PY212 Lecture 25

Interference and Diffraction

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12/3/09

Fun Link: Diffraction with Ace Ventura

Summary from last time

- The wave theory of light is strengthened by the interference and diffraction of light.
- Young's double-slit experiment demonstrated interference.
- In the double-slit experiment, constructive interference occurs when

$$d \sin\theta = m\lambda, \qquad m = 0, 1, 2, \cdots. \qquad \begin{bmatrix} \text{constructive} \\ \text{interference} \\ \text{(bright)} \end{bmatrix}$$

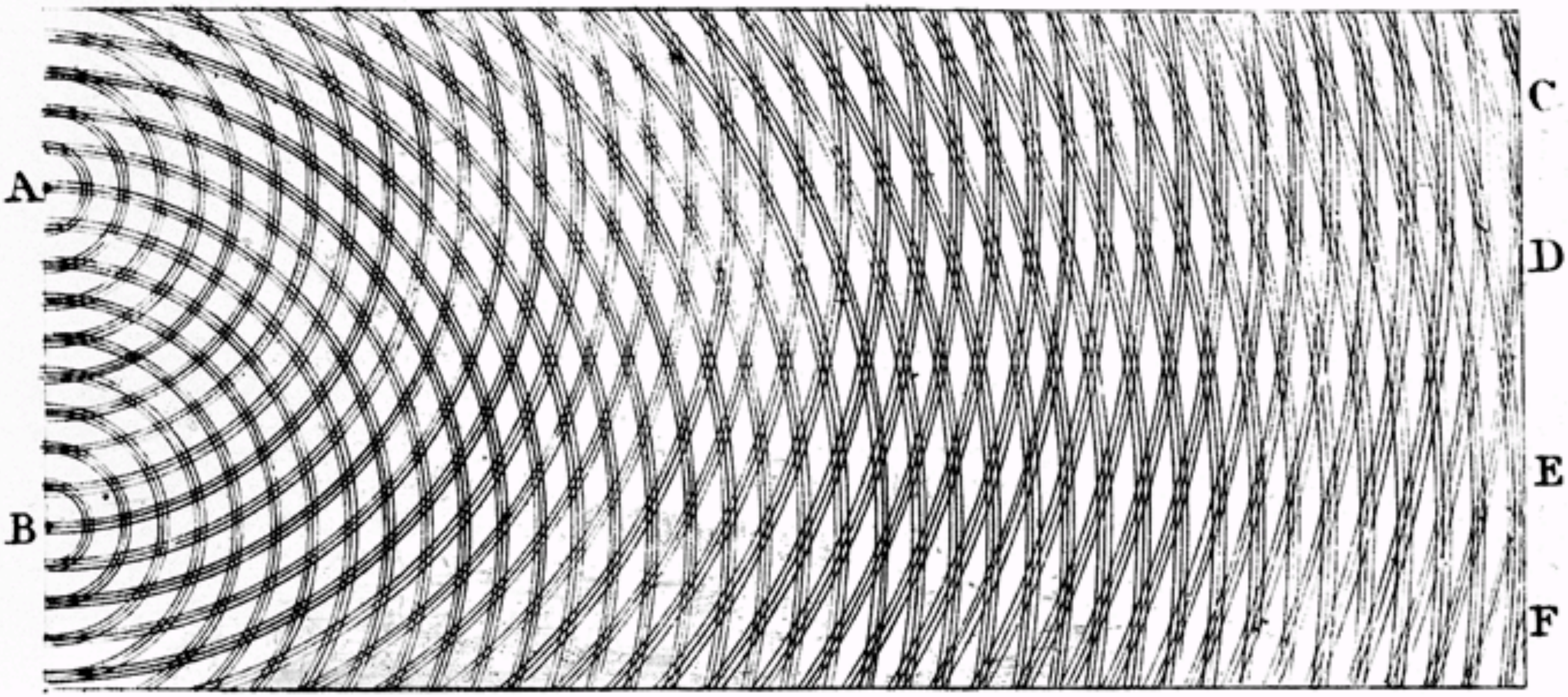
- and destructive interference when

$$d \sin\theta = (m + \tfrac{1}{2})\lambda, \qquad m = 0, 1, 2, \cdots. \qquad \begin{bmatrix} \text{destructive} \\ \text{interference} \\ \text{(dark)} \end{bmatrix}$$

- Two sources of light are coherent if they have the same frequency and maintain the same phase relationship.

Summary from last time

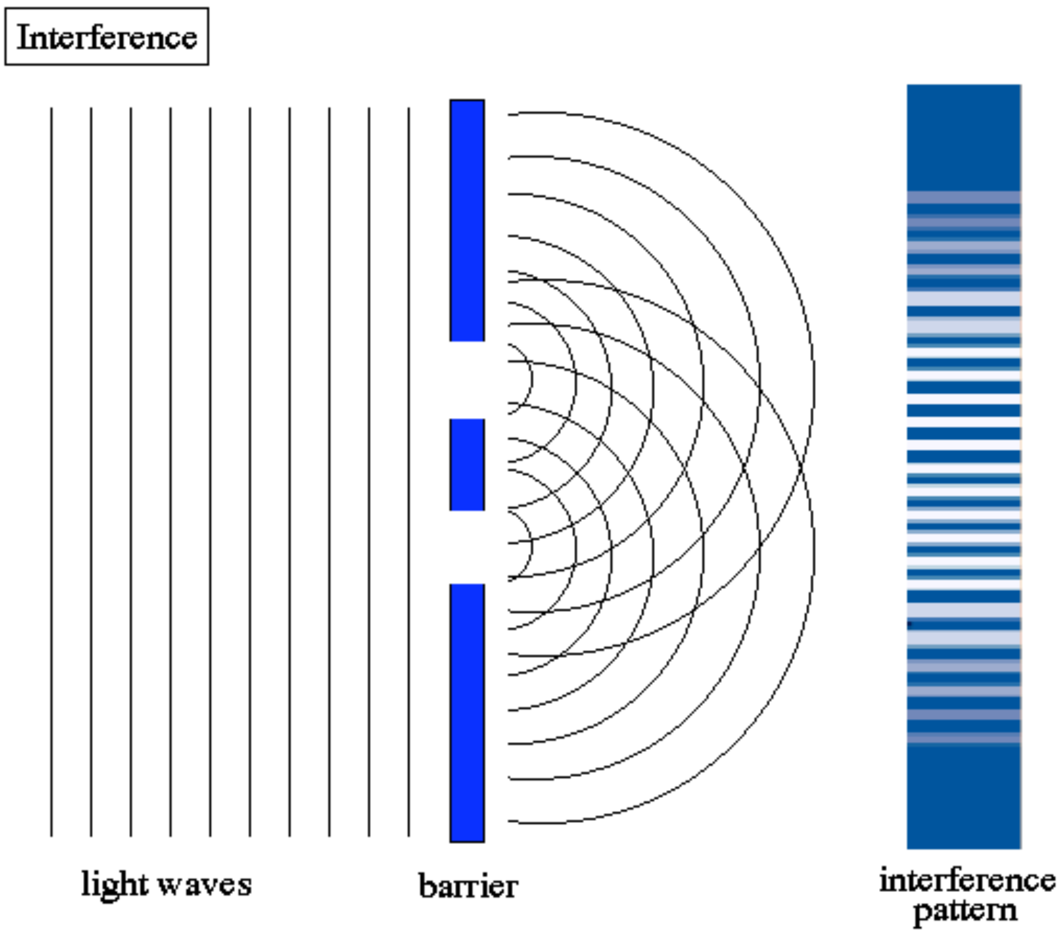
- Young realized that he could create two coherent sources of light waves that produce an interference pattern



Thomas Young's sketch of two-slit interference,

Summary from last time

- Young realized that he could create two coherent sources of light waves that produce an interference pattern



$$y_{con} = \frac{\lambda L}{d} m$$

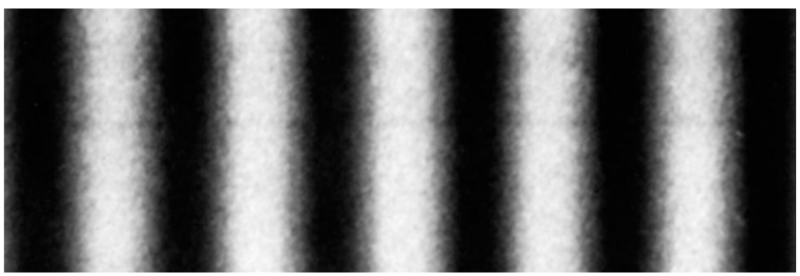
Summary from last time

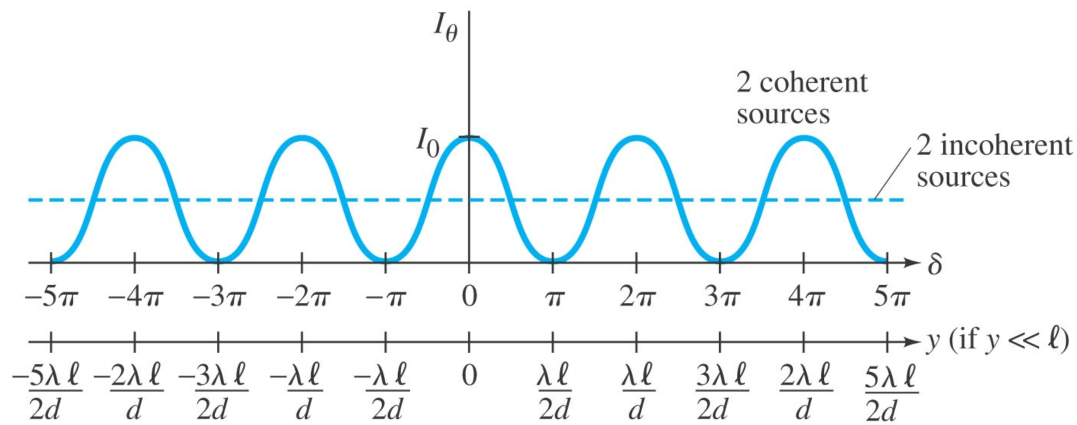
Intensity distribution of double-slit interference pattern

$$E_P = 2E_o \cos\left(\frac{\delta}{2}\right) \sin\left(\omega t + \frac{\delta}{2}\right)$$

