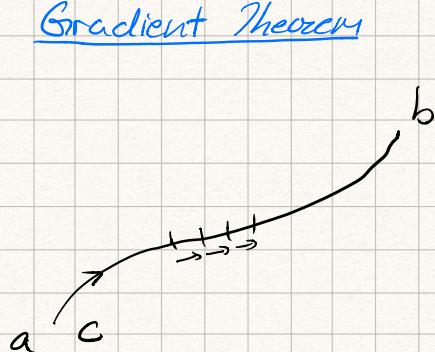


Fundamental Theorems of Vector Calculus

- Relate d dim integrals to $d-1$ dim integrals using derivative

Gradient Theorem

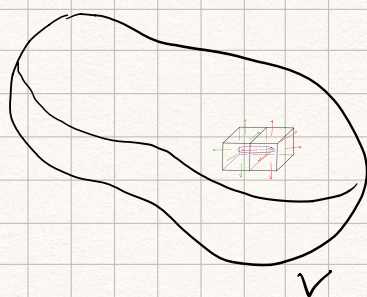


$$\int_{a, C}^b \nabla f \cdot d\vec{r} = \underbrace{f(b) - f(a)}_{\text{change in } f \text{ along } d\vec{r}} = \underbrace{f(x) \Big|_a^b}_{\text{total change in } f}$$

1d 0d

Cor $\int_a^b \nabla f \cdot d\vec{r}$ is independent of path C (don't need to write C)

Gauss/Divergence Theorem

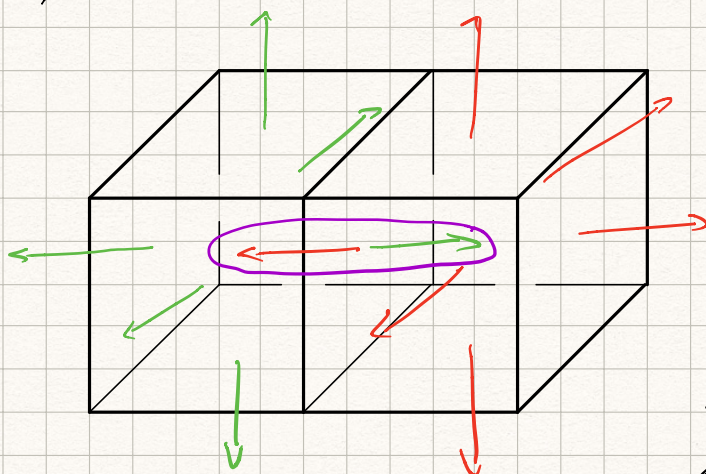


$$\int_V \nabla \cdot \vec{A} d\tau = \oint_{\partial V} \vec{A} \cdot d\vec{\sigma}$$

3d 2d

indicates surface is closed
"boundary of" V

Why? Subdivide volume into tiny boxes



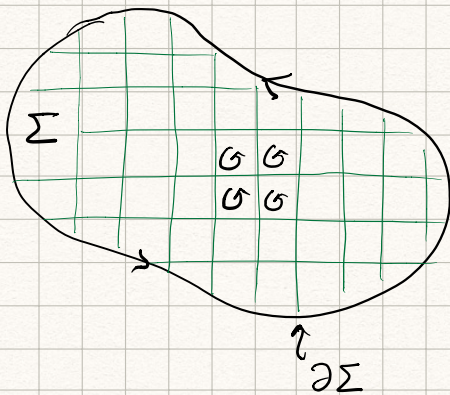
$$\begin{aligned} \nabla \cdot \vec{A} d\tau &= \text{flow out of cell} \\ &= \sum \text{flow thru face faces} \end{aligned}$$

$$\vec{A} \cdot \hat{n} dA = \text{flow thru face of cell}$$

All internal flows cancel.

Only flow through outer boundary remains.

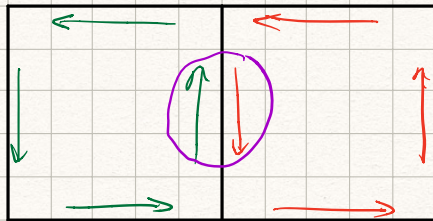
Stokes' Theorem



$$\int_{\Sigma} \underbrace{\nabla \times \vec{A} \cdot d\vec{s}}_{\substack{\text{circulation} \\ \text{around element} \\ d\vec{s}}} = \oint_{\partial\Sigma} \underbrace{\vec{A} \cdot d\vec{r}}_{\substack{\text{circulation on } \partial\Sigma}}$$

total circulation in Σ

Idea is same as Gauss: internal flows cancel on neighboring cells



Orientation: Direction of integral on $\partial\Sigma$ given by right hand rule:
w/ $\hat{n}$ in thumb direction, fingers curl around $\partial\Sigma$

Cor $\oint_{\Sigma} \nabla \times \vec{A} \cdot d\vec{s} = 0$

$\leftarrow$ closed surface

No net vorticity allowed on closed surface.