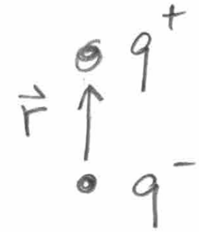
# The strong CP problem

electric dipole moment of neutron

classically: $\vec{d} = q\vec{r}$




neutron:

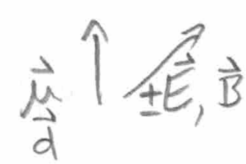

$$r_n \sim \frac{1}{m_\pi} \sim 10^{-13}\,\mathrm{cm} \qquad q \sim e$$

$$\Rightarrow |\vec{d}_n| \sim 10^{-13}\, e\,\mathrm{cm}$$

direction? d is a vector, needs to point in some direction, expect it to align with the only other vector, spin of the neutron.

spin precession in $\vec{E}, \vec{B}$ field with frequency

$$\nu_\pm = 2|\mu B \pm dE|$$



$\Rightarrow |d_n| \lesssim 10^{-26}$ ecm

solution?

$$\underset{-1/3}{d} - \underset{2/3}{u} - \underset{-1/3}{d} \; \curvearrowleft \theta$$

$\Rightarrow d_n \propto 1 - \cos\theta$

for small enough $\theta$, $d_n$ small enough.

QM: average angle over wave-function

seems like $\langle\theta\rangle = 0$ is preferred by symmetry.

what symmetry? $\overset{\text{assume}}{T}$ time reversal

$T: \vec{d} \to \vec{d}$

$\quad \vec{s} \to -\vec{s}$

$\vec{d} \uparrow\uparrow \vec{s} \xrightarrow{T} \vec{d} \uparrow\downarrow \vec{s}$

$\Rightarrow$ 2 degenerate neutron states

but there is no such thing in nature

$\Rightarrow d = 0$.

note: CPT is symmetry $\Rightarrow$ T equivalent to CP

but CP is broken $\to$ T also $\Rightarrow$ no $\overset{\text{symmetry}}{\text{argument}}$ for $d = 0$

but CP is only broken by weak interactions $V_{CKM}$ phase.

$\Rightarrow$ suppression by $\frac{1}{M_W^4}$, small enough? yes.

but there is another CP violating coupling

kinetic term: $F^a_{\mu\nu} F^{\mu\nu a} + \theta \frac{g_s^2}{32\pi^2} \overbrace{\epsilon^{\mu\nu\alpha\beta} F^a_{\mu\nu}}^{\tilde{F}} F^a_{\alpha\beta}$

$\underbrace{\epsilon^{\mu\nu\alpha\beta} F^a_{\mu\nu} F^a_{\alpha\beta}}_{\text{P and T odd}}$

$\vec{x} \to -\vec{x}$ $\quad \vec{A} \to -\vec{A}$

$t \to -t$ $\quad A_0 \to -A_0$

F gluons $\Rightarrow$ CP violating coupling to gluons,

causes $d_n \propto \frac{\theta g_s^2}{32\pi^2} \sim 10^{-3}\theta$

$\Rightarrow d_n \propto \theta\, 10^{-16}$ ecm

$\Rightarrow$ strong CP problem,

why is $\theta \lesssim 10^{-10}$

axion solution:

pseudo-scalar

postulate field, axion $a \xrightarrow{P} -a$

with coupling $\underbrace{\left(\frac{a}{f_a} + \theta\right)}\, g_s^2 \frac{F\tilde{F}}{32\pi^2}$

if $\langle a \rangle = -\theta f_a$ then $\theta_{eff} = 0$

in fact QCD vacuum energy depends on $\theta_{eff}$

$$V_{QCD} \sim \Lambda_{QCD}^4 \sin^2\theta_{eff}$$

issue: - where does this coupling come from?

physics at scale $f_a$

- any other potential for $a$ screws this up

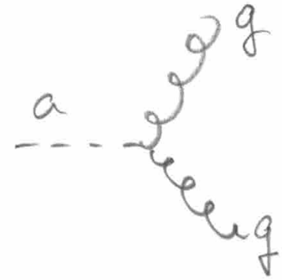
e.g. $V \sim m^2 a^2 \rightarrow$ wants $a = 0$ and not $a = -\theta f_a$

a-mass? $V_{QCD} \sim \frac{\Lambda_{QCD}^4}{f_a^2} (\delta a)^2$

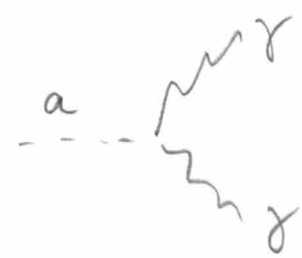
$a = \langle a \rangle + \delta a$

$m_a = \Lambda^2 / f_a$

axion coupling? $\frac{g_s^2}{32\pi^2 f_a}$




but also $\frac{a}{f_a} \frac{e^2}{32\pi^2} \underbrace{F_{EM} \tilde{F}_{EM}}_{\vec{E}\cdot\vec{B}}$