$$I = I_{max} \cos^2 \frac{\delta}{2} = I_{max} \cos^2 \left(\frac{\pi d \sin\theta}{\lambda}\right)$$

$$= I_{max} \cos^2 \left(\frac{\pi d}{\lambda L} y\right)$$





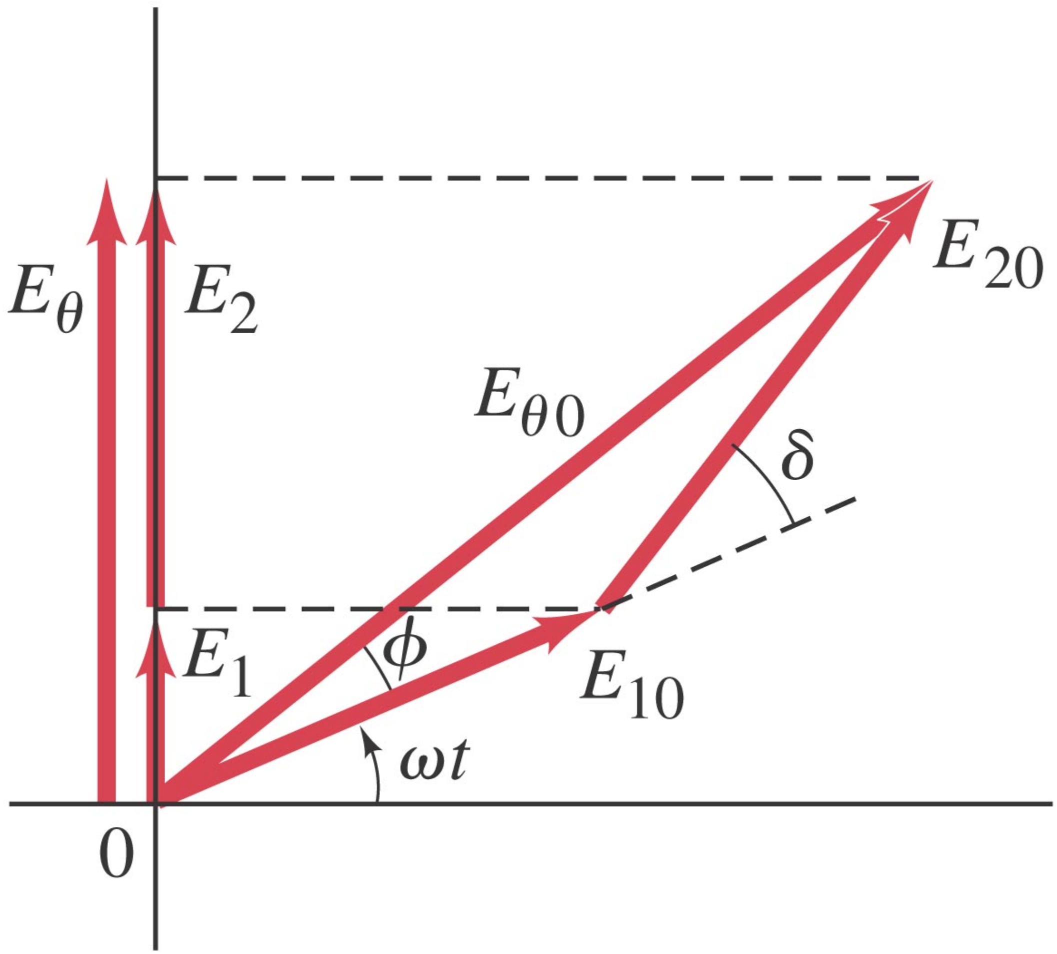
$$\delta = \frac{2\pi}{\lambda} d \sin\theta$$

Note this is not the full picture;
will study diffraction today….

Phasor Analysis

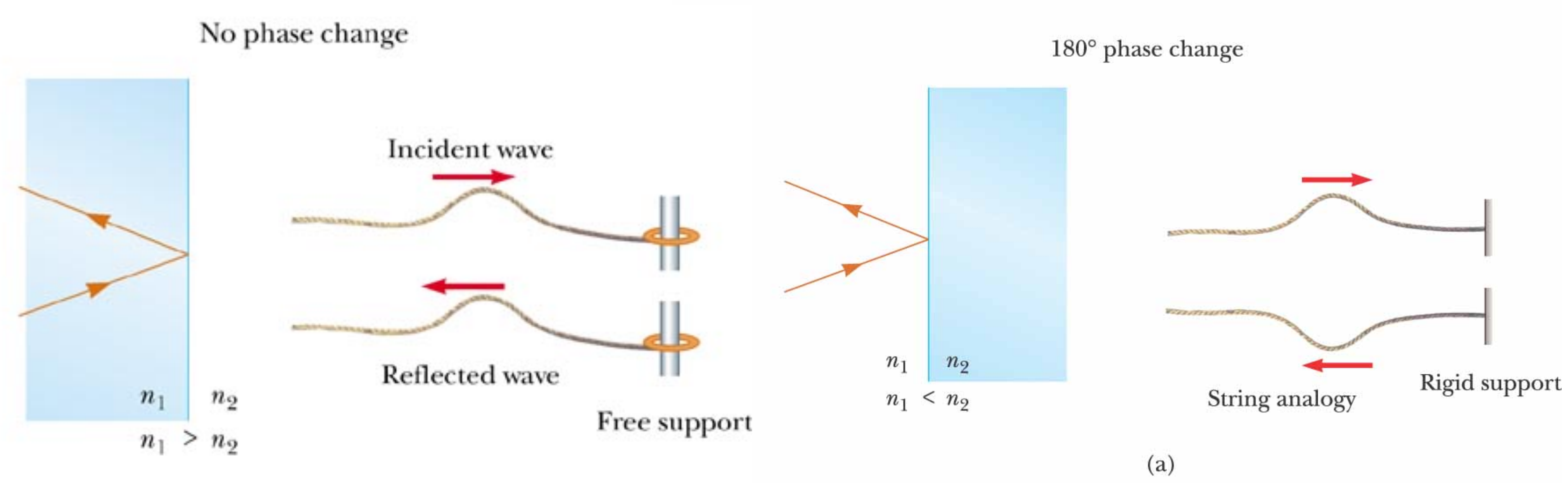
The two waves can be added using phasors, to take the phase difference into account:

$$E_\theta = 2E_0 \cos\frac{\delta}{2} \sin\left(\omega t + \frac{\delta}{2}\right).$$



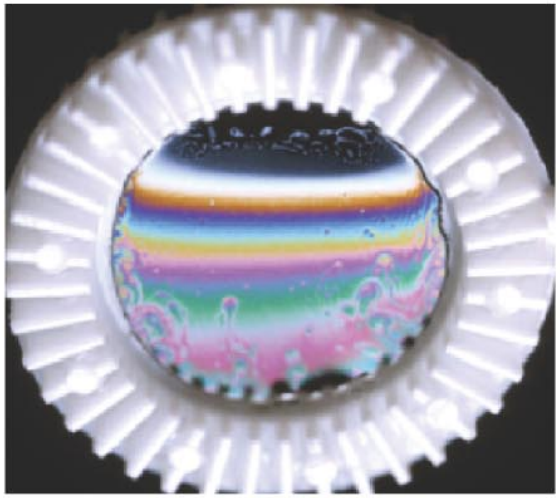
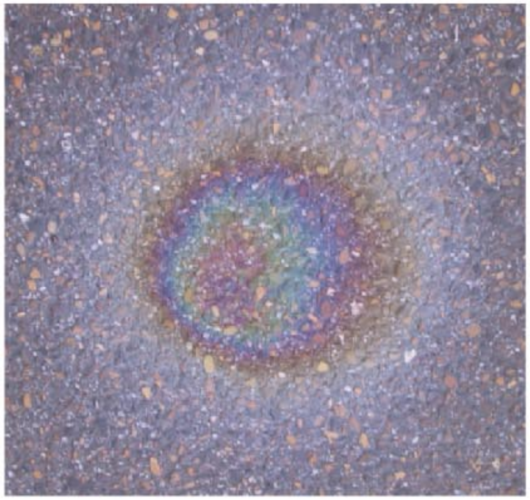
Summary from last time

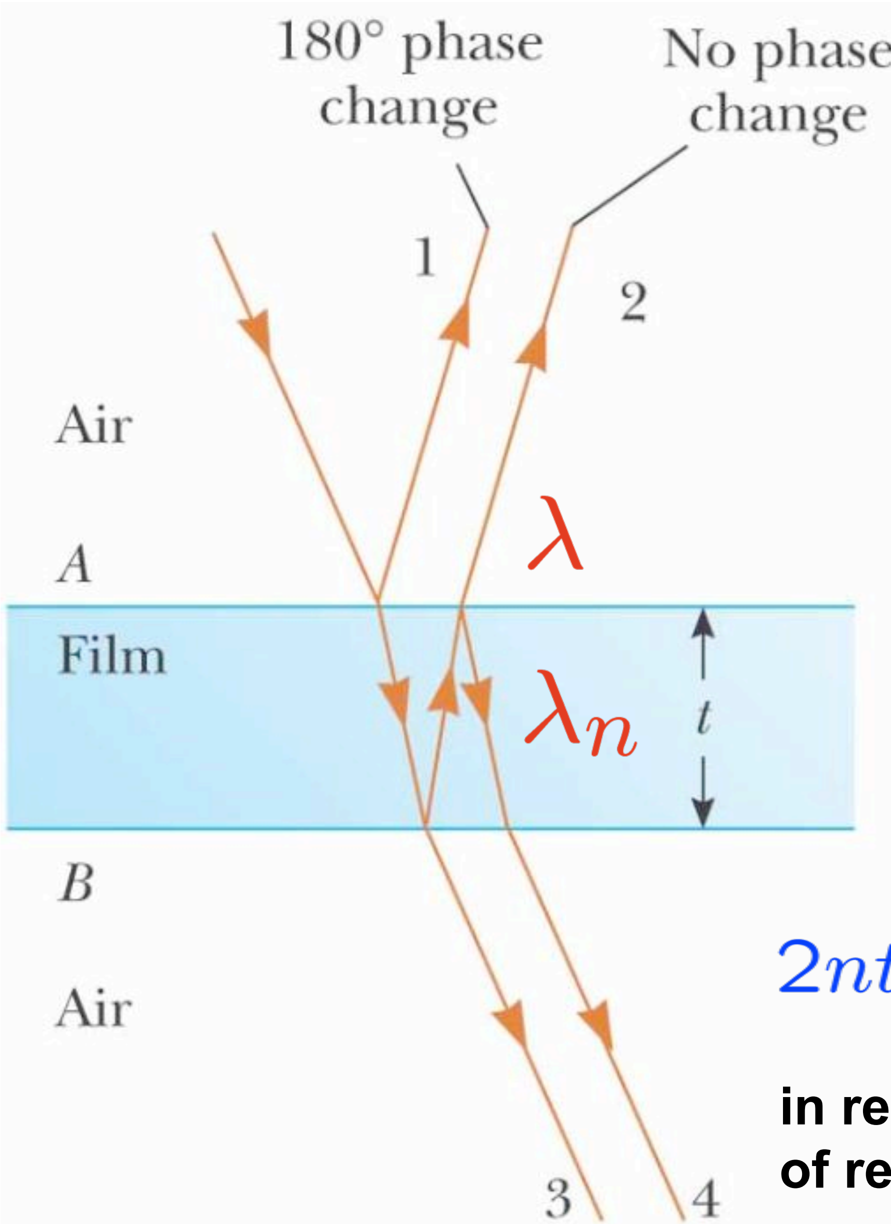
• Light undergoes a 180° phase change if it reflects from a medium of higher index of refraction.



