PY581 - Advanced Laboratory

Exploring the Properties of p-doped Germanium with the Hall Effect

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October 29, 2012

Abstract

An effective observation of the Hall effect is made in p-doped Germanium. Measurements of the hall voltage, V_H, agree with the linearity of theory. A measurement of the hall coefficient is found to be $2.62 \times 10^{-3} C^{-1}$. From this the number density of charge carriers is calculated $N = 2.38 \times 10^{21}\ m^{-3}$. A measurement of the mean time between collisions gives $5.11 \times 10^{-14}\ sec$, and agrees to known values within %18. From this we approximate a mean path length of $\ell \simeq 0.24\ nm$.

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1 Introduction

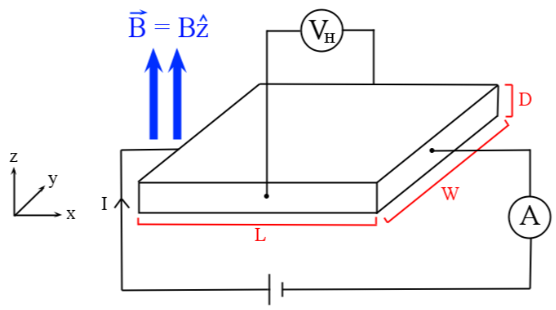


Figure 1: The basic Hall experiment set up

To describe the Hall effect classically, we first consider a conducting crystal structure of dimensions $L \times W \times D$, shown in Figure 1. The crystal is connected to a power supply providing a current I, such that charge carriers of charge q move in the $\hat{x}$ direction with a speed v_x. A magnetic field, $\vec{B}$, in the $\hat{z}$ direction will induce a Lorentz force,

$$\vec{F} = qv_x\hat{x} \times \vec{B} = -qv_xB\hat{y} \tag{1}$$

This force will cause charges to collect on the two faces of the crystal separated by W. When the system is in equilibrium, the force of the electric field, E_y, due to this charge accumulation will equal that due to the magnetic field such that $E_y = v_xB$. It follows that the voltage difference between these faces (the Hall Voltage), V_H, can be written: $V_H = -v_xBW$.

With the number density of charge carriers, N, we can consider the current density,

$$J_x = Nqv_x = \frac{I}{DW} \qquad \Rightarrow \qquad v_x = \frac{I}{NqDW} \tag{2}$$

so we can rewrite the Hall Voltage:

$$V_H = -\frac{BI}{NqD} = -\frac{BIR_H}{D} \tag{3}$$

where we define the Hall coefficient, $R_H \equiv (Nq)^{-1}$. As I, D, B are known, the sign of the V_H will tell us the sign of the charge carriers, q, and the magnitude of V_H will tell us the number density of charge carriers, N.

Some other interesting properties of materials can be probed if we consider the Drude model of conduction[1]. The acceleration of charge carriers in a material can be approximated dividing the drift velocity - we've been using v_x - by the mean free time between collisions, τ. For a charge carrier of mass m, Newton's second law tells us:

$$F_x = ma_x = m\frac{v_x}{\tau} = qE_x \tag{4}$$

From Ohm's law, $J_x = Nqv_x = \sigma E_x$ where σ is the conductivity of the metal and can be found given a measured resistance, R:

$$\sigma \equiv \frac{LR}{WD} = \frac{Nq^2\tau}{m} \tag{5}$$

Noting a measured value for $R_H = (Nq)^{-1}$, we can easily determine:

$$\tau = \frac{m\sigma R_H}{q} \tag{6}$$

Now knowing τ, we can approximate the mean length between collisions, ℓ:

$$\ell \simeq v_{th}\tau \tag{7}$$

where v_{th} is the thermal velocity of conduction electrons and can be approximated by the fermi velocity, v_F given by:

$$v_F = \frac{\hbar}{m}(3\pi^2 N)^{\frac{1}{3}} = \frac{\hbar}{m}\left(\frac{3\pi^2}{qR_H}\right)^{\frac{1}{3}} \tag{8}$$

$$\therefore \quad \ell \simeq \frac{\hbar}{m}(3\pi^2 N)^{\frac{1}{3}}\tau = \frac{\hbar}{m}\left(\frac{3\pi^2}{qR_H}\right)^{\frac{1}{3}}\frac{m\sigma R_H}{q} \tag{9}$$

2 Description of Apparatus

A board containing a $6mm \times 4mm \times 0.5mm$ P-Doped Germanium crystal is connected to a Hewlett Packard 6214A power supply that provides a current across the crystal. The current and voltage provided by the power supply is measured by a Keithley 177 MicroVolt Digital Multimeter and a Keithly 181 nanoVoltMeter respectively. The induced hall voltage is measured by a Keithly 181 nanoVoltMeter. A schematic of the basic I/O of the crystal is detailed in Figure 2. The crystal is subjected to a magnetic field of a 2 x 400 turn U-shaped electromagnet powered by a Kenwood PR36-3A DC Power Supply. The field strength for a given supply current is measured by a Cenco Digital Gauss Meter. The polarity of the field can be verified using an AlphaLab GM-1-ST DC Gaussmeter[2]. A labeled photograph of the testbench is provided in Figure 3.

