Problem Set #5 Due: Tuesday, March 28, 9:15 am

1 Relic Baryon Density

(a) Consider massive particles and antiparticles with mass m and number densities $n(m,t)$ and $\overline{n}(m,t)$. If they interact with cross section σ at velocity v, explain why the evolution of $n(m,t)$ is described by

$$\frac{\partial n}{\partial t} = -3Hn - n\overline{n}\langle\sigma v\rangle + P(t),$$

and identify the physical origin of each of the terms of the right-hand-side.

(b) By considering the evolution of the antiparticles, show that $(n-\overline{n})a^3 = const.$

(c) Assuming initial particle-antiparticle symmetry, show that

$$\frac{1}{a^3}\frac{d(na^3)}{dt} = -\langle\sigma v\rangle\left[n^2 - n_{eq}^2\right],$$

where n_{eq} denotes the equilibrium number density.

(d) Define $Y \equiv n/T^3$ and $x \equiv m/T$, and show that

$$\frac{dY}{dx} = -\frac{\lambda}{x^2}\left[Y^2 - Y_{eq}^2\right],$$

where $\lambda \equiv m^3\langle\sigma v\rangle/H(T=m)$. If λ is constant, show that at late times Y approaches a value given by

$$Y_\infty = \frac{x_f}{\lambda},$$

where x_f is the freeze-out time. Explain the dependence of Y_∞ on $\langle\sigma v\rangle$ and sketch the schematic evolution of Y versus x for a more strongly and a more weakly interacting population of annihilating particles and antiparticles. If there was a speed-up in the expansion rate of the universe caused by the addition of extra low-mass neutrino species what would happen to the abundance of surviving massive particles and why?

(e) Now apply this to proton-antiproton annihilation. Proton-antiproton annihilation proceeds via the strong interactions. The cross section is well-approximated by the geometric cross section, $\sigma = \pi r_p^2$ where $r_p \sim 1$ fm is the proton radius. Show that $T_f \approx 20$ MeV.

(f) Show that

$$\frac{n}{n_\gamma} = \frac{\overline{n}}{n_\gamma} = 10^{-19}\ .$$

How does this compare with observational data? What do you conclude about the abundances of protons and antiprotons in the early universe?