Interference in thin films

- Interference effects are commonly observed in thin films
  - Examples include soap bubbles and oil on water
- The varied colors observed when white light is incident on such films result from the interference of waves reflected from the two surfaces of the film



$$\lambda_n = \frac{\lambda}{n}$$

constructive

$$2t = \left(m + \frac{1}{2}\right)\lambda_n$$

$$2nt = \left(m + \frac{1}{2}\right)\lambda$$

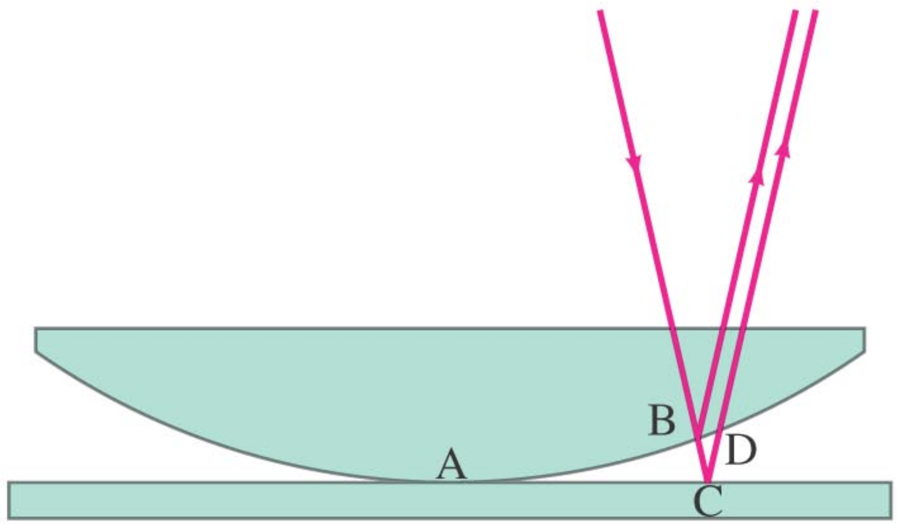
destructive

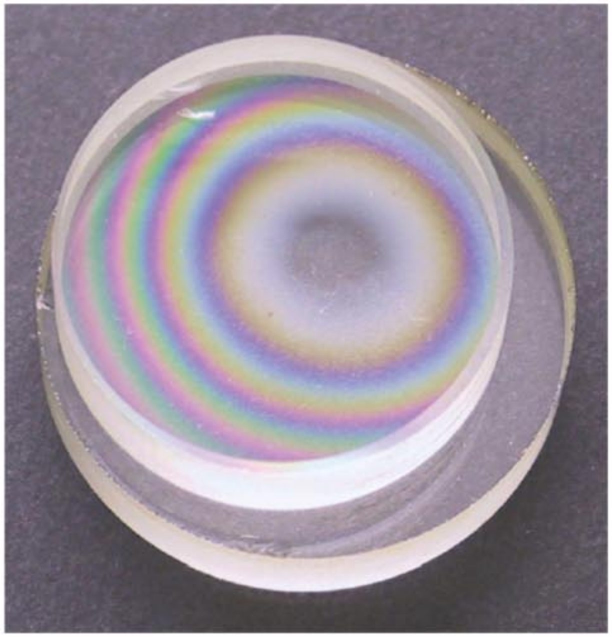
$$2nt = m\lambda \quad m = 0, 1, 2 \ldots$$

in reality there are an infinite number of reflections.

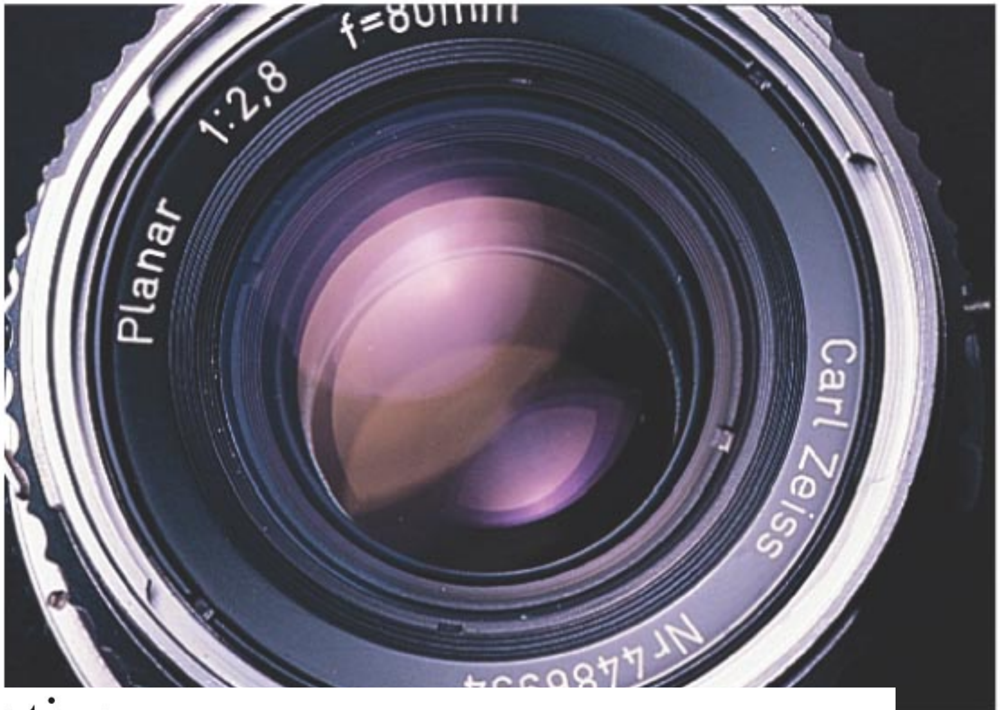
Interference in thin films

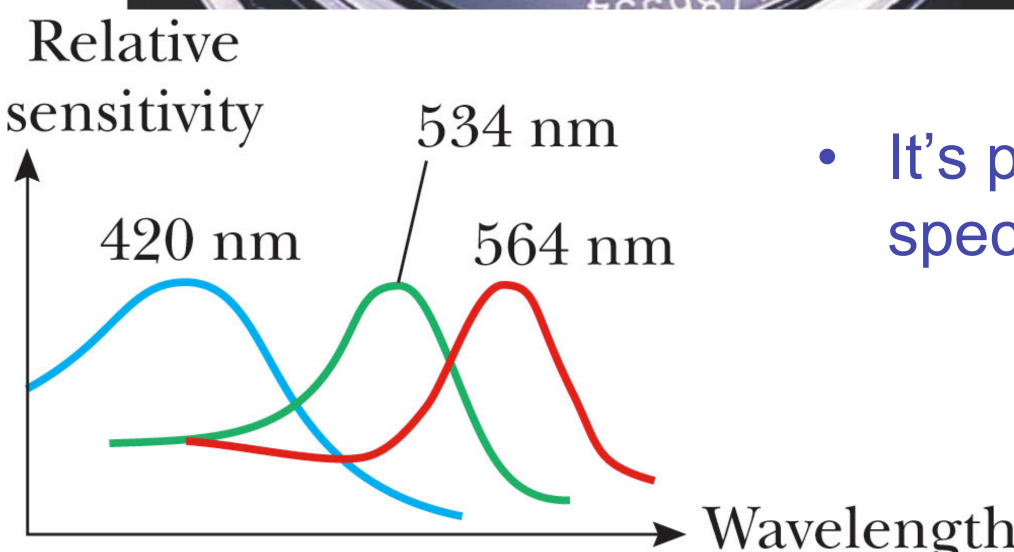
A similar effect takes place when a shallowly curved piece of glass is placed on a flat one. When viewed from above, concentric circles appear that are called Newton's rings.





