$$V_{CKM} \equiv \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \approx \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho+i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

Wolfenstein parameterization (approximate)

$$\lambda \equiv V_{us} = \sin\theta_{\text{cabibbo}} \simeq 0.22$$

A, ρ, η are order 1 numbers.

PDG parametrization:

$$V_{CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{13} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

with $s_{ij} = \sin\theta_{ij}$ $c_{ij} = \cos\theta_{ij}$ $\delta =$ CP violating phase

can choose $\theta_{ij} \in [0, \frac{\pi}{2}] \Rightarrow s_{ij}, c_{ij} \geq 0$

Compare to Wolfenstein ...

$$S_{12} = \lambda \qquad S_{23} = A\lambda^2 \qquad S_{13} e^{i\delta} = A\lambda^3 (\rho + i\eta)$$

by phase-redefinition of quark field the CKM phase can be moved around inside V_{CKM} but it cannot be eliminated.

A redefinition invariant measure of CP violation is given by the Jarlskog invariant J_{CKM}

$$\operatorname{Im}\left(V_{ij} V^{\dagger}_{jk} V_{k\ell} V^{\dagger}_{\ell i}\right) = J_{CKM} \sum_{m,n=1}^{3} \epsilon_{ikm} \epsilon_{j\ell n} \quad (\text{no sum } ijk\ell)$$

in PDG param:
$$J_{CKM} = c_{12} c_{23} c_{13}^2 s_{12} s_{23} s_{13} \sin\delta$$
$$\approx \lambda^6 A^2 \eta \approx 10^{-4} \eta$$

$\Rightarrow$ CP violation is small effect.

Higgs couplings? $\quad m^{u,D}_{ij} = \lambda^{u,D}_{ij} \frac{v}{\sqrt{2}}$

in CKM basis $m^{u,d}$ are diagonal $\Rightarrow \lambda^{u,D}_{ij}$ are diagonal.

$\Rightarrow$ Higgs couplings $\frac{\lambda_{ij}}{\sqrt{2}}(v+h)$ are also diagonal.

$h \to b\bar{s}$? Beyond the SM!

Cool argument (symmetry) for counting physical parameters when there is freedom of basis choice:

example: magnetization vector in 3D

$$\vec{M} = (M_x, M_y, M_z) \quad 3 \text{ parameters}$$

can use rotational symmetry to change coordinates

$$\Rightarrow \quad \vec{M}' = (0 \;\; 0 \;\; |\vec{M}|) \;, \quad |\vec{M}| = \sqrt{\textstyle\sum_i M_i^2}$$

$\Rightarrow$ there is only 1 physical parameter $|\vec{M}|$.

relation to symmetries:

in absence of $\vec{M}$, have $SO(3)$ rotation symmetry

3 "generators", rotations about x, y, z

with $\vec{M}$ have only $SO(2)$

1 generator, rotations about $\vec{M}$ axis

$\Rightarrow$ 2 generators are "broken" by M

$$\#\text{ physical parameters} = \#\text{ parameters} - \#\text{ broken generators}$$

$$1 \qquad = \qquad 3 \qquad - \qquad 2\,.$$

Example: quark masses: $\lambda^u, \lambda^D = 36$ parameters

$U(3)^3$ symmetry acting on $(Q, u^c, D^c) = 27$ generators
(3×9)

How many generators are broken?

