PY - Cosmology Spring 2017

Problem Set #3 Due: Friday, February 24, 10:00 am

1 Scalar Field Dynamics

The Lagrangian for a scalar field in a curved spacetime is

$$L = \sqrt{-g}\left[\frac{1}{2}\, g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

where $g \equiv \det(g_{\mu\nu})$ is the determinant of the metric tensor.

1. Evaluate this Lagrangian for a homogeneous field $\phi = \phi(t)$ in a flat FRW spacetime. Using the Euler-Lagrange equation applied to your Lagrangian determine the equation of motion for the scalar field. You should recognize it as an old friend, the KG equation in a FRW universe.

2. Near the minimum of the inflaton potential, we can expand $V(\phi) = V_{min} + \frac{1}{2}m^2\phi^2 + \cdots$. Making the Ansatz $\phi(t) = a^{-3/2}(t)\chi(t)$, show that the KG equation becomes

$$\ddot{\chi} + \left(m^2 - \frac{3}{2}\dot{H} - \frac{9}{4}H^2\right)\chi = 0\ .$$

   For the remainder of this problem assume that $m^2 \gg H^2 \sim \dot{H}$. Find $\phi(t)$, expressing your answer in terms of the maximum amplitude of the oscillations. Determine the energy density ρ_ϕ and verify the claim from lecture that ρ_ϕ scales like matter during this oscillating phase after inflation.

2 Slow-Roll Inflation

The equations of motion of the homogeneous part of the inflaton are

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0\ , \quad 3M_{pl}^2 H^2 = \frac{1}{2}\dot{\phi}^2 + V\ .$$

1. For the potential $V(\phi) = \frac{1}{2}m^2\phi^2$, use the slow-roll approximation to obtain the inflationary solutions

$$\phi(t) = \phi_S - \sqrt{\frac{2}{3}}mM_{pl}\, t\ , \quad a(t) = a_S\, \exp\left[\frac{\phi_S^2 - \phi^2(t)}{4M_{pl}^2}\right]\ ,$$

   where $\phi_S > 0$ is the field value at the start of inflation ($t_S = 0$).

2. What is the value of ϕ when inflation ends? Find an expression for the number of e-folds. If $V(\phi_S) \sim M_{pl}^4$, determine a condition on m to obtain at least 60 e-folds of inflation. What does this imply for the starting value of the inflaton field ϕ_S?