

Prof. Chris Laumann A: EPC 207 TR 12:30 - 1:45

TA Elliot Yu D1: CAS 204B M 3:35 - 4:25

LA Angel Morales D2: " M 4:40 - 5:30

Grader TBD

Website: physics.bu.edu/~claumann/py355 ← See here.

Q+A: piazza

Homework: weekly due Monday
Written asst due in Gradescope by 11:59 pm
Jupyter asst due in Jupyter "

- 20% per day late, not accepted after Wednesday

Office hours: TBD, probably Friday

Grading: HW 20% Py HW 20%
 Midterm 1 15%
 Midterm 2 15%
 Final 30%

Overview: syllabus is online

- Calculus, Complex functions, vector calculus, linear alg, ODEs
- complementary numerical methods in Python

Materials

- Altland + von Delft Mathematics for Physicists
- Shankar Basic Training in Mathematics
- Arfken, Weber, Harris Mathematical Methods for Physicists
- Lecture notes
- Online tutorials especially for computational stuff
- Lin et al Introduction to Python Programming for Scientists and Engineers

Poll → Programming experience?

→ Lin alg experience / concurrent?

→ Laptops for next lecture

HW Warm up HW due Monday.

Mathematical Preliminaries

cf AVO L1

What are numbers?

Pretty complicated. Can add, multiply etc.

Come in different types: integer, real, complex

Useful to build up a picture of how they fit together.

1. Language of modern math/physics
2. Know which numbers to use & why
3. Know how to represent them in a computer

Sets

A set contains elements.

Ex • $\mathbb{Z}_2 = \{0, 1\}$ curly braces denote "the set of"

$$0 \in \mathbb{Z}_2$$

↑
is an element of

$$2 \notin \mathbb{Z}_2$$

↑
not an element of

$$\{0\} \subset \mathbb{Z}_2$$

↑
subset of

$$\bullet \text{ The empty set } \emptyset = \{\}. \quad \emptyset \subset \mathbb{Z}_2$$

$$\bullet \mathbb{Z}_2 = \{0\} \cup \{0, 1\} = \{0, 1\}$$

↑
union

$$\bullet \{0\} = \{0\} \cap \mathbb{Z}_2 = \mathbb{Z}_2 \cap \{0\}$$

↑
intersection

$$\bullet \{1\} = \mathbb{Z}_2 - \{0\}$$

↑
difference

The Cartesian product $A \times B = \{(a, b) \mid a \in A, b \in B\}$

"the set of tuples (a, b) such that a is an element of A and b is an element of B "

Ex. $\mathbb{Z}_2 \times \mathbb{Z}_2 = \{(0, 0), (0, 1), (1, 0), (1, 1)\}$

$\mathbb{R} \times \mathbb{R} = \mathbb{R}^2 = \{(x, y) \mid x \in \mathbb{R}, y \in \mathbb{R}\}$ 2D real plane

↑
real numbers

The cardinality of a set is the number of elements.

• $|\mathbb{Z}_2| = 2$

• $|\mathbb{Z}_2^2| = 4$

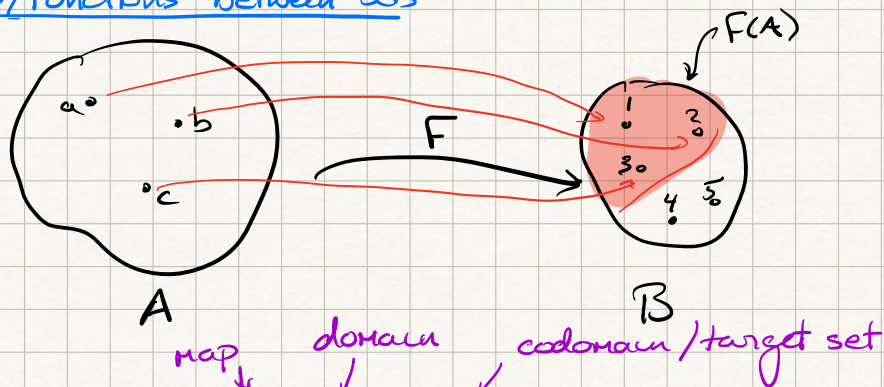
• $\mathbb{Z}$ is countably infinite ← can label every element with a natural number

Eg. $0, 1, -1, 2, -2, 3, -3, \dots$

↑ ↑ ↑
0th element 1st 2nd ...

