Faraday Rotation in SF-59 Glass

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Direct measurement of the verdet constant has been found in SF-59 glass. A semi-classical model for the interaction of light with matter yields qualitative agreement with the observed results. We measured a verdet constant of 26.4 rad $\mathrm{T^{-1}\ m^{-1}}$ which is in good agreement with the anticipated result of 25.9 rad $\mathrm{T^{-1}\ m^{-1}}$ at 300 K, for our 632 nm source [2]. The results have an excellent fit to the expected form, with a slight discrepancy at the final data point considered.

I. INTRODUCTION

External magnetic fields acting on a dielectric brake symmetry within the normal distribution of electrons. This broken symmetry yields differing propagation speeds for right handed and left handed circularly polarized plane waves, and thus induces rotation in an incident linearly polarized plane wave. In 1845 Michael Faraday first noticed this effect through a subtle observation [1]. This was the first link observed between light and magnetism. Since then, the theory of classical electricity and magnetism has been formulated, which and is now complete. Potent descriptions of materials are commonly written in terms of their interaction with light, separated into bulk electric and magnetic parts. Over one decade ago, materials with designer electric and magnetic components, or metamaterials, were first demonstrated, which may possess properties which cannot be found in nature. This is one area keeping the interaction of light with matter on the forefront of research [3]. Further, material properties studied for research and application are commonly studied through spectroscopic techniques. The Faraday Effect itself, has found been exploited in near field imaging [4], measurement of effective mass and energy bands of materials [5], and study of the interstellar medium [6]. Much of our understanding of materials is owed to Michael Faraday's discovery.

II. THEORY

In this section, we present a semi-classical derivation, provided by Professor Ahlen, which arrives at the anticipated angle of rotation for an incident linearly polarized light beam after passing through a dielectric medium under the influence of a magnetic field. This derivation assumes that electrons within atoms act as harmonic oscillators, subject to some level of damping in the system. This motivation for this treatment is due to the spring like coupling within atoms, which is damped in a real material. We find that these results correctly present the qualitative picture found from measurements of Faraday rotation.

Faraday Rotation arises in materials due to a magnetically induced difference in propagation speeds for circular polarized light of different handedness. In general, linear polarized light consists of an electric and magnetic field orthogonal to the propagation direction. This is a combination of right hand and left hand circularly polarized light. When an external magnetic field is placed on a sample, the motivations described above drive us to write that atoms respond to a Lorentz force via:

$$m\ddot{\vec{x}} = -m\omega_0^2\vec{x} + q\vec{v}\times\vec{B} + q\vec{E} - \gamma\dot{\vec{x}} \qquad (1)$$

Where m is the mass of an electron ω_0 is the characteristic frequency of the electron in the atom, q is the charge of an electron, and γ a dissipation constant.

We now consider linearly polarized light entering the sample. This can be written as a combination of right and left handed circularly polarized light.

$$\overrightarrow{E_\pm} = E_0\hat{\imath}e^{i\omega t} \pm iE_0\hat{\jmath}e^{-i\omega t} \qquad (2)$$

Where + refers to right handed circularly polarized and – refers to left handed circularly polarized light respectively. Solving equation (1) with (2) as the acting field, we obtain

$$\vec{x} = x\hat{\imath} + y\hat{\jmath} \qquad (3a)$$

$$x_\pm = \frac{9D \pm q^2\omega B_0}{D^2 - q^2\omega B_0^2}\vec{E} \qquad (3b)$$

Where $D = m(\omega_j^2 - \omega^2) - i\gamma\omega$. This motion will induce a macroscopic polarization in the material, and thus an electric displacement field different from that in free space

$$\vec{D} = \varepsilon_0\vec{E} + \overrightarrow{P_\pm} = (\varepsilon_0 + Nq\sum_j \overrightarrow{x_{j\pm}}f_j)\vec{E} \qquad (4)$$