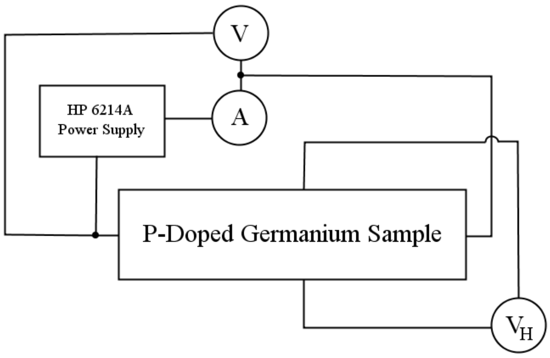


Figure 2: Image of the B4C detector, contained within a Faraday cage.

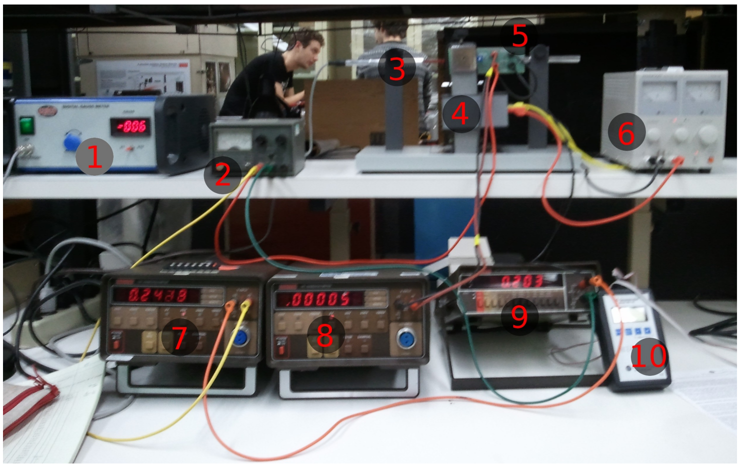


Figure 3: The Hall effect testbench: 1. Cenco Digital Gauss Meter. 2. HP Power supply for current through crystal. 3. Hall probe for Cenco Gauss Meter. 4. Electromagnet. 5. Board containing P-Doped germanium crystal. 6. Power Supply to Electromagnet. 7. Multimeter reading voltage from (2). 8. Multimeter reading hall voltage, V_H. 9. Multimeter reading current from (2). 10. Gauss meter for determining magnetic polarity

3 Safety

There are relatively few safety risks present in measuring the Hall effect. The power supply to the crystal sample provides current on the order of milliamps, and therefore poses little risk to humans. The power supply for the electromagnet, however, can provide up to 2 amps of current and could pose a health risk. It only takes about 10 miliamps across the heart to cause fibrillation, and this could be easily achieved by allowing a path to ground that passes across the chest. Special care should therefore be taken to make sure that the power supply is in good working condition, to avoid exposure to users. All connecting cables should be undamaged and in full working order. Users of the equipment should avoid jewelry that could potentially contact circuitry and should wear rubber bottomed shoes to prevent current running through a user and to ground. Before moving or modifying equipment, residual charges should be discharged by touching contacts with a safe grounding probe.

Magnetic field can also have a nontrivial effect on the body. This experiment makes an electromagnet capable of providing a .2 T DC magnetic field. DC magnetic fields are considerably safer than AC fields. The DC field exposure limit for workers is 2 T, so we are within the safe range. However, persons with artificial pacemakers should be nonetheless wary because magnetic fields as small as 5 Gauss (5 x 10^{-4} T) can have an adverse effect and influence the normal operation of the pacemaker.

The final safety measures should be noted not for their effect on humans, but to avoid damage to the hardware used in this experiment. Users should for no reason exceed the maximum allowed supply currents to the following hardware:

Equipment	Maximum Current (A)
Power supply to Ge Crystal	5 x 10^{-3}
Power supply to Electromagnet	3.5

4 Data Acquisition

A first measurement of voltage across the crystal as a function of current is taken with zero magnetic field to determine the resistance as a function of current. Measurements of V_H are later made by subjecting the crystal to a magnetic field. V_H is measured by first holding the current constant and varying the magnetic field from 0 to -1500 Gauss. This is done three times for currents of 1, 2, and 3 mA. Measurements are also made by holding the magnetic field fixed and varying the current from 0 to 3.6 mA. This is again done three times for B values of -500, -1000 and -1500 Gauss. Data is collected by hand and digitized for post processing in ROOT.

5 Data Analysis

We first examine the resistance behavior as a function of input current with zero magnetic field applied. A plot of this behavior can be seen in Figure 4.

The decrease in resistance for currents above 1 mA is likely due to heating. For higher currents, the crystal warms, and the extra thermal energy allows more charge carriers to move into the condution band, reducing the resistance of the sample.

We next verify the linearity of the relationship between the hall voltage V_H, the current and the magnetic field strength. Figure 5 confirms equation 3.

