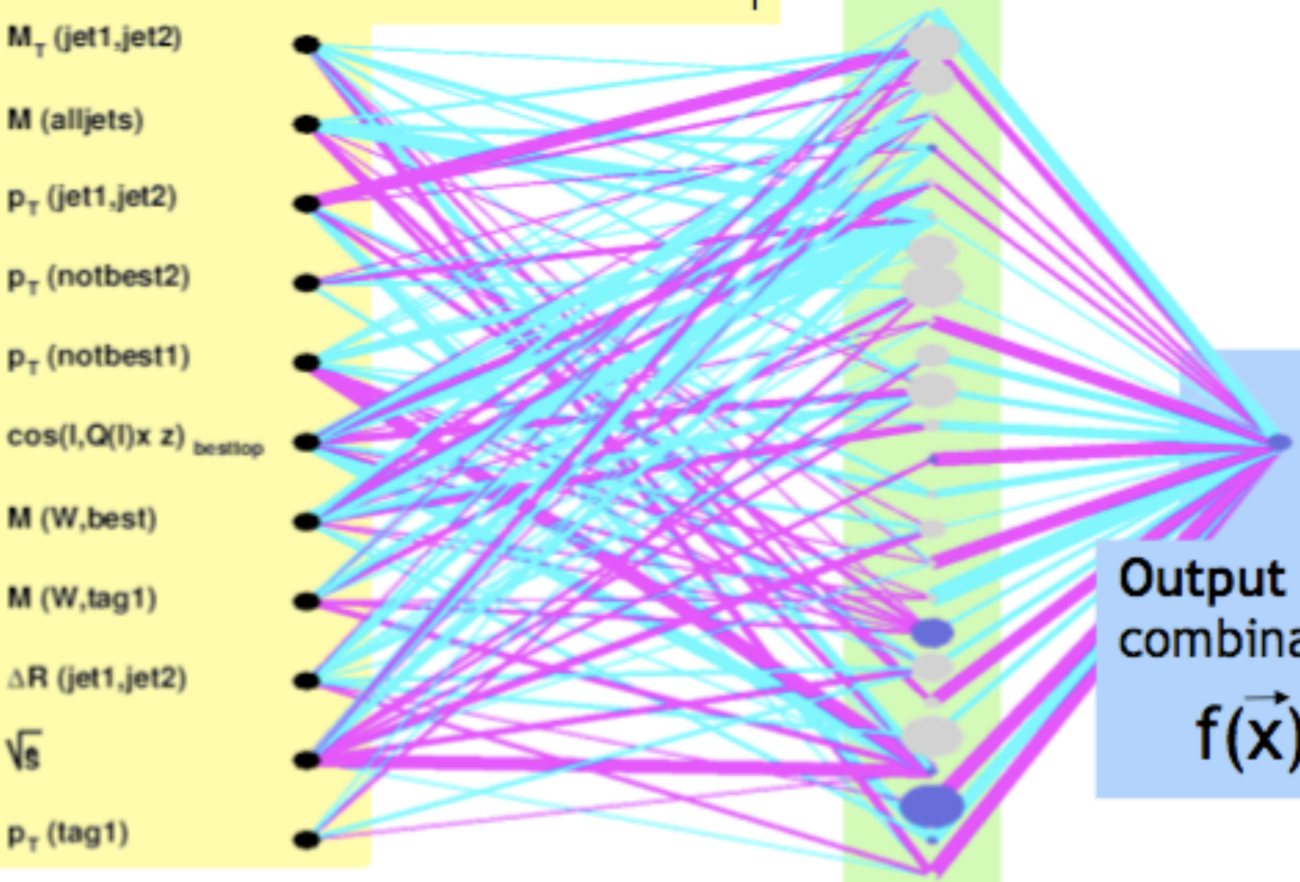
$$y_k = x_o{}' + \sum_i^n w_i \cdot x_i$$

$$z = x_o{}'' + \sum_k^m w_k \cdot y_k$$

- The activity of the input units represents the raw info that is fed into the network.
- The activity of each hidden unit is determined by the activities of the input units and the weights on the connections between the input and the hidden units.
- The behavior of the output units depends on the activity of the hidden units and the weights between the hidden and output units.