Where f_j represents the oscillator strength of an atom. Since this is effectively described by the permittivity of free space with an added component, which differs for right handed and left handed circularly polarized light, those components of linearly polarized light will propagate at different speeds in a material

$$\begin{aligned}\vec{E} &= E_0(\hat{\imath} + i\hat{\jmath})e^{i(k_+z-\omega t)} \\ &\quad + iE_0(\hat{\imath} - i\hat{\jmath})e^{-i(k_-z-\omega t)} \\ &= E_0e^{-i(k_-z-\omega t)}(2\hat{\imath} - \delta z\hat{\jmath})\end{aligned} \qquad (5)$$

Where $\delta = k_+ - k_-$ and thus, this constant leads to a rotation of the polarization by an angle

$$\beta = \frac{-\delta z}{2} = \frac{N\pi\omega B_0 q^3 z}{\bar{n}\varepsilon_0}\sum_j \frac{f_j}{D_j^2} \qquad (6)$$

As linearly polarized light travels along its axis of propagation through the material.

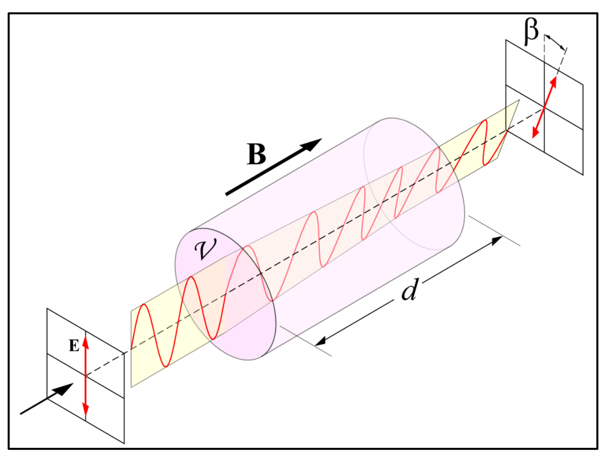

Figure 1: Faraday rotation schematic. Light which enters the crystal is rotated by an angle β due to the different indices of light experienced by the right handed and left handed circularly polarized components of incident linearly polarized light. Taken from Wikipedia [6].

III. <u>Instrumentation</u>

In order to measure the verdet constant we utilized a predesigned setup for measurement of Faraday rotation from TeachSpin. The apparatus consists of a JDS Uniphase Model 1108P 632 nm laser, a solenoid from Teachspin [7], which is placed in between a polarizer and analyzer and serves as the housing for SF-59 glass. The output intensity is measured by a photo detector connected in series with a 1k resistor, which could be switched to 3 or 10k if desired. The AC signal is fed into SR560 low noise preamplifier and then to a Kiethly model 177 Microvolt DMM. The DC signal is fed to a Pigilent 34401A 6 ½ digital multimeter.

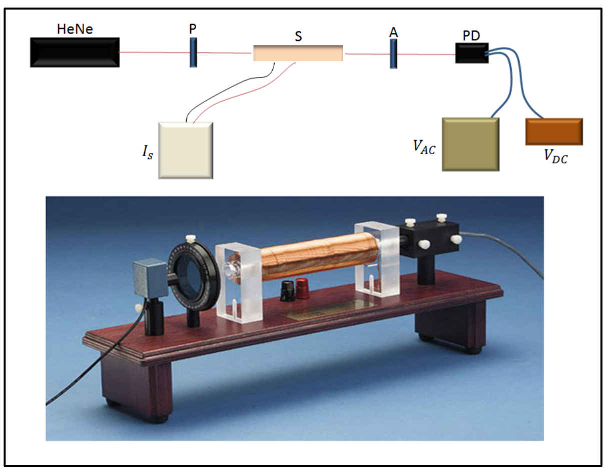

Figure 2: Experimental apparatus. The light source is a HeNe laser. The initially unpolarized beam enters a polarizer (P) propagates through the sample, housed in a solenoid (S) which is supplied AC current (I_S). The output beam passes through an analyzer (A) whose angle θ is varied. The output intensity is measured by a photodetector (PD) and sent to an AC voltmeter (V_{AC}) and a DC voltmeter (V_{DC}). Picture on bottom was taken from Teachspin.

The AC current reading from the lock-in disregards DC signal from the photo detector, and thus provides a modified RMS of the output intensity. The DC measurement provides only the DC response of the output intensity.