Non-reflective coatings: work for a range of wavelengths

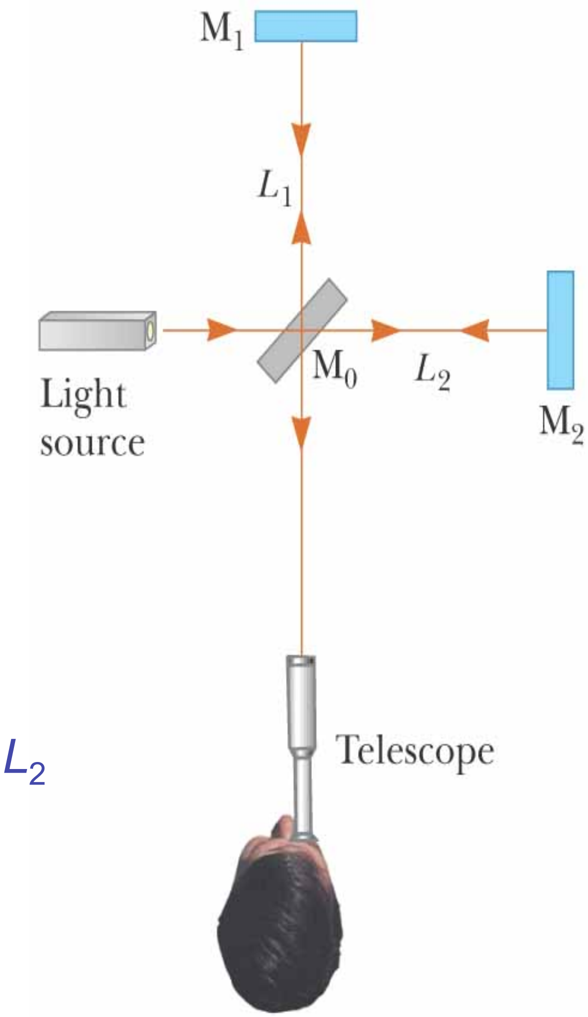




- It's purple because the visible spectrum is centered on green

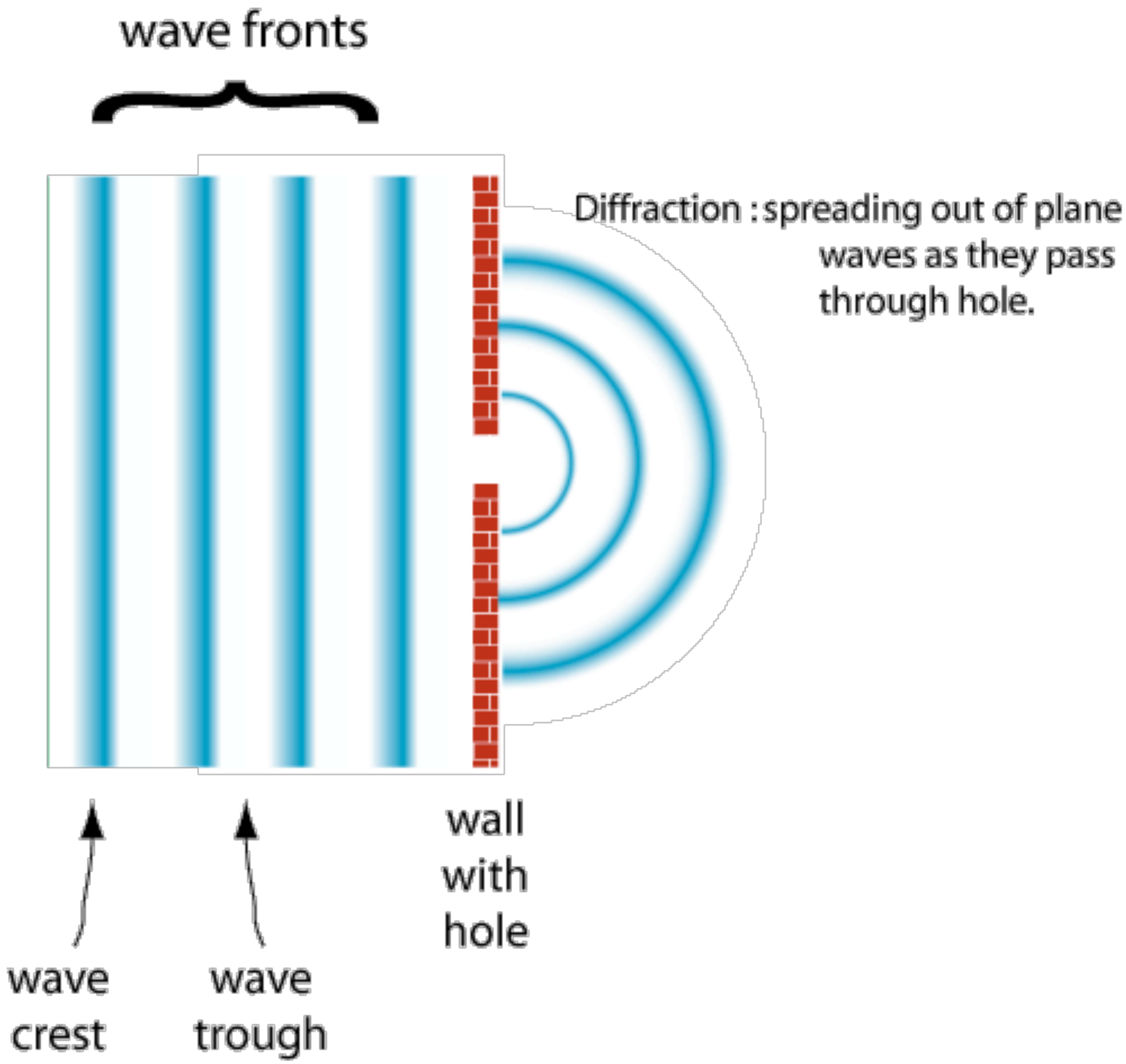
Michelson Interferometer, Schematic

- A ray of light is split into two rays by the mirror M_o
  - The mirror is at 45^o to the incident beam
  - The mirror is called a *beam splitter*
- It transmits half the light and reflects the rest



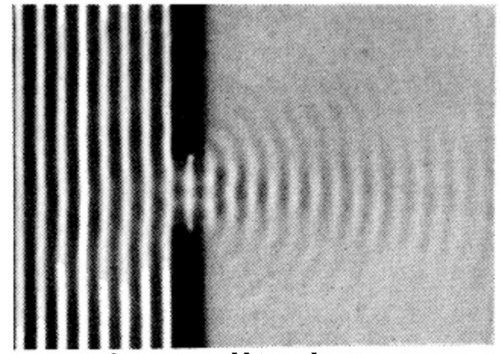
- The reflected ray goes toward mirror M_1
- The transmitted ray goes toward mirror M_2
- The two rays travel separate paths L_1 and L_2
- After reflecting from M_1 and M_2, the rays eventually recombine at M_o and form an interference pattern

Diffraction

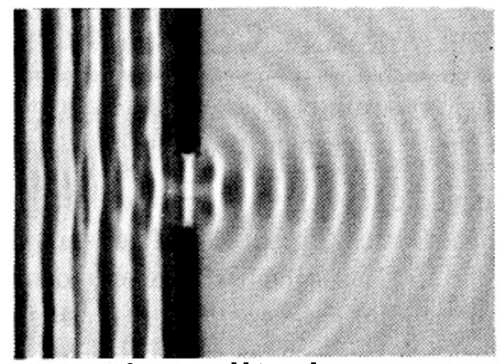


Diffraction of a wave by a slit

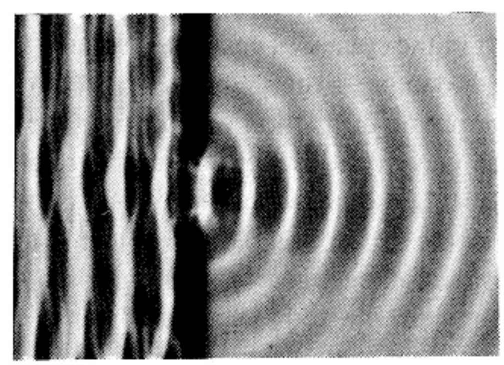
Whether waves in water or light in air, passage through a slit yields a diffraction pattern that will appear more dramatic as the size of the slit approaches the wavelength of the wave.



$\lambda \ll$ slit size



$\lambda <$ slit size

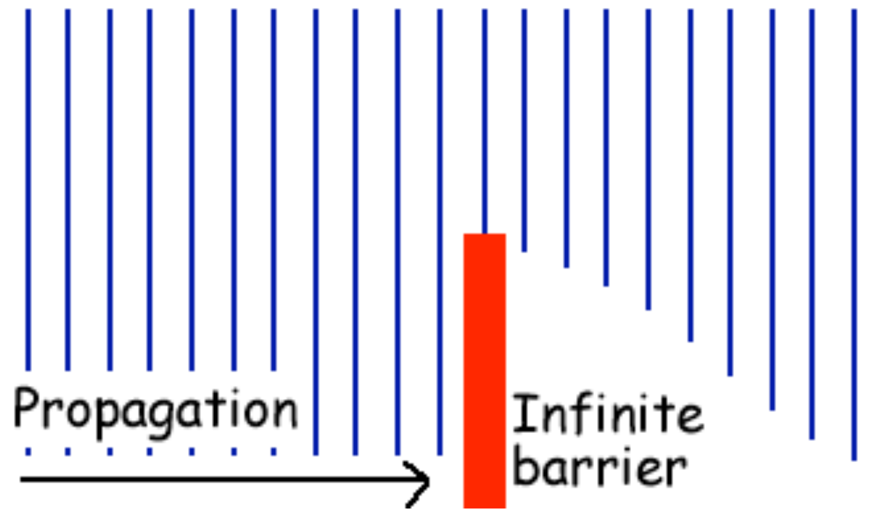


$\lambda \approx$ slit size

Diffraction by an Edge

Exaggerated view

Even without a smal slit, diffraction can be strong.

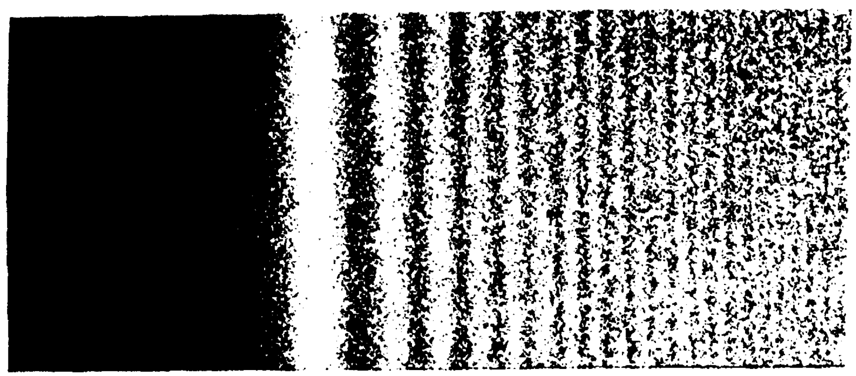


Simple propagation past an edge yields a diffraction pattern.

