

PY 355: Methods of Theoretical Physics
Spring 2024

Name (Print): _____
ID: _____

Midterm 1

Time Limit: 75 minutes

- Do not turn this page until told to do so.
- Hand your exam in before leaving the room.
- Please write your name on every page and your student ID on the first page.
- This exam contains 11 pages (including this cover page) and 8 problems.
- This is a closed book exam. **No books, notes or calculators allowed.**
- Write all your work on these sheets. If you need more space, you may use the back of the front page; clearly indicate when you have done this.
- **Organize your work**, in a reasonably neat and coherent way.
- **Mysterious or unsupported answers will not receive full credit.**

Problem	Points	Score
1	15	
2	10	
3	20	
4	25	
5	10	
6	10	
7	20	
8	10	
Total:	120	

Extra work page. Please indicate clearly which problem your work is for if you want it graded.

Practice

1. Write the first 3 non-zero terms in the Taylor series about the origin for

(5) (a) $\sinh x$

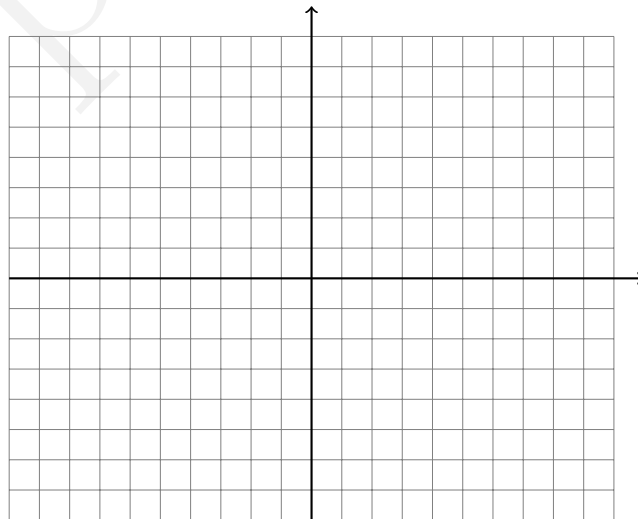
(5) (b) $e^{\frac{x}{1-x}}$

(5) (c) $\frac{1}{1-x^2}$

(10) 2. Sketch the real function,

$$f(x) = \frac{\tanh x}{x^2 - 1}$$

and label all important features and asymptotes.



3. Compute the following integrals (Hint: the basic Gaussian integral over the real line is $\int_{-\infty}^{\infty} dx e^{-x^2} = \sqrt{\pi}$.)

(5) (a) $\int_{-\infty}^{\infty} dx x e^{-x^2}$

(5) (b) $\int_{-\infty}^{\infty} dx x^2 e^{-x^2}$

(5) (c) $\oint_C dz e^{-z^2}$ where C is the circle of radius 1 about the origin in the complex plane

(5) (d) $\oint_C dz \frac{e^{-z^2}}{\sin z}$ where C is the circle of radius 1 about the origin in the complex plane

4. A block with mass m attached to a spring with spring constant k slides back and forth on a table with effective friction constant γ . Newton's law gives the equation of motion for the position $x(t)$:

$$m \frac{d^2 x}{dt^2} = -kx - \gamma \frac{dx}{dt}$$

A solution can be found by plugging in the guess

$$x(t) = e^{zt}$$

and solving for the values of z which satisfy the resulting equation.

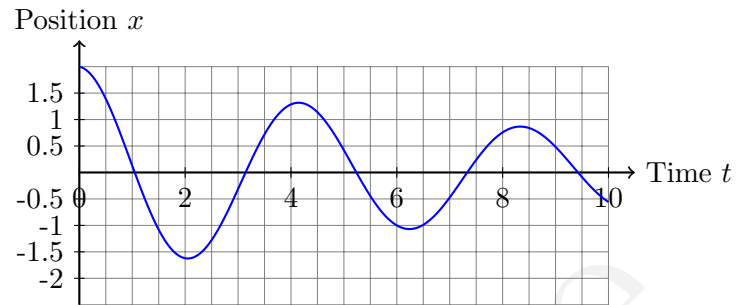
- (10) (a) Find all values of z which satisfy the equation.

- (5) (b) What do the real and imaginary part of z tell you about the oscillation physically?