The AC and DC signals read by the voltmeters were recorded by hand. While taking data we ensured that the AC and DC signals recorded did not fluctuate. This needed to be cared for since the output intensity of the laser might have fluctuated throughout the course of measurement. Equation (10) ensures that we receive no additional error from these fluctuations, provided that the DC and AC measurements correspond to the same input intensity.

The polarizer's angle was never adjusted during the course of measurement. The Analyzers angle was adjusted with a small potential for error in regards to the angle that was recorded, and that which was physically set (<2%). Data was taken at 5^o increments for four values of input current.

All values used in the calculation of the magnetic field, with the exception of wire diameter, were measured. These include crystal length (10.16 cm), and solenoid length (15.2 cm). The outside radius of the solenoid was also measured so that wire diameter could be inferred. This value was checked against the diameter of #18 AWG gauge wire, and found good agreement.

IV. <u>Safety Issues</u>

Since a laser was used to obtain data, safety glasses were worn at all times while handling the experiment. The solenoid has a maximum current of 3A which can be sustained for 30 seconds. The maximum current sent to the solenoid was 2.5A, while the maximum current, which contributed to our data was 1A. Although there was no damage from sending 2.5A of current, this is not recommended for future students preforming this experiment. Care was taken when handling the polarizers and detector, ensuring that the surfaces of these devices were never touched by hand. Further, when measuring the glass rod, care was taken to ensure that it was never touched by hand. The reflection of the laser off of the glass rod was blocked before it propagated out of the area provided for measurements. Further caution was taken when turning the current supply on and off, this was done slowly, and no wires were connected or disconnected while the instrument was on. The current supply was tracked using an ammeter, and never changed when the ammeter was not being carefully watched.

V. Data taking

We would like to obtain a measurement of the verdet constant, υ, which describes the proportionality of the rotation angle to the field present in the medium

$$\beta = \upsilon \int_{-l/2}^{l/2} \vec{B}(t) \cdot d\vec{z} \quad (7)$$

Where the magnetic field from a finite solenoid with 10 layers is given by:

$$\overrightarrow{B(t)} = \sum_{i=1}^{10} \frac{\mu_0 N I_{AC}}{2} \left[\frac{\frac{L}{2} - z}{\sqrt{r_i^2 - (\frac{L}{2} - z)^2}} + \frac{\frac{L}{2} + z}{\sqrt{r_i^2 - (\frac{L}{2} + z)^2}} \right] \hat{k} \quad (8)$$

In this equation, N is the number of turns per unit length, r_i is the radius of the i^{th} layer of the solenoid, L is the length, and I_{AC} is the AC current, responsible for the time dependence of β. This equation was solved and integrated using MATLAB. The results appear to be reasonable from comparison to the 11 mT/A specification from Teachspin and the measured magnetic field at the end of the coil. Using our setup, we are able to measure the RMS and DC components of the output intensity:

$$I_2 = I_1 cos^2(\theta - \beta) \quad (9)$$

Where I_1 is the intensity after exiting the sample. By expanding this, we are able to separate the DC, and RMS components of the field, taking care to neglect any DC contribution to the RMS field. The result is

$$I_2^{DC} = I_1 cos^2(\theta)$$

$$I_2^{AC} = 2\, I_1\, \upsilon \sin(\theta) cos(\theta) \int_{-l/2}^{l/2} \vec{B}(t) \cdot d\vec{z} \quad (10)$$

Since we may measure everything needed in the above equations, apart from υ we may solve for the verdet constant. Furthermore, since laser intensity tends to fluctuate and since the variation of the angle will cause an imperfect contribution to the intensity present on the sample, I_1, this is not a truly reliable constant. Using both parts of equation 10, we may eliminate it.

VI. Data Analysis

Data was taken for four input currents, in addition to a reference scan. The results can be seen in figure 1. In figure 1a) the only fitted parameter was the amplitude of the intensity, which was used in figure 1b). The verdet constant was found to be 26.4 rad/(T m) from a fit to the data in c). The values obtained fluctuated from 25.9-27.0 rad/(T m), which compares well with the known value of 25.9 rad/(T m) for SF-59 at 300K with a 632nm probe. This fit finds excellent agreement with our data.

