

Discussion 2

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1 Integration Exercises

Evaluate the following integrals.

(a) $\int_0^\infty \frac{r^3}{(r^2+a^2)^3} dr$

(b) $\int \frac{dx}{\sqrt{b^2+x^2}}$ (Recall that $1 + \sinh^2 x = \cosh^2 x$ and $\frac{d}{dx} \sinh x = \cosh x$.)

(c) $\int_0^\infty e^{-\alpha x} \sin x dx$ (Use integration by parts twice.)

2 Gaussian Integrals

The Gaussian is an extremely prevalent function in physics, so it's a good idea to get to know it better.

Recall that

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}.$$

Use this to prove that

$$\int_{-\infty}^{\infty} e^{-ax^2+bx+c} dx = \sqrt{\frac{\pi}{a}} e^{\frac{b^2}{4a}+c}.$$

(Hint: Complete the square.)

3 Leibniz Rule: Differentiating under the Integral Sign

The Leibniz integral rule for differentiation under the integral sign states that if an integrand depends on some extra parameter, we can differentiate the integral by taking a partial derivative of the integrand, that is

$$\frac{d}{dx} \left(\int_a^b f(x, t) \, dt \right) = \int_a^b \frac{\partial}{\partial x} f(x, t) \, dt.$$

Use this fact to evaluate the integral

$$I = \int_0^1 \frac{t - 1}{\ln t} \, dt$$

by thinking of it as a special case of the integral

$$I(\alpha) = \int_0^1 \frac{t^\alpha - 1}{\ln t} \, dt.$$

- (a) Take the α derivative of both sides.
- (b) Evaluate the t integral.
- (c) Integrate the result with respect to α . (This is an indefinite integral, so remember the integration constant.)
- (d) Determine the integration constant by evaluating $I(0)$.
- (e) Finally, set $\alpha = 1$ to get I .

4 Additional Questions

4.1 Integration Exercises II

- Integration by Parts: $\int u \, dv = uv - \int v \, du$.
- U-Substitution: $\int f(g(x))g'(x) \, dx = \int f(u) \, du$.

Here are some integrals to warm yourselves up on these techniques,

- (a) $\int b^{ax} \, dx$

(b) $\int t^a \, dt$

(c) $\int \ln x \, dx$

4.2 Leibniz Rule II

Using the Leibniz rule, prove that the integral

$$F(n) = \int_0^\infty x^n e^{-x} \, dx$$

satisfies $F(n) = nF(n-1)$. This time, treat it as a special case of the integral

$$F(n, a) = \int_0^\infty x^n e^{-ax} \, dx.$$