Easiest to count in mass basis $u^c \begin{pmatrix} m^u & & \\ & m^c & \\ & & m^t \end{pmatrix} \langle H \rangle Q$

$\Rightarrow$ rotational symmetries all broken (assuming $m^u \neq m^c \neq m^t$)

$\Rightarrow$ can still redefine phases $u^c, u \to e^{i\alpha}\, u^c, u$

but this is broken by CKM matrix

only one phase redefinition where all quarks rotate the same $Q \to e^{i\alpha} Q$, $u^c, D^c \to e^{-i\alpha} u^c, D^c$

is unbroken. $\Rightarrow$ 26 broken generators

$\Rightarrow 36 - 26 = 10$ physical parameters (6 masses, 3 angles, 1 phase)

<u>Charged leptons</u> (ignore ν - masses)

$$\lambda^E_{ij}\, E^c_i \langle \tilde{H} \rangle^T L_j \Rightarrow m^E_{ij} = \lambda^E_{ij} \frac{v}{\sqrt{2}}$$

diagonalize with $V_{E^c}, V_L \Rightarrow \begin{pmatrix} m^e & & \\ & m^\mu & \\ & & m^\tau \end{pmatrix}$

no flavor change in W-couplings in absence of ν-masses

$$L^\dagger i \bar{\sigma}^\mu W_\mu L \qquad L_i = \begin{pmatrix} \nu_e \\ e \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$$

[with ν-masses? ν-mass eigenstates ν_1, ν_2, ν_3

$\nu_e = \alpha \nu_1 + \beta \nu_2 + \gamma \nu_3$ etc]

parameter counting? ($m_\nu = 0$)

$\lambda^E = 18$, $U(3)^2 = 18$ broken to $U(1)^3$

$\Rightarrow$ 15 broken parameters $\Rightarrow$ $18 - 15 = 3$ physical (masses)

note that there is no "CKM" because we rotated $L' = V_L L$, possible because no ν-masses.

ν-masses: why not add Dirac partner for ν_L?

$$\left(H^T \begin{pmatrix} \nu \\ E \end{pmatrix} \right) \text{ has charge } (-,-)_0 \quad \text{completely neutral!}$$

$\Rightarrow$ introduce ν^c with no charge

$$\lambda^\nu \, \nu^c H^T \begin{pmatrix} \nu \\ E \end{pmatrix} \to m_\nu \, \nu^c \nu \qquad \text{Dirac mass}$$

$$m^\nu = \lambda^\nu \frac{v}{\sqrt{2}} \qquad m^\nu \lesssim 0.1\,\text{eV}$$

$$v \approx 246\,\text{GeV}$$

$$\Rightarrow \lambda_\nu < 10^{-10} \quad \text{tiny!}$$

ν^c has no gauge interactions.

Higgs interactions

$\approx -i\lambda^\nu$

Higgs width to neutrinos

$$\frac{\Gamma_{\nu\nu^c}}{\Gamma_{bb^c}} \propto \left(\frac{\lambda_\nu}{\lambda_b} \right)^2 \approx \left(\frac{10^{-10}}{10^{-2}} \right)^2 \approx 10^{-16}$$

current bound on $Br(h \to \text{invisible}) \lesssim 20\%$

$\Rightarrow \Gamma_{\nu\nu^c}$ not measurable $\Rightarrow$ cannot prove existence of ν^c this way.

Note: $m^\nu \nu^c \nu$ preserves lepton # symmetry

$$\nu^c, E^c \to e^{-\alpha} \nu^c, E^c$$

$$L \to e^{\alpha} L$$

3 generations: $\lambda^\nu_{ij} \Rightarrow M^\nu = V^+_{\nu^c} \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix} V_\nu$

change basis $\nu^{c\prime} = V^*_{\nu^c} \nu^c$

$\nu' = V_\nu \nu$ } rotate 2 components of

$E' = V_E E$ } $L = \binom{\nu}{E}$ separately

$\Rightarrow$ Masses now diagonal but W-couplings not

$$\nu^+ \bar{\sigma}^\mu W^+_\mu E \to \nu'^+ \bar{\sigma}^\mu W^+_\mu V_{PMNS} E'$$

$V_{PMNS} = V_\nu V^+_E$ = leptonic CKM

(first defined by Glashow)

Q: What's wrong with all this?

A: nothing, this might be the SM of neutrinos

weird: $\lambda_\nu \lesssim 10^{-10}$

but $\lambda_{e^-} \sim 10^{-5}$ already