Knife-edge diffraction pattern

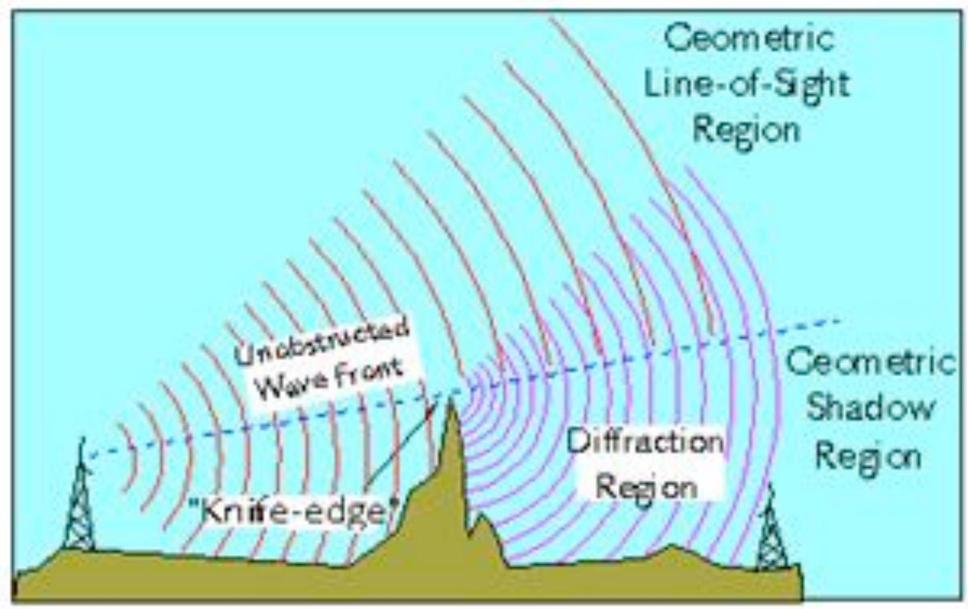
- If a wave passes over a sharp corner, there is a diffraction pattern, that changes the intensity around the region of the geometrical shadow.
- We can demonstrate that with the laser and a knife edge.
- The wave bends around the edge, and some intensity appears in the region where we would expect a geometric shadow.

Light passing by edge



Radio waves diffract around mountains

When the wavelength is km long, a mountain peak is a very sharp edge!



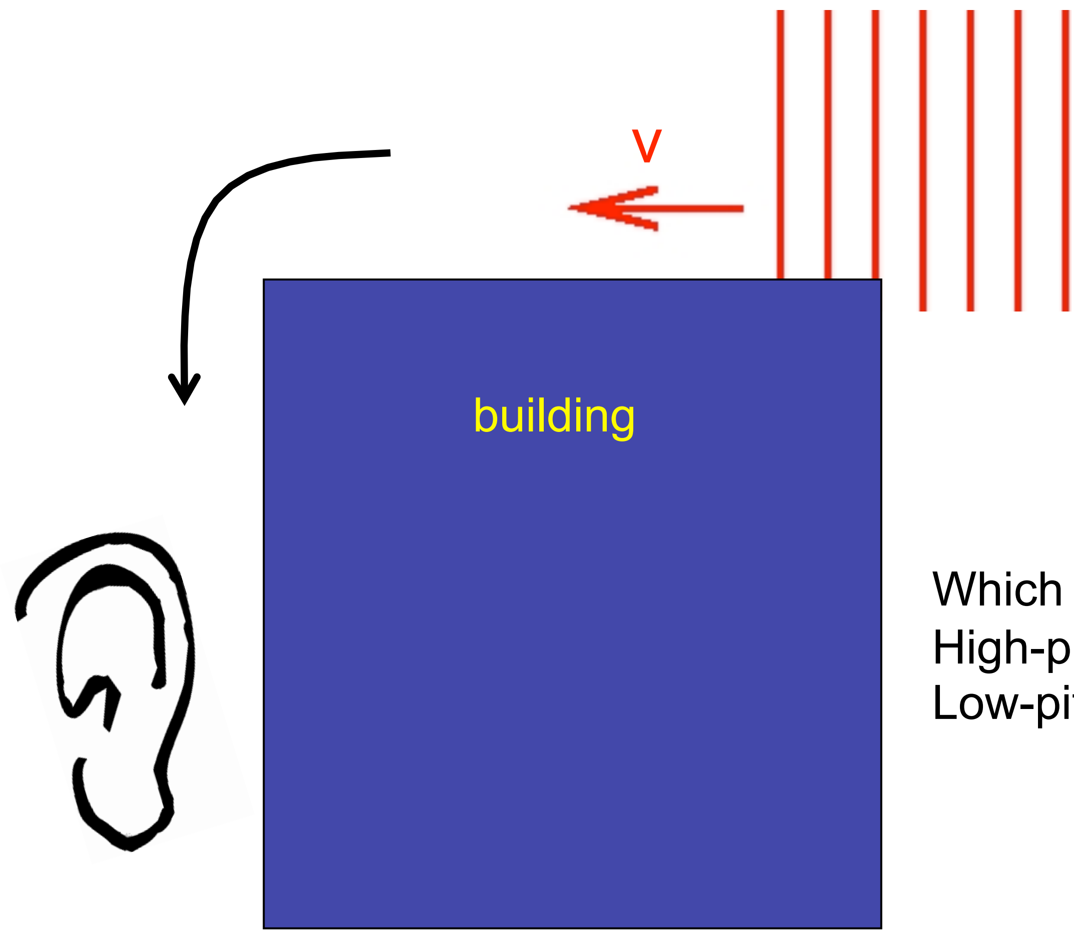
knife-edge effect is explained by Huygens' principle:

a well-defined obstruction to an electromagnetic wave acts as a secondary source, and creates a new wavefront.

This new wavefront propagates into the geometric shadow area of the obstacle.

Diffraction of sound around a sharp edge

- You may have noticed that if you are around the corner from a marching band, you can still hear some sound from it



Which instrument will you hear first ?
High-pitched guitar ?
Low-pitched bass drum ?

Diffraction of sound around a sharp edge

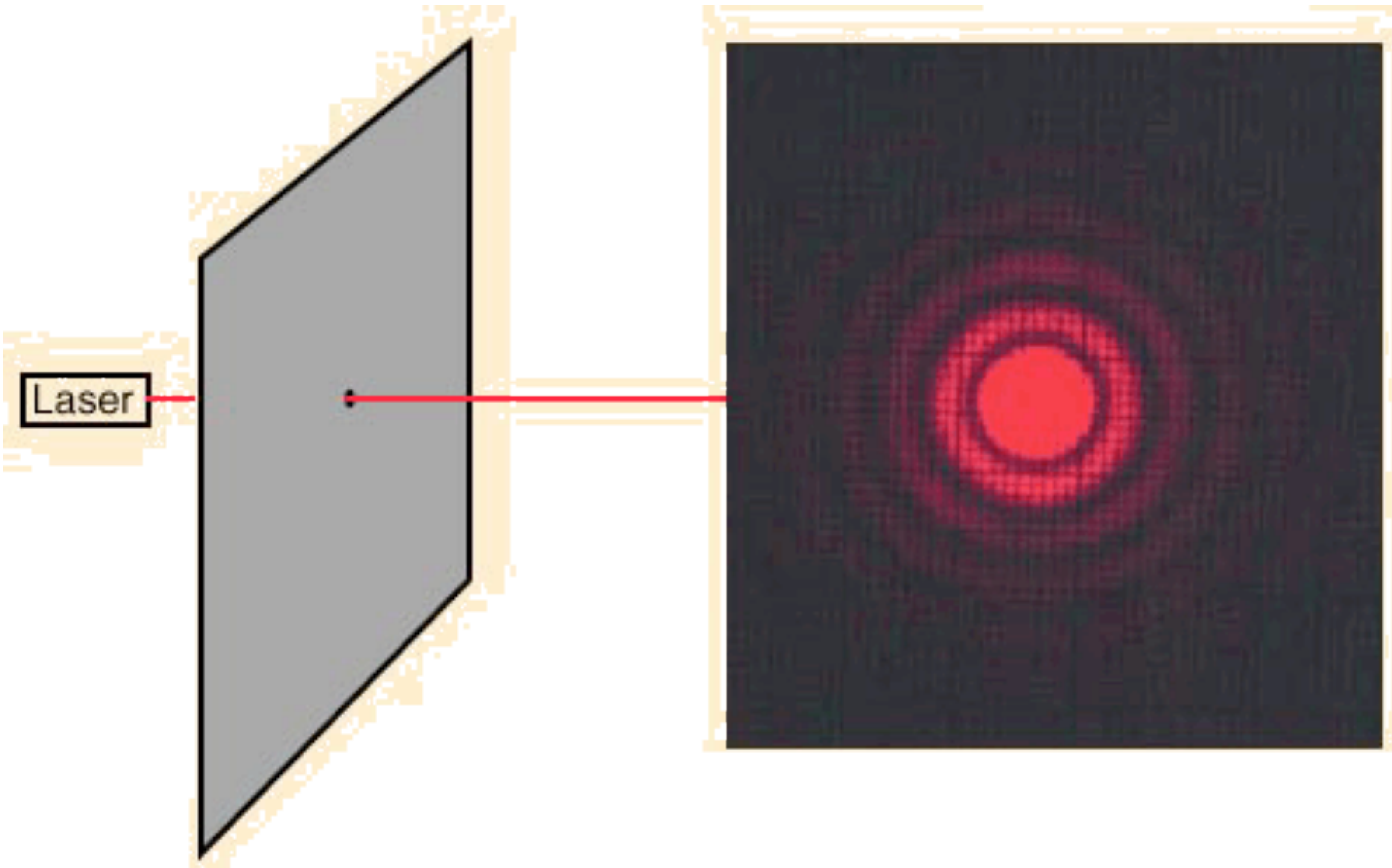
The long wavelength sounds of the bass drum will diffract around the corner more efficiently than the more directional, short wavelength sounds of the higher pitched instruments.