- (10) (c) Somebody comes along and kicks the mass at time $t = 0$. The resulting motion is shown in the figure, and given by (the real part of)

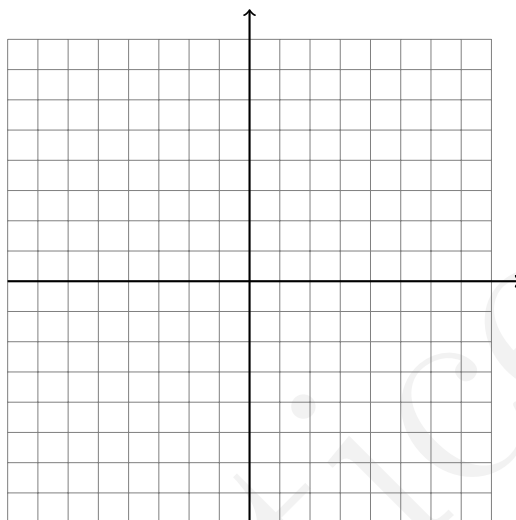
$$Ce^{zt}$$

Determine the complex numerical values of the constants C and z from the figure.



- (10) 5. In the complex plane, identify all singularities and branch points for the following function. If there are branch points, sketch an allowed choice of branch cut.

$$f(z) = \frac{\sqrt{z-i}}{\sin z}$$



- (10) 6. Which, if any, of the following functions are analytic as a function of $z = x + iy$ either everywhere, or with at most isolated singularities?

- A. $f_1 = x^2 - y^2 + 2ixy - 1$
- B. $f_2 = |z|^2$
- C. $f_3 = z^*/|z|^2$
- D. $f_4 = \frac{1}{3}e^{2x}(\cos 2y + i \sin 2y)$

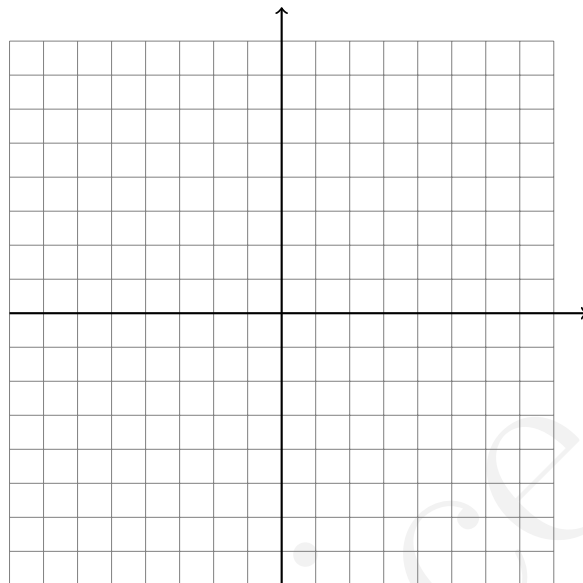
7. Suppose $f(z)$ is a complex analytic function on the complex plane. Let $f(z) = u(x, y) + iv(x, y)$ where u and v are the real and imaginary parts of f , and x, y are the real and imaginary parts of z .
- (5) (a) Show that u and v satisfy the 2-D Laplace equation when thought of as real scalar fields on the 2D x - y plane:

$$\nabla^2 u = (\partial_x^2 + \partial_y^2)u = 0$$

$$\nabla^2 v = (\partial_x^2 + \partial_y^2)v = 0$$

- (5) (b) Show that the contours of u are perpendicular to the contours of v .

(10) (c) Sketch the contours of u and v for $f(z) = \ln z$.



- (5) 8. (a) Evaluate $\epsilon^{ijk}\epsilon_{ijk}$ where ϵ is the 3D Levi-Civita symbol and we use the Einstein summation convention.

- (5) (b) In n dimensions, the Levi-Civita symbol has n indices, each of which takes values $1 \cdots n$, and is defined by the property that it is totally antisymmetric (that is, that it flips sign if any pair of indices is swapped) and that $\epsilon^{12 \cdots n} = +1$. What is the value of $\epsilon^{i_1 i_2 \cdots i_n} \epsilon_{i_1 i_2 \cdots i_n}$, where all n of the indices are paired and summed?