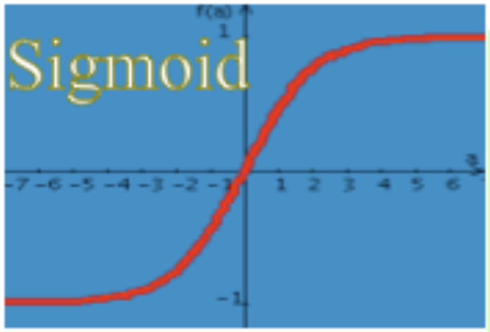
Neural networks

Input Nodes: One for each variable x_i



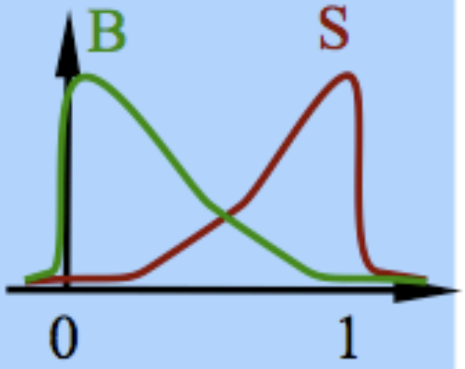
Output Node: linear combination of hidden nodes

$$f(\vec{x}) = \Sigma\ w'_k\ n_k(\vec{x},\vec{w}_k)$$



Hidden Nodes: Each is a sigmoid dependent on the input variables

$$n_k(\vec{x},\vec{w}_k) = \frac{1}{1 + e^{-\Sigma\, w_{ik}\, x_i}}$$



What method is best?

- The “no free lunch” theorem tells you that there is no one method that is superior to all others for all problems.

- In general, one can expect neural networks (NN), Boosted decision trees (BDT) and random forests (RF) to provide excellent performance over a wide range of problems.

The Buzz about Deep Learning

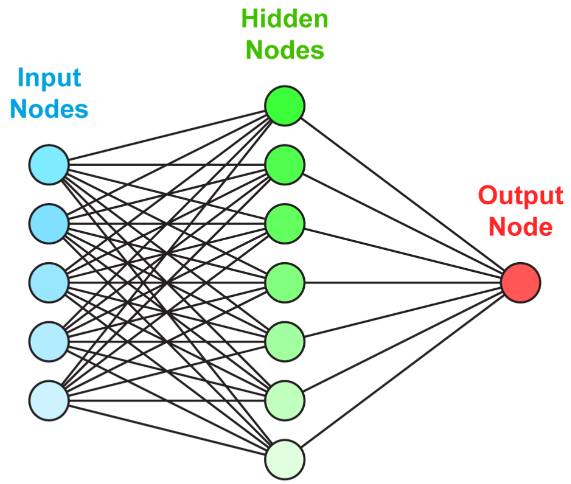
- A lot of excitement about “Deep Learning” Neural Networks (DNN) in the Machine Learning community
  - Spreading to other areas!
  - Some studies already in HEP!

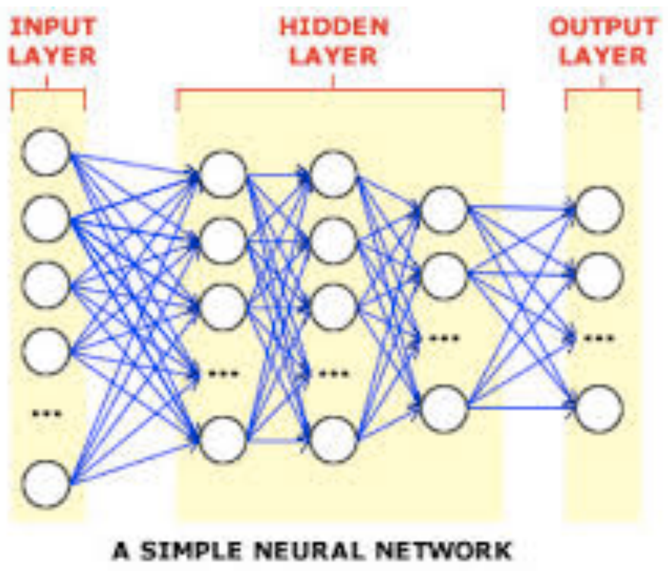
- Multiple non-linear hidden layers to learn very complicated input-output relationships

- Huge benefits in applications in computer vision (image processing/ID), speech recognition and language processing

Deep Learning

Single hidden layer NN





Multiple hidden layer NN

Use raw data inputs instead of derived "intelligent" variables (or use both)

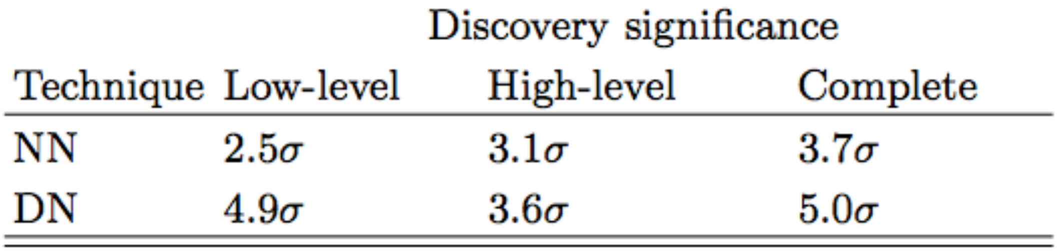
Final learning better than shallow networks, particularly when inputs are unprocessed raw variables!