Introduction to diffraction patterns

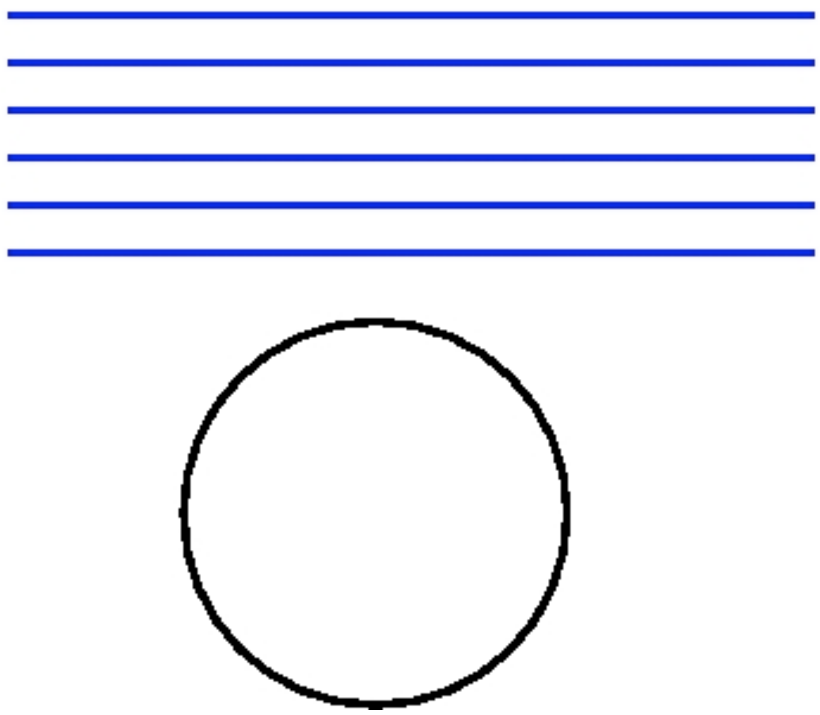
- Since a slit consists of two edges, we have to take this into account when discussing diffraction/interference from a slit, or set of slits.
- In fact, waves from different parts of the opening can also interfere, so the situation is even more complicated than just the slit edges.
- On the next few projections we show some diffraction patterns.

diffraction through a circular opening

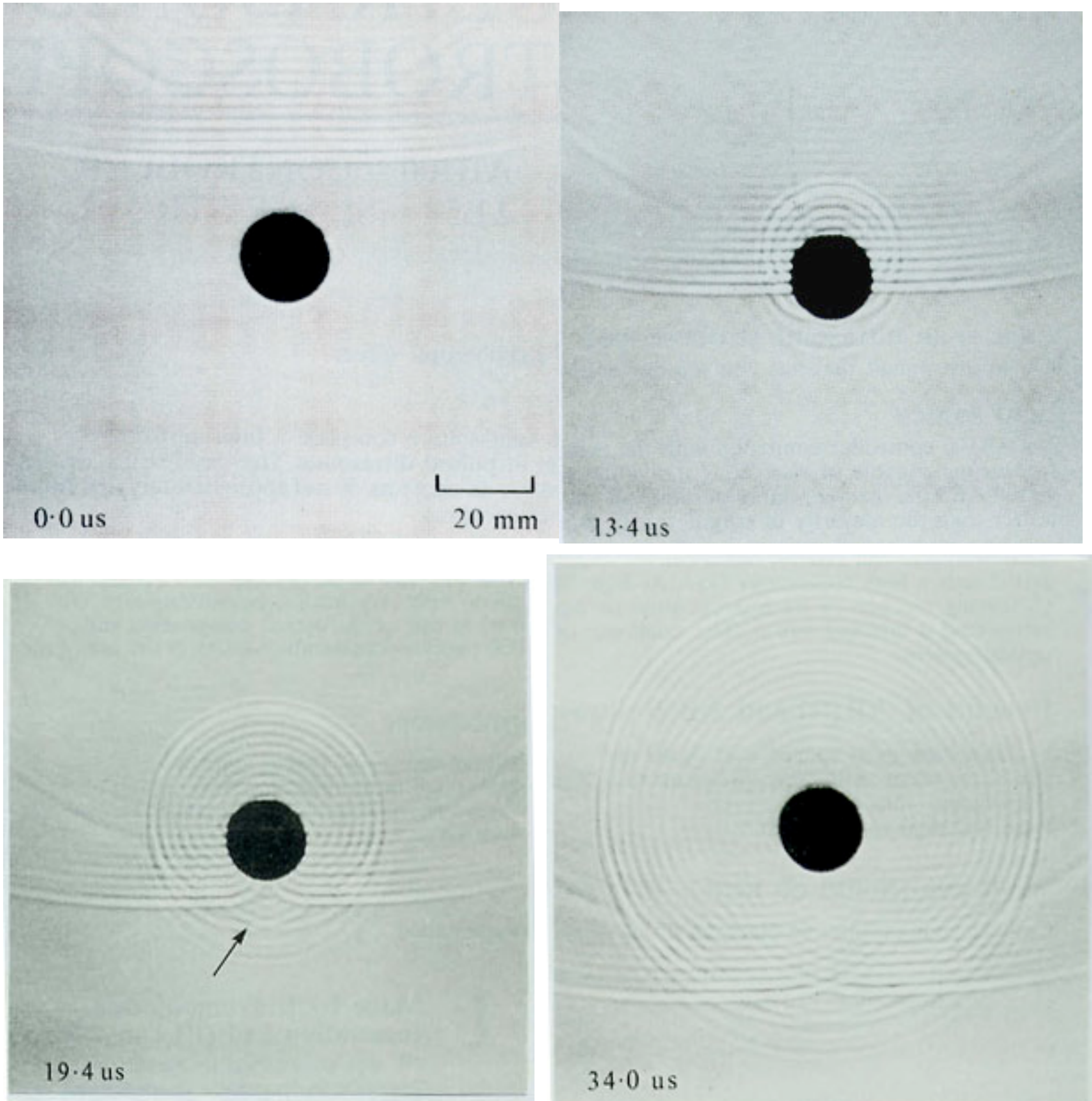
a laser beam passes through an opening and hits a screen



diffraction off an object



wave incident on a cylinder. the incident wave comes from above. Notice how the circular wave comes off of the cylinder asymmetrically since it strikes the top first

