

HW 12

PY 355

S2010

1. Using Frobenius, solve $xy' = y$

(don't give up if you get something confusing, the resolution - and the solution - is quite simple)

2. a.) Using the recursion relation from lecture construct the first 6 $P_\ell(x)$. i.e. $P_0 \dots P_5$

b.) Show that $P_\ell(-1) = (-1)^\ell$ for all ℓ (easy)

c.) Write $f(x) = x^4$ as a linear combination of the $P_\ell(x)$. Note, the $P_\ell(x)$ form a basis for all polynomials on the interval $x \in [-1, 1]$.

d.) using Rodriguez' formula

$$P_\ell(x) = \frac{1}{2^\ell \ell!} \left(\frac{d}{dx} \right)^\ell (x^2 - 1)^\ell$$

verify your result for $P_5(x)$ from part a.)

3. A sphere of radius a has potential $\phi(a, \theta, \varphi) = \phi_0$ on its surface. Find the potential for $r > a$ using our general formula for the solution to $\nabla^2 \phi(r, \theta, \varphi) = 0$ in spherical coordinates.

4. A sphere of radius a is held at a constant surface temperature $T = T_0 \cos^2 \theta$. Find the equilibrium temperature inside and outside the sphere. i.e. solve $\nabla^2 T(r, \theta, \varphi) = 0$ subject to the BC at $r = a$ for $r > a$ and $r < a$.