• $\mathbb{R}$ is uncountable

Maps/Functions between Sets



map $F: A \rightarrow B$
 $a \mapsto F(a)$
 domain A codomain/target set B
 element of A image or value of F at a

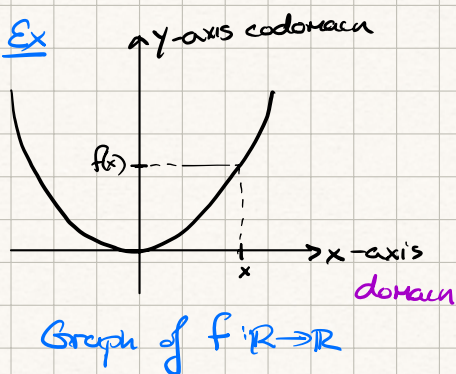
- Image of A : $F(A) = \{F(a) \mid a \in A\} \subset B$
- F is **injective** or one-to-one if every value in B hit at most once
- F is **surjective** or onto if every value in B is hit ($F(A) = B$)
- F is **bijective** if both injective & surjective



$\Leftrightarrow F$ has **inverse map**

$$F^{-1}: B \rightarrow A \quad \text{s.t.} \quad F^{-1}(F(a)) = a \quad \& \quad F(F^{-1}(b)) = b$$

Ex



$$f: \mathbb{R} \rightarrow \mathbb{R} \\ x \mapsto x^2$$

Q? injective
No

surjective
No

bijective
No

$$f: [0, \infty) \rightarrow \mathbb{R}$$

half-open interval
from 0 to ∞
inclusive

Yes

No

No

$$f: [0, \infty) \rightarrow [0, \infty)$$

Yes

Yes

Yes

$$f^{-1}(y) = \sqrt{y}$$

The Hierarchy of "Numbers"

AVO L1.4

Numbers come with more structure than just the set. Can do operations: add, subtract, multiply, invert, etc. A set is "closed" under an operation if the result always lies in the set.

$\mathbb{N} = \{0, 1, 2, \dots\}$ natural numbers

closed under $+$, $\cdot$
not under $-$, $/$

$\cap$

$\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$ integers

closed under $-$
not under $/$

$\cap$

$\mathbb{Q} = \{p/q \mid p \in \mathbb{Z}, q \in \mathbb{Z} - \{0\}\}$ rational numbers

closed under $/$
no $\sqrt{2}$, no limits

countable
dense in $\mathbb{R}$

$\cap$

$\mathbb{R}$

real numbers

closed under limits
no $\sqrt{-1}$

uncountable
"continuous line"