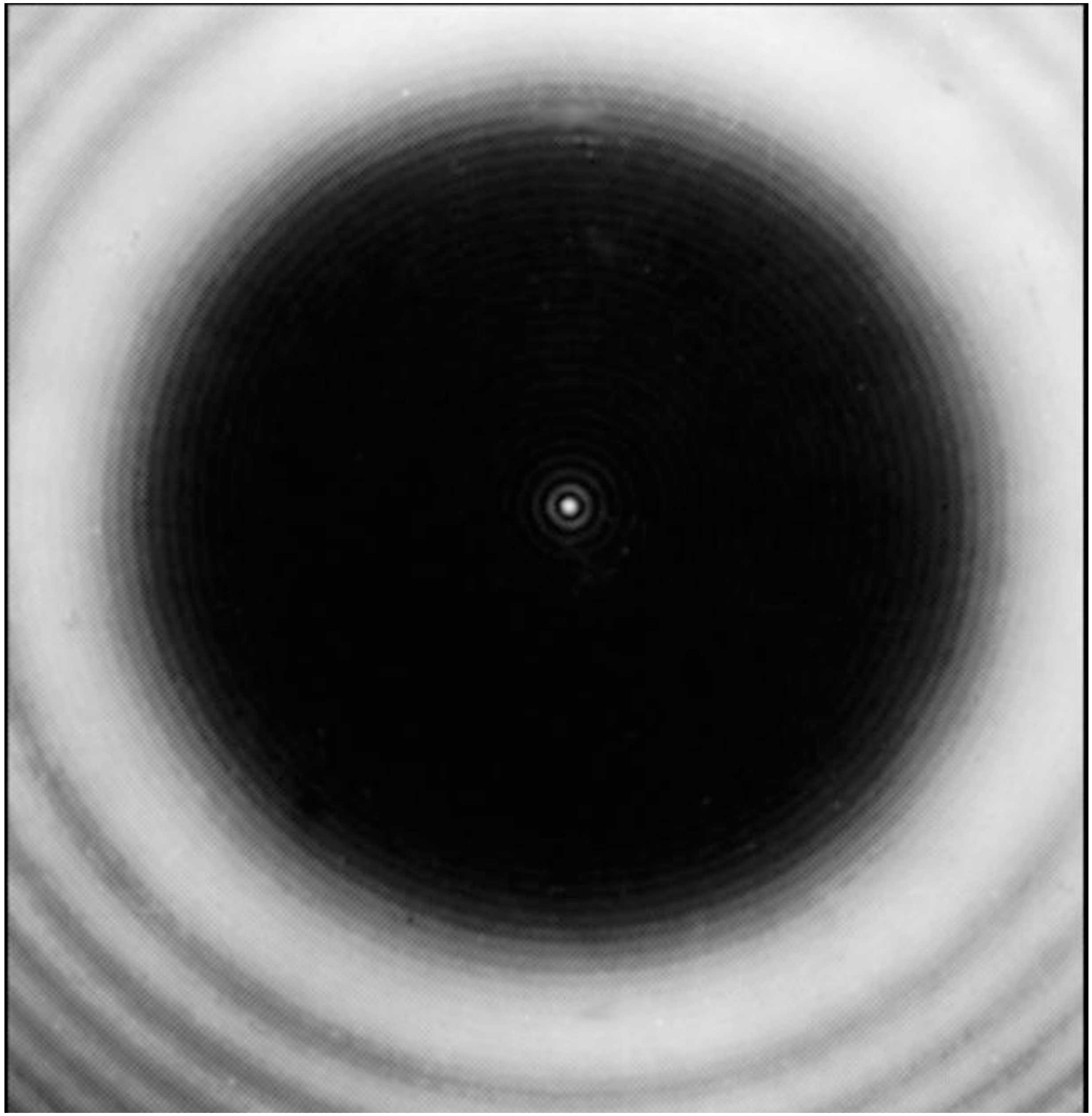


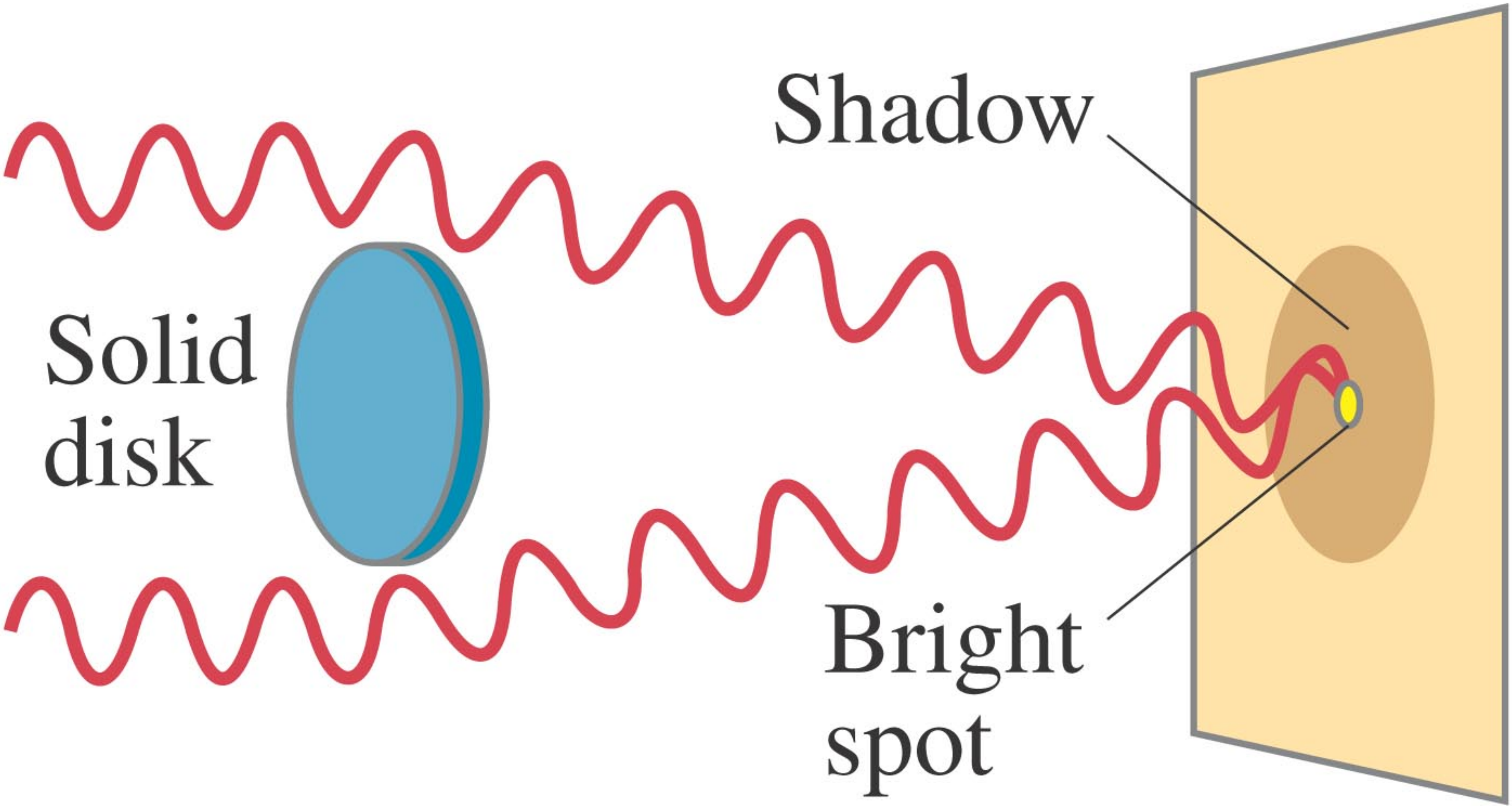
Note the interference between the incoming plane-wave and the outgoing circular wave

Diffraction pattern off of a penny

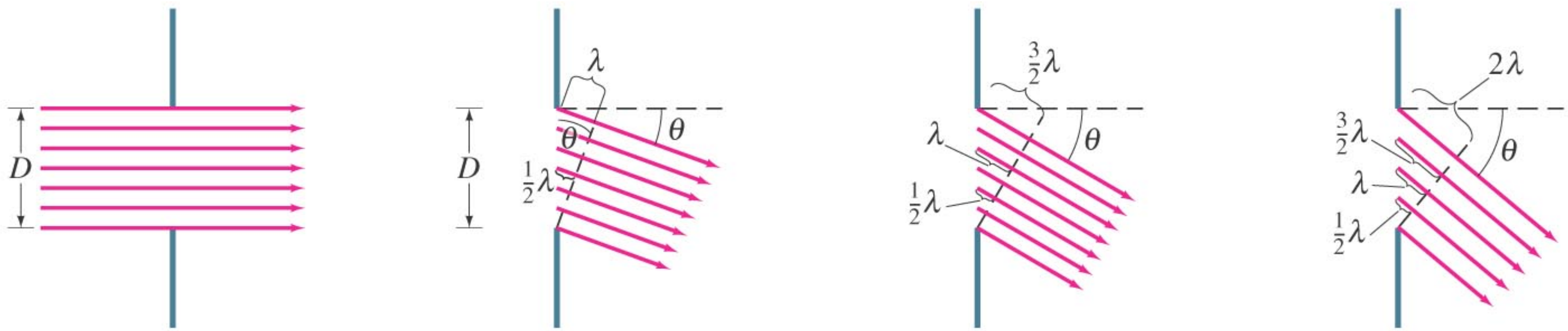
The penny is between the source and the screen.

Note the constructive interference at the center of the geometrical shadow region.




Shadow
Solid
disk
Bright
spot

Narrow slit: Fraunhofer diffraction pattern



(a) $\theta = 0$ Bright

(b) $\sin\theta = \frac{\lambda}{D}$ Dark

(c) $\sin\theta = \frac{3\lambda}{2D}$ Bright

(d) $\sin\theta = \frac{2\lambda}{D}$ Dark

- Each portion of the slit acts as a source of light waves!
  - these different sources can interfere since they are in phase and coherent
- (b) If we divide the opening into halves, then the wave from the top half can interfere with the bottom half.

$$\sin\theta = \frac{\lambda}{D}$$ [first minimum]

- In general the minima occur at

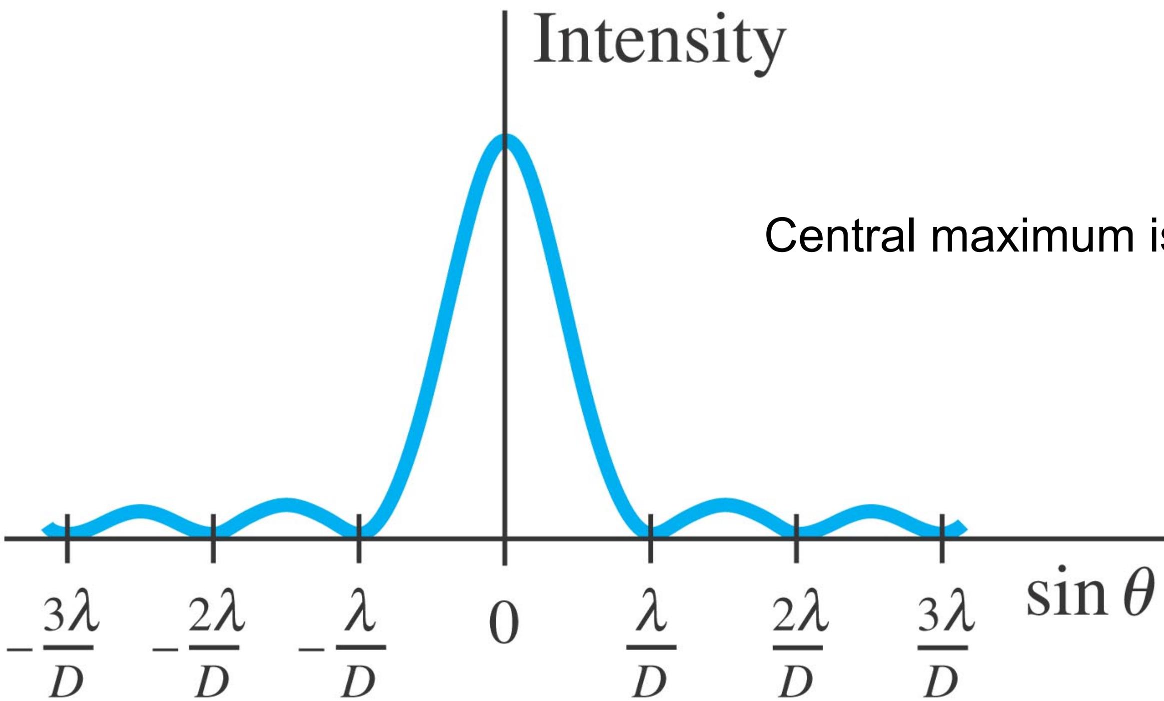
$$D\sin\theta = m\lambda, \qquad m = \pm 1, \pm 2, \pm 3, \cdots. \qquad \text{[minima]}$$

Diffraction pattern intensity

$$D \sin\theta = m\lambda, \qquad m = \pm 1, \pm 2, \pm 3, \cdots. \qquad \text{[minima]}$$

NOTE:

- Minima for a diffraction pattern satisfy a criterion that looks similar to that for the maxima for double-slit interference
- D is the slit width while d was the slit separation
- m=0 does not represent a dark fringe



Central maximum is twice as wide

Intensity

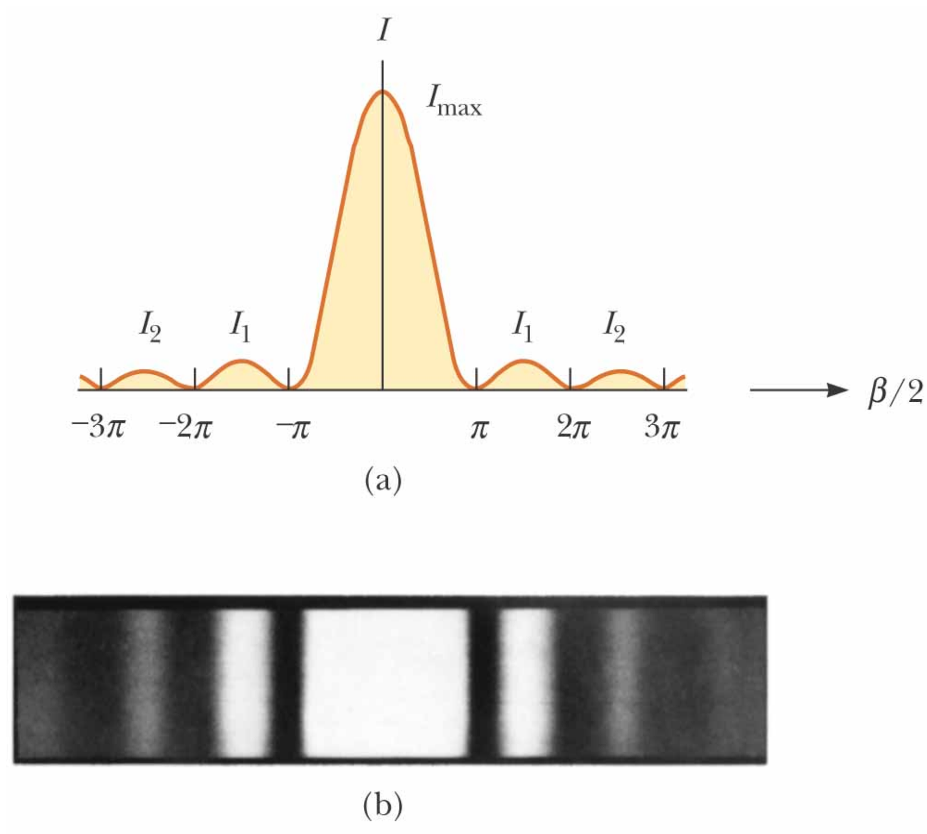
- Intensity as a function of angle θ

$$I_\theta = I_0\left(\frac{\sin\beta/2}{\beta/2}\right)^2.$$

where β is the phase difference:

$$\beta = \frac{2\pi}{\lambda} D\sin\theta.$$

- Most of the light intensity is concentrated in the central maximum
- The graph shows a plot of light intensity vs. $\beta/2$