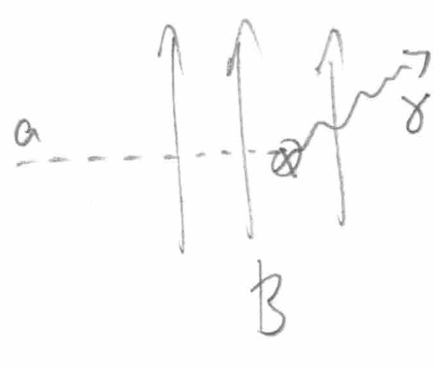



a prediction $m_a \sim \frac{\Lambda_{QCD}^2}{f_a}$ typical $f_a \sim 10^{12}$ GeV

coupling $\propto \frac{1}{f_a} \frac{e^2}{32\pi^2}$ $f_a \overset{exp}{>} 10^{10}$ GeV

axion detection:

if axion is DM:

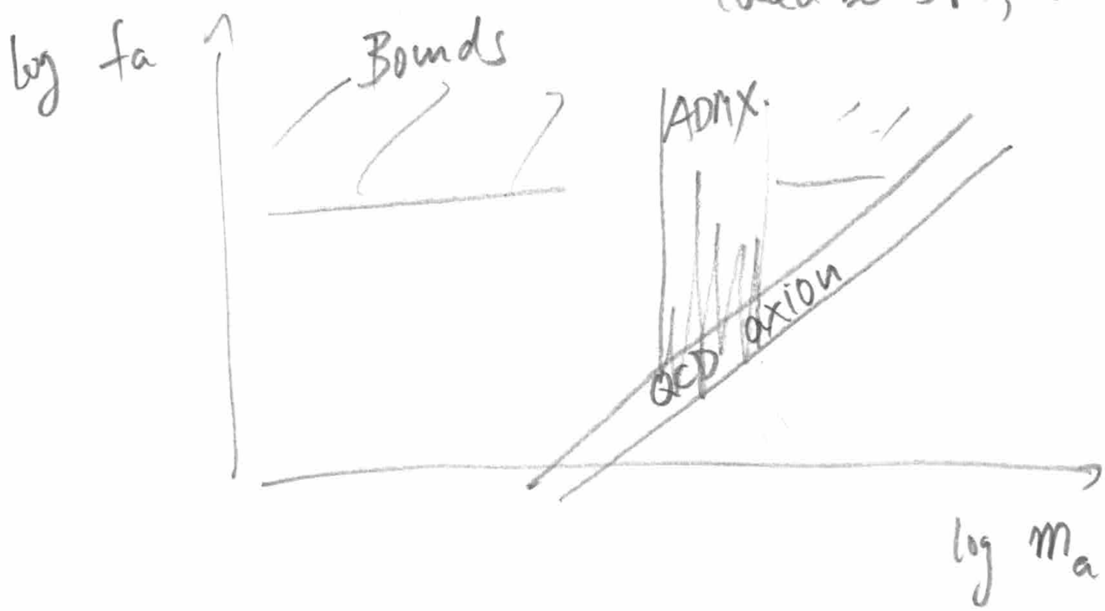


if axion is not DM:

$$pp \to X + a$$


axion-like particles ALPs: $m_{ALP}$ unrelated $f_{ALP}$

could be DM, or not



ADM.

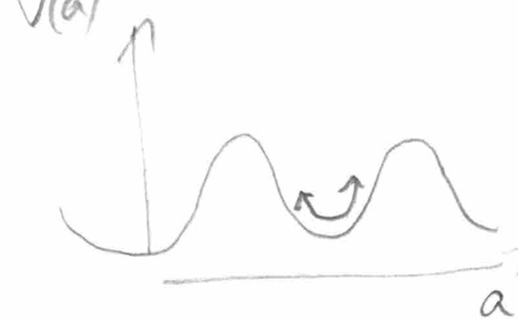
$$\mathcal{L} \sim \frac{1}{2}\left((\partial_\mu a)^2 - m^2 a^2\right)$$

EOM $$(\partial_t^2 - \partial_{\vec{x}}^2 + m^2)\, a = 0$$

space independent solution

$$a \sim a_0 e^{\pm imt}$$

$\xRightarrow{\text{real}} a = a_0 \cos mt$




energy density? $\rho_{DM}$

$$H \sim \dot{a}^2 + m^2 a^2 \longrightarrow m^2 a_0^2 (\sin^2 + \cos^2)$$

in cosmological setting .. Hubble friction

$$\ddot{a} + 3H\dot{a} + m^2 a = 0$$

solution: fast time dependence $\cos(mt)$

slow " " $a_0 \propto 1/R(t)^{3/2}$

$$\Rightarrow \rho_a \propto 1/V \text{ like DM}$$