Rearranging equation 3. slightly, we can easily determine the hall coefficient, R_H given specific values of the current and the hall voltage. Considering these values for all six runs (3 x I_{fixed} and 3 x B_{fixed}), we obtain the distribution shown in 6a.

From equation 6, we can calculate the mean free time between collisions, τ. The distribution of values for each datapoint is shown in Figure 6b.

This data allows us to determine a number of properties of our P-Doped Germanium sample. Our mean value for $R_H = 2.62 \pm 0.015 \times 10^{-3}$ C^{-1} first tells us that the sign of our charge carriers is positive. Assuming that the magnitude of a charge carriers charge is $e = 1.6 \times 10^{-19} C$, we can determine the number of charge carriers per unit volume, $N = 2.38 \times 10^{21}\ m^{-3}$. Given $N = 2.38 \times 10^{21}\ m^{-3}$ and $\tau = 5.11 \times 10^{-14} sec$, we calculate the mean path length, ℓ, from equation 9. So we measure $\ell \simeq 0.24\ nm$.

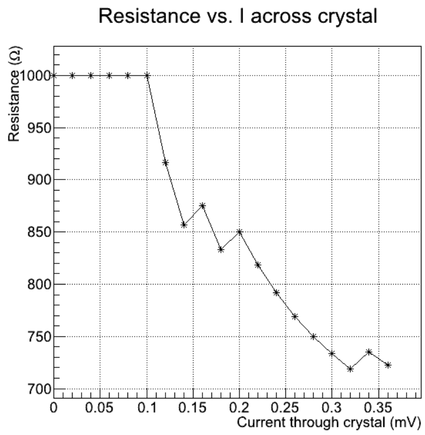
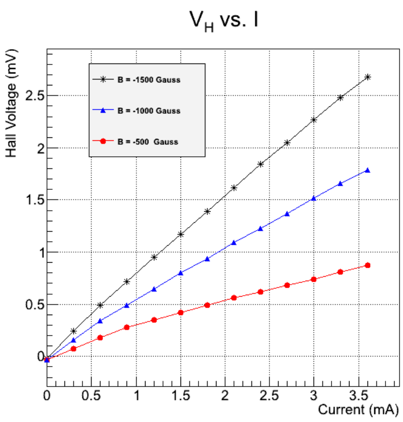
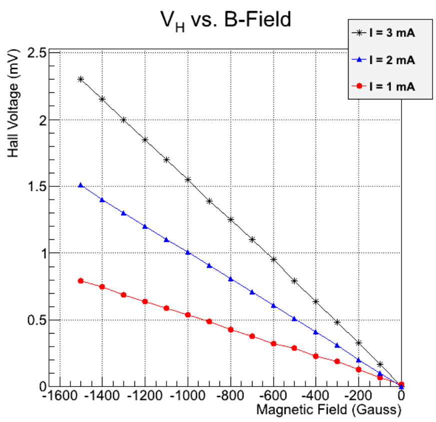


Figure 4: Resistance as a function of input current

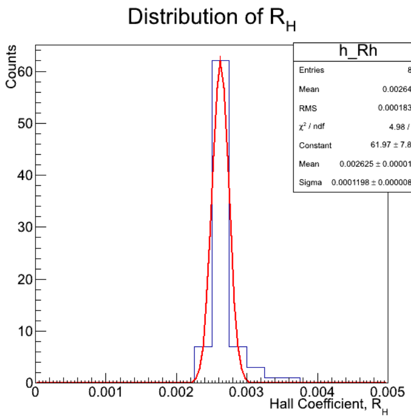


(a) Hall Voltage as a function of input current. Runs are shown for three different values of field strength

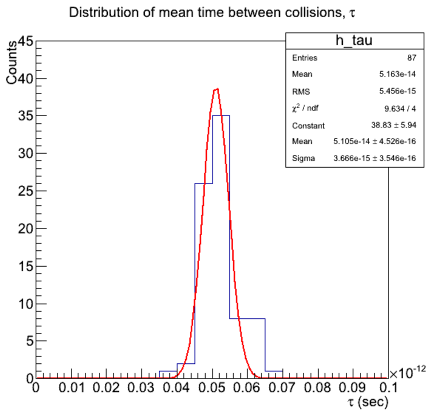


(b) Hall Voltage as a function of applied magnetic field. Runs are shown for three different input currents.

Figure 5



(a) Distribution of values for the hall coefficient.



(b) Distribution of mean time between collisions, τ

Figure 6

6 Conclusions

It is difficult to assess the acuracy of our measurement of the number density of charge carriers, N, as we are not given the rate of impurities in the doped Germanium sample. Nonetheless, assuming a charge carrier density of $2.5 \times 10^{19}\ m^{-3}$[3], our value of $2.38 \times 10^{21}\ m^{-3}$ seems reasonable. Another valuable check of accuracy is the fact we successfully determined the sign of charge carriers (positive - holes). Previous experiments determine the mean collision time τ to be $6.2 \times 10^{-14}\ sec$[4], so our value of $\tau = 5.11 \times 10^{-14} sec$ agrees to within %18.

References

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[3] Rajput, R.K. A Textbook of Electrical Engineering Materials. 2006.

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