However, need a lot of processing power (implement in GPUs) time….

Deep Neural Networks for HEP

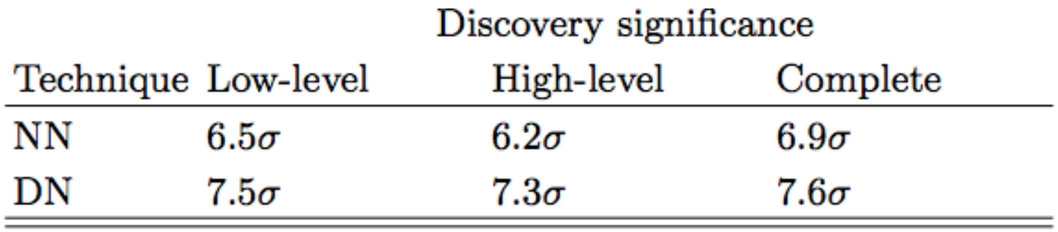
- Baldi, Padowski, Whiteson arXiv:1402.4735v2
- Studied two benchmark processes
  - Charged Higgs vs ttbar events
  - SUSY: Chargino pairs vs WW events into dilepton+MET final state

Significant improvement in Higgs case, not so dramatic in case of SUSY



	Discovery significance		
Technique	Low-level	High-level	Complete
NN	2.5σ	3.1σ	3.7σ
DN	4.9σ	3.6σ	5.0σ

Exotic Higgs

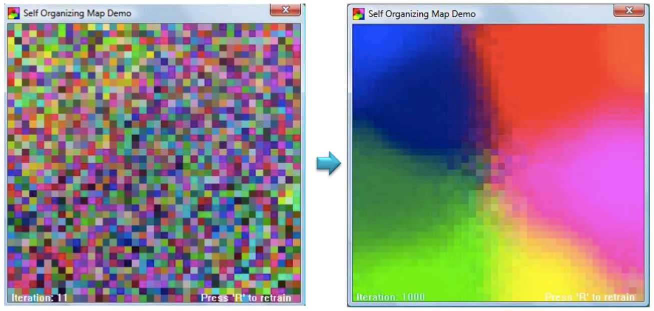


	Discovery significance		
Technique	Low-level	High-level	Complete
NN	6.5σ	6.2σ	6.9σ
DN	7.5σ	7.3σ	7.6σ

SUSY Study

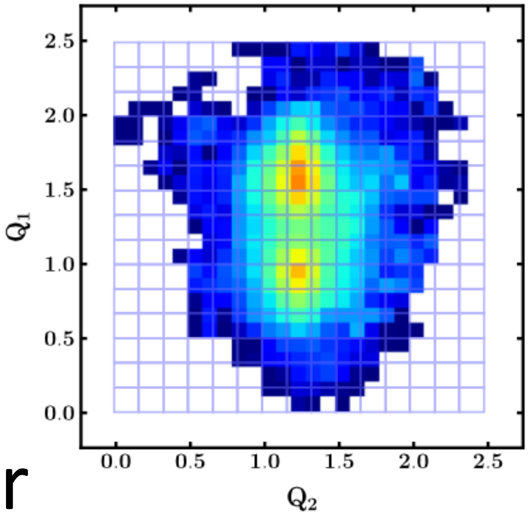
Unsupervised Learning

- The most common approach is to find clusters or hidden patterns or groupings in data



http://chem-eng.utoronto.ca/~datamining/Presentations/SOM.pdf

Cogan, Kagan, Strauss & SchwarRman (arXiv:1407.5675)



- We have not tapped these methods for identifying unknown components in data, unsupervised classification, for exploratory data analysis
- Could be useful in applications for topological pattern recognition
  - Use in Jet-substructure, boosted jet ID

