PY1000: Cosmology Spring 2017

Problem Set #1 Due: Friday, February 3, 10:00 am

Please feel encouraged to ask me questions and discuss with your classmates. Some parts of problem 4 will be covered in class on Tuesday and can be found in the notes.

1. Consider a Tensor $X^{\mu\nu}$ and Vector V^μ with components

$$\mathbf{X} = \begin{pmatrix} 2 & 0 & 1 & 0 \\ -1 & 0 & 3 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad \mathbf{V} = (-1, 2, 0, -2)$$

   and assume that the metric is the Minkowski metric $diag(1, -1, -1, -1)$. Calculate

   (a) $X^\mu{}_\nu$
   (b) $X_\mu{}^\nu$
   (c) $X^{(\mu\nu)}$
   (d) $X_{[\mu\nu]}$
   (e) X^μ_μ
   (f) $V^\mu V_\mu$
   (g) $V_\mu X^{\mu\nu}$

2. In a particular reference frame in which the metric is simply Minkowski the 4-velocity of a particle is given by the vector field U^μ with components

$$\mathbf{U} = (1 + f^2t^2/2, f^2t^2/2, -ft, 0)$$

   where t is the time coordinate of the reference frame and f is a constant with units of time^{-1}.

   (a) Verify that $U^2 = 1$
   (b) Compute the compoents of $U_\mu \partial_\nu U^\mu$. Give a simple argument supporting your result.
   (c) Find the 3-velocity of the particle as a function of t.

3. The Universe contains a blackbody spectrum of photons (the Cosmic Microwave Background) with a temperature of 2.7 Kelvin. Energetic charged particles, such as a proton, will scatter off of such photons thereby losing energy. The most important energy loss reaction for protons and neutrons is "photoproduction" of pions:

$$p + \gamma \rightarrow p + \pi$$

   Protons with energy greater than the threshold energy for this reaction will rapidly lose energy through this process. Thus cosmic protons from distant sources should not have energies larger than this threshold. This is called the GZK (Griesen-Zatsepin-Kuzmin) cutoff energy for cosmic rays. You may ignore the photon redshift due to the expansion of the universe because the mean free path between scatters is small compared with the size of the universe.

(a) Estimate this cutoff energy. (The microwave background photons have a distribution in energy, to simplify use the average energy for the given temperature in your estimate.)

(b) High energy cosmic rays are observed via the air showers which they generate when they hit the top of our atmosphere. Observations indicate that there may be very rare cosmic ray events with higher energies than the GZK bound. Can you give an explanation for how this might be possible? (this is an area of current active research)

4. Basic physics of a FRWL universe.

   (a) What is conformal time? Why is it useful?

   (b) How do the energy densities in radiation ρ_r, matter ρ_m, and cosmological constant ρ_Λ evolve with the scale factor $a(t)$?

   (c) How does $a(t)$ scale with t for a flat universe dominated by radiation, matter or cosmological constant?

   (d) Find the redshift of matter-radiation equality if $\Omega_r = 9.4 \times 10^{-5}$ and $\Omega_m = 0.32$.

   (e) What is H_0^{-1} in *sec* and in *cm*?