$\cap$

$\mathbb{C}$

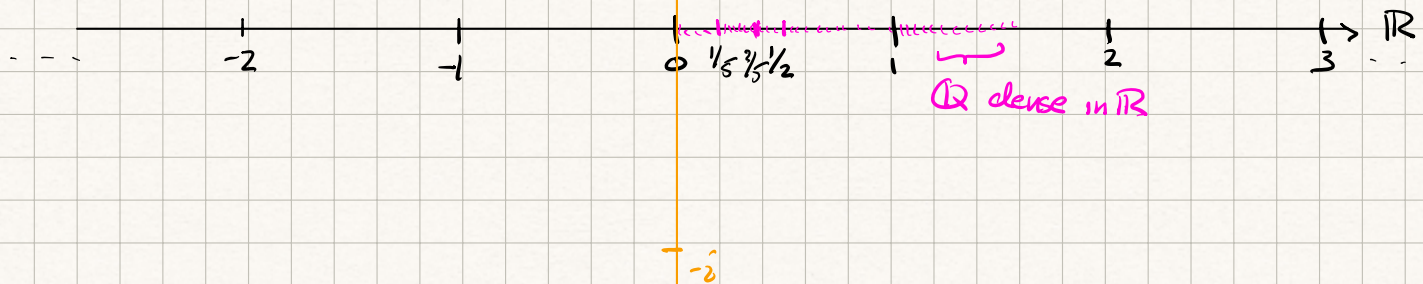
complex numbers

closed under $\sqrt{\quad}$

$\mathbb{C} \simeq \mathbb{R} \times \mathbb{R}$

↑ imaginary axis

$-i = \sqrt{-1}$

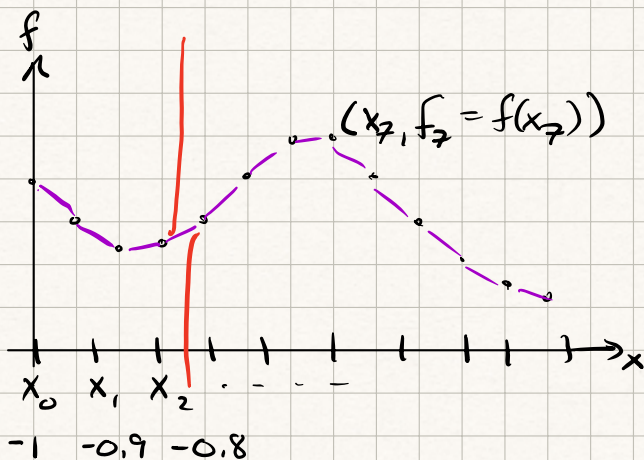


Later: complex numbers more carefully

Visualizing Real Functions

$$f: \mathbb{R} \rightarrow \mathbb{R}$$

A plot or graph of $f(x)$ can be constructed by evaluating $f(x_i)$ at a discrete set of points x_i and connecting them.



This is what a computer does.

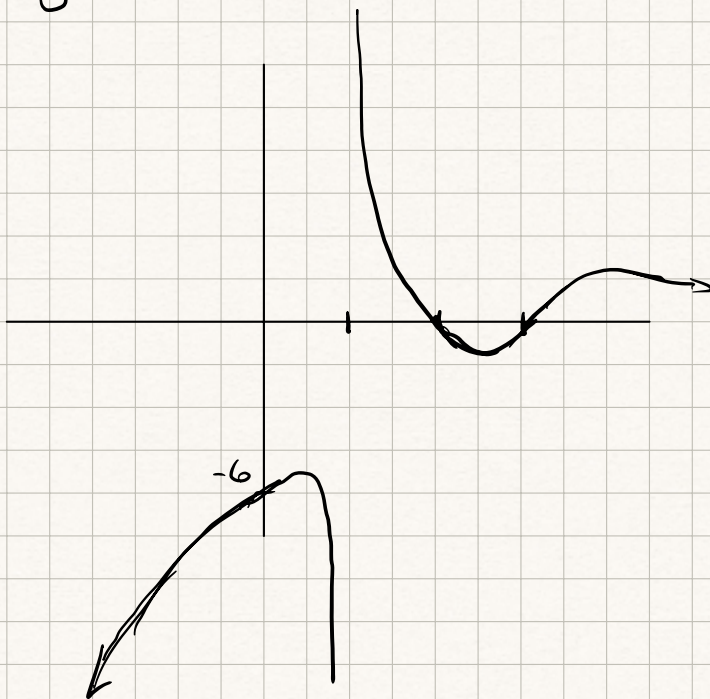
→ Tedious by hand.

→ But can miss features.

for this class, we have high standards for sketching

To sketch a function, need to identify special points:

zeros, minima, maxima, divergences, behavior as $x \rightarrow \pm\infty$



$$f(x) = \frac{x^2 - 5x + 6}{x - 1} e^{-x/5}$$

$$= \frac{(x-2)(x-3)}{x-1} e^{-x/5}$$

Important points:

$$x \rightarrow -\infty: \sim \frac{x^2}{x} e^{-x/5} \sim x e^{-x/5} \rightarrow -\infty$$

$$x = 0: f(x) \approx -6 + x/5$$

$x = 1$: divergence

$x = 2, 3$: zeros

$$x \rightarrow +\infty: x e^{-x/5} \sim \frac{x}{e^{x/5}} \rightarrow \frac{1}{5} e^{-x/5} \rightarrow 0$$

L'Hopital's rule