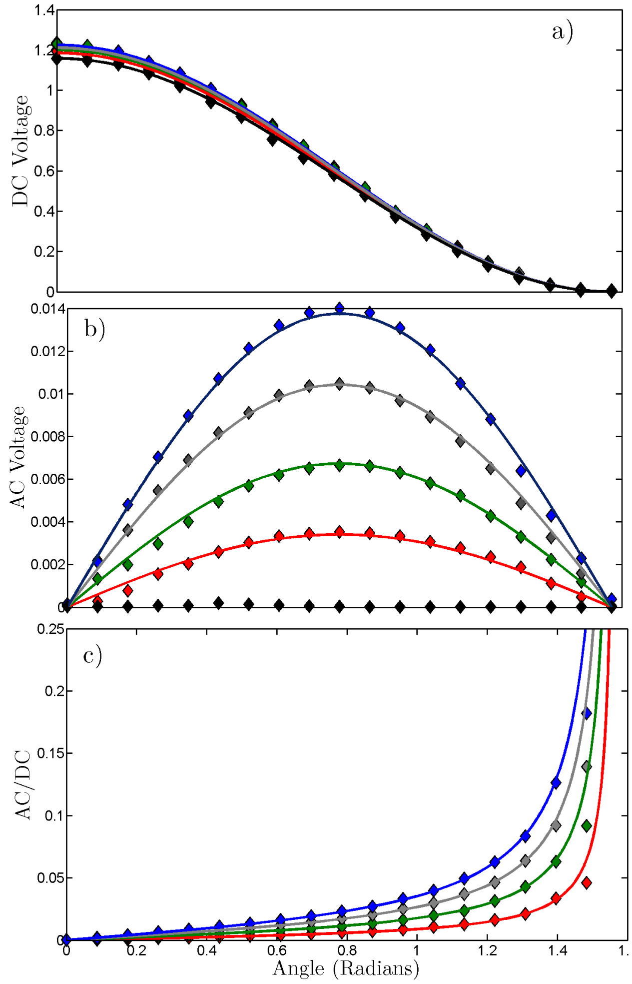

Figure 3: Data taken. a) Measurement of DC voltage. Note the trend for increased peak amplitude as current increases. b) Measurement of AC voltage. c) Division of AC and DC current values. Current input for data: 0 A (Black), 0.25 A (Red), 0.5 A (Green) 0.75 A (Gray), and 1 A (Blue).

There are a few noteworthy points for these data sets. Firstly, in the case of the DC current, there is an apparent trend for values recorded to be larger for larger currents. The raw data, however, fluctuates from reading to reading in this respect for small angles, leaving some values for lower currents higher than those for higher currents. For small angles, we attribute this to the warm up time for the laser, not a physical reading from the crystal. There is a definite trend for large angle readings, which we attribute to the small inclusion of mV or less from the alternating intensity, which is averaged in the DC reading. This stands as a potential source of error in our readings, however, since the intensity is divided out of the equation used in our fits, this cannot affect the fit used for the final determination of the verdet constant.

There is a concerning discrepancy between our final data points and the fit employed, as can be seen in figure 3c. We removed these data points in the fits shown, but included them in figure 3 for the purpose of discussion when future results are taken. Given the consistency in the trend for this data, we preformed fits with these points included. This yielded a verdet constant of 22.8 rad/(T m), which compares poorly with the value of 25.9 rad/(T m) for SF-59 under our conditions [2]. Further, these fits found extremely poor agreement with the AC data (not shown) and most other data points in figure 3c. Because of these considerations, the results shown have been chosen for this report.

VII. Conclusion

We have preformed a measurement, and analysis of Faraday rotation in SF-59. The results fit excellently to the data taken, and are found to be in reasonable agreement with the anticipated value. The behavior observed was found to hold qualitative agreement with predictions from a simplistic model described in section I.

I would like to thank Professor Ahlen for helpful discussions, and for providing the model described herein.

VIII. References

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