Intensity of Two-Slit Diffraction Patterns

- When more than one slit is present, consideration must be made of
  - The diffraction patterns due to individual slits
  - The interference due to the wave coming from different slits

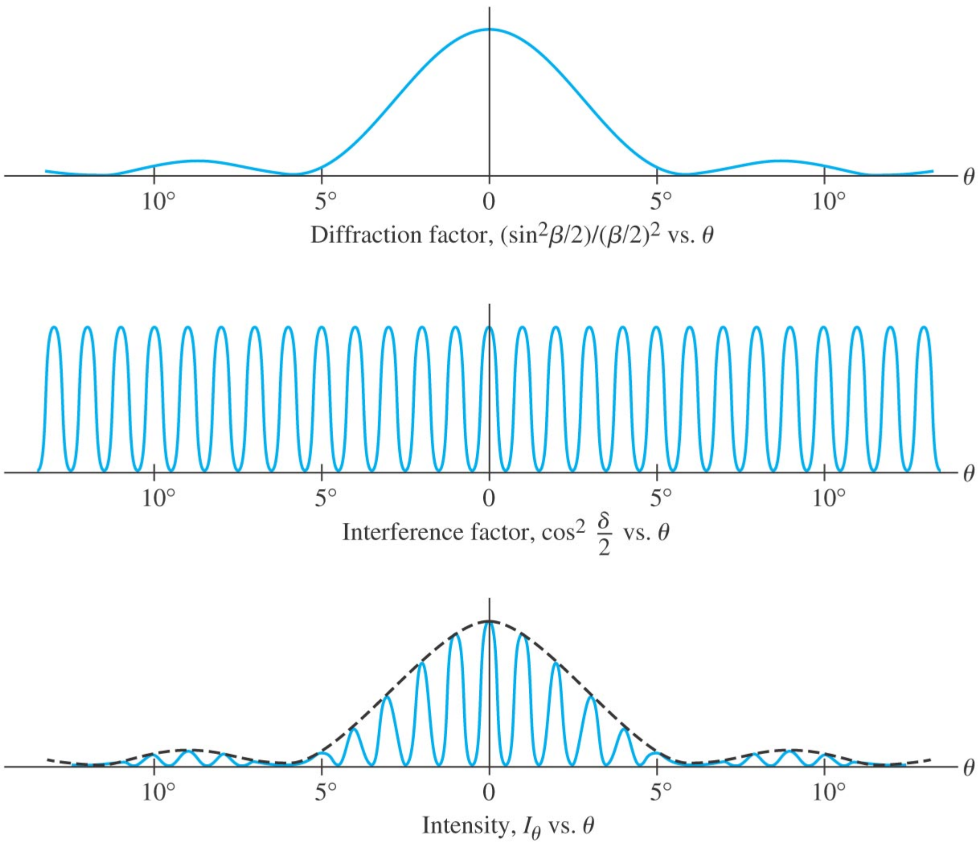
$$I_\theta = I_0 \left(\frac{\sin \beta/2}{\beta/2} \right)^2 \boxed{\left(\cos \frac{\delta}{2} \right)^2}$$

Interference term

$$\beta = \frac{2\pi}{\lambda} D \sin\theta. \qquad \delta = \frac{2\pi}{\lambda} d \sin\theta.$$

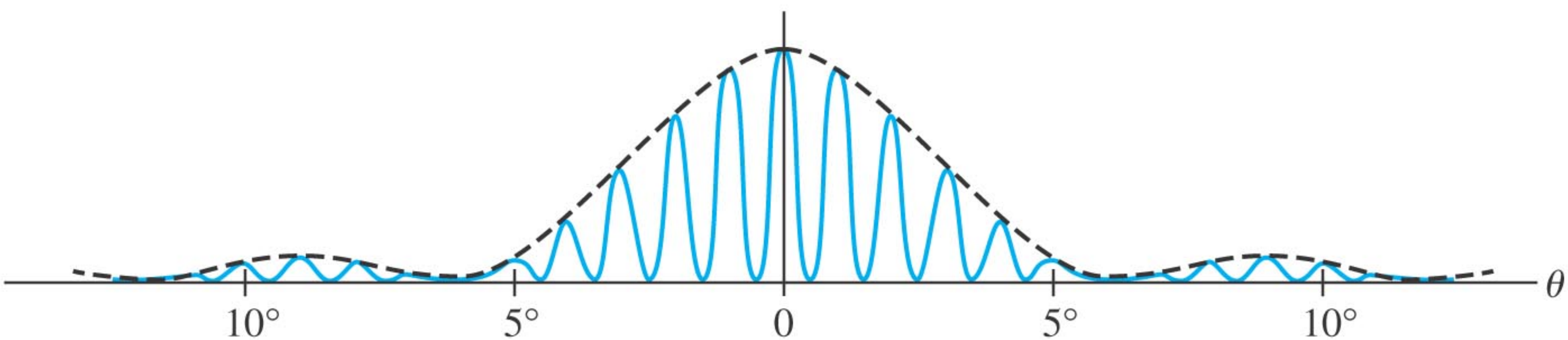
Intensity of Two-Slit Diffraction Patterns

The diffraction factor (depends on β) appears as an "envelope" modifying the more rapidly varying interference factor (depends on δ).



$$I_\theta = I_0\left(\frac{\sin\beta/2}{\beta/2}\right)^2\left(\cos\frac{\delta}{2}\right)^2$$

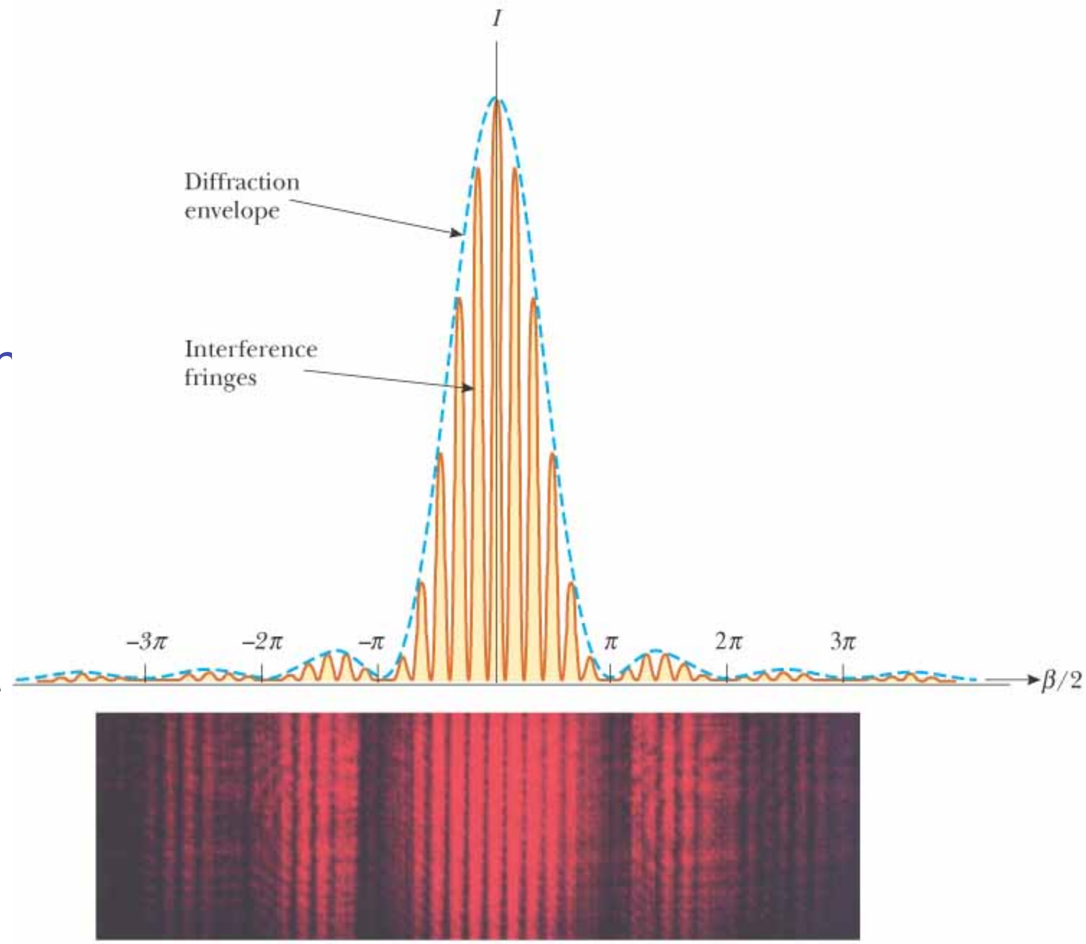
Why the central diffraction peak shown, plotted for the case where $d = 6D = 60\lambda$, contains 11 interference fringes ?



Intensity, I_θ vs. θ

Intensity of Two-Slit Diffraction Patterns, Graph of Pattern

- The broken blue line is the diffraction pattern
- The red-brown curve shows th cos² term
  - This term, by itself, would result in peaks with all the same heights
  - The uneven heights result from the diffraction term (square brackets in the equation)



single and double slit diffraction