Summary

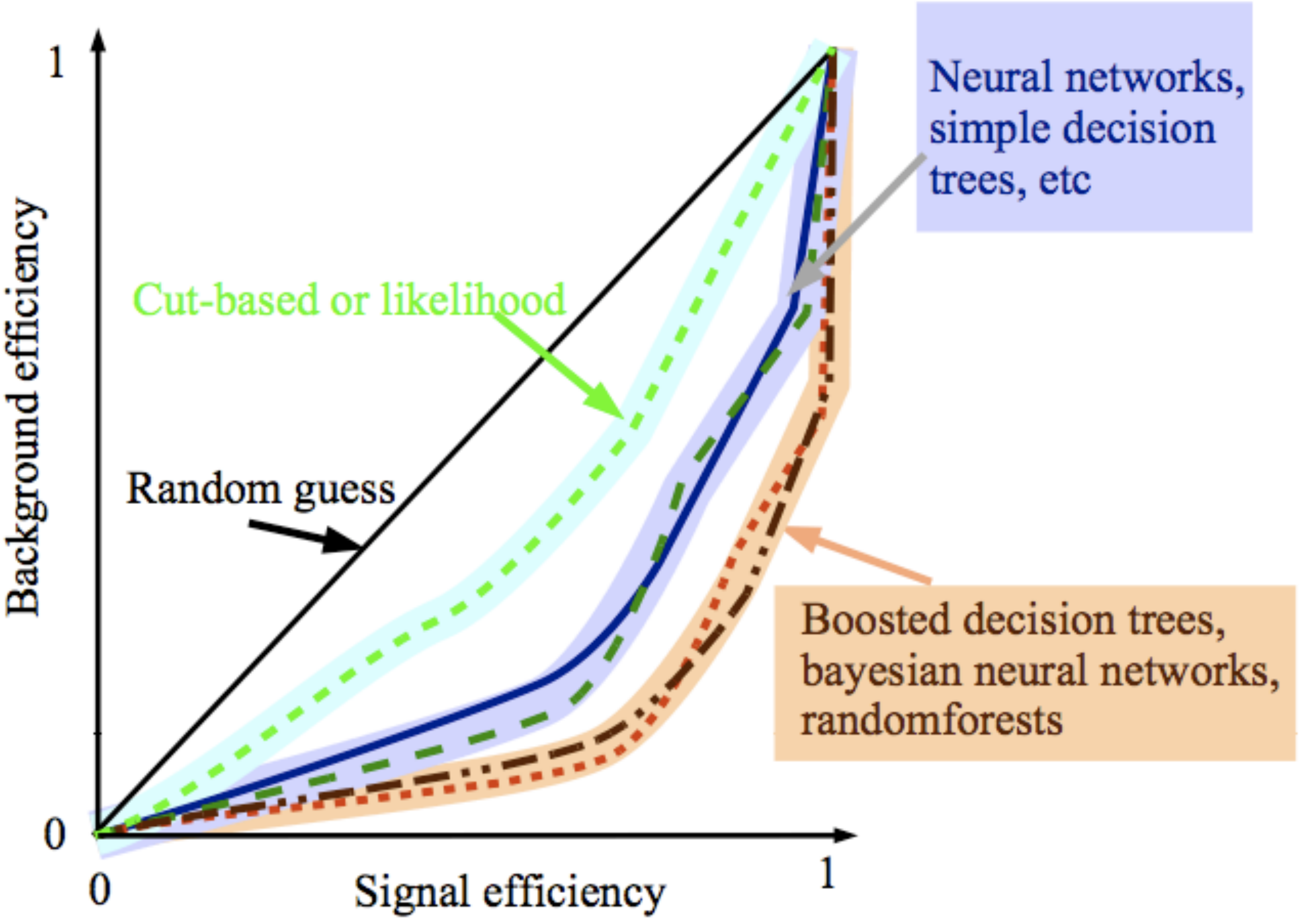
- Multivariate methods brought a paradigm shift in HEP analysis ~10 years ago. Now they are state of the art.
- Applications of new ideas/algorithms such as deep learning should be explored, but the resources involved may not justify the use in every case.
- Well established techniques of the past – neural networks, Boosted Decision Trees will continue to be the ubiquitous general purpose MVA methods.

Extra

Resources

- PhyStat code repository
  https://plone4.fnal.gov:4430/P0/phystat/
- PhyStat 2007 conference
  http://phystat-lhc.web.cern.ch/phystat-lhc/
- Jim Linnemann's collection of statistics links:
  http://www.pa.msu.edu/people/linnemann/stat_resources.html
- Statistical analysis tool R
  http://www.r-project.org/
- TMVA (multivariate analysis tools in root)
  http://tmva.sourceforge.net/
- Neural Networks in Hardware
  http://neuralnets.web.cern.ch/NeuralNets/nnwInHep.html
- Boosted Decision Trees in MiniBoone
  http://arxiv.org/abs/physics/0508045
- Decision Tree Introduction
  http://www.statsoft.com/textbook/stcart.html
- GLAST Decision Trees
  http://scipp.ucsc.edu/~atwood/Talks%20Given/CPAforGLAST.ppt

Summary


1
0
Background efficiency
0
1
Signal efficiency
Random guess
Cut-based or likelihood
Neural networks, simple decision trees, etc
Boosted decision trees, bayesian neural networks, randomforests