CP violation

$$CP: \quad \Gamma(A \to f) = \Gamma(\bar{A} \to \bar{f}) \equiv \bar{\Gamma}$$

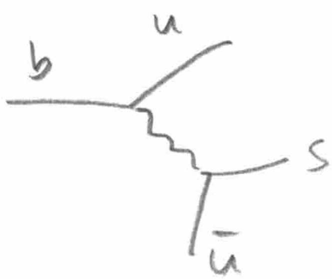
How NOT to observe $\cancel{CP}$:

1. 

$$\sim V_{ub} V_{us}^* \Rightarrow \Gamma \sim |V_{ub}|^2 |V_{us}|^2$$

$$\sim V_{ub}^* V_{us} \Rightarrow \Gamma = \bar{\Gamma}$$

2. need interference



$+$

"Penguin" diagram

$$= A e^{i\alpha} + B e^{i\beta}$$

$$\Rightarrow \Gamma \propto A^2 \left| 1 + \frac{B}{A} e^{i(\beta - \alpha)} \right|^2$$

CP conjugate:

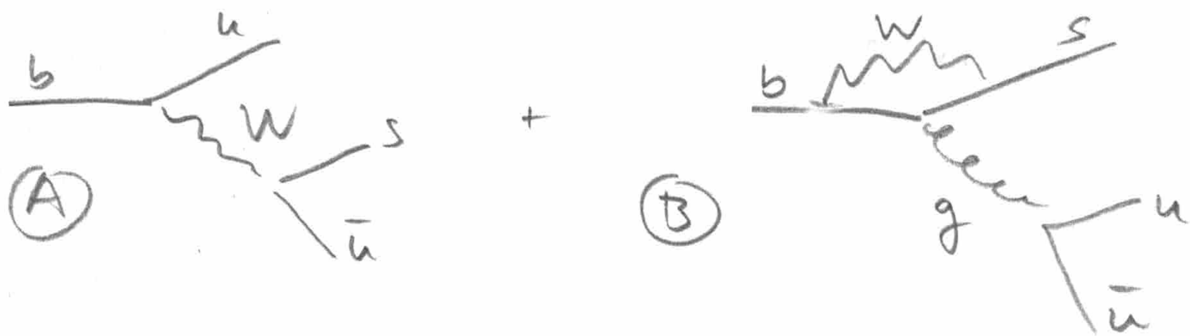
$$\bar{\Gamma} \propto A^2 \left| 1 + \frac{B}{A} e^{-i(\beta-\alpha)} \right|^2 \Rightarrow \bar{\Gamma} = \Gamma \ !$$

$\Rightarrow$ need another phase which does not flip sign under CP, so-called "strong phase".

- e^{-iEt} time evolution phase from on-shell particle evolution

- phases from hadronization in QCD i.e. turning $b\bar{u}$ into B. These are the same between B and $\bar{B}$.

e.g.



with hadronization get a strong phase δ

$$\Gamma(B\to\pi K) \sim \left| A e^{i\alpha+i\delta_A} + B e^{i\beta+i\delta_B} \right|^2$$

$$\bar{\Gamma}(\quad) \sim \left| A e^{-i\alpha+i\delta_A} + B e^{-i\beta+i\delta_B} \right|^2$$

weak phase
CP odd

$$\Rightarrow \Gamma \propto A^2 \left| 1 + \frac{B}{A} e^{i\phi} e^{i\delta} \right|^2 \quad \text{with} \quad \phi = \beta - \alpha, \quad \delta = \delta_B - \delta_A$$

strong phase
CP even

$$= A^2 \left(1 + \left(\frac{B}{A}\right)^2 + 2\frac{B}{A}\cos(\phi+\delta) \right)$$

$$\bar{\Gamma} = A^2 \left(1 + \left(\frac{B}{A}\right)^2 + 2\frac{B}{A}\cos(-\phi+\delta) \right)$$

$$a_{CP} \equiv \frac{\bar{\Gamma}-\Gamma}{\bar{\Gamma}+\Gamma} \simeq 2\frac{B}{A}\,\frac{\cos(\phi-\delta)-\cos(\phi+\delta)}{2} = 2\frac{B}{A}\sin\phi\sin\delta$$

lesson, we need:
1. interference
2. different weak phases, $\phi \neq 0$
3. strong phase $\delta \neq 0$

to see CP violation.

Note: strong phases are not calculable in perturbation theory (sometimes measurable via similar process, or lattice). Thus if we don't know what δ is we cannot measure the CP violating phases ϕ, we can only establish $\phi \neq 0$. i.e. $\cancel{CP}$ if $a_{CP} \neq 0$

What is ϕ? 

$\sim V_{ub} V_{us}^*$

$\sim V_{tb} V_{ts}^*$ + smaller

$$\Rightarrow \quad \phi = \arg\left(\frac{V_{ub} V_{us}^*}{V_{tb} V_{ts}^*}\right)$$

asymmetry in $B \to K\pi$ vs $\bar{B} \to \bar{K}\pi$ is called "direct" CP violation. "Indirect"? CP violation in meson mixing.

Indirect CP violation

consider processes

$$\begin{tikzcd} & B \arrow[dr, "A_f"] & \\ \text{CP conjugate initial particles} \arrow[ur] \arrow[dr] & & f \\ & \bar{B} \arrow[ur, "\bar{A}_f"'] & \end{tikzcd}$$

$A_f \leftarrow \langle f | H_{weak} | B \rangle$

$f \leftarrow$ CP eigenstate

assume (for simplicity $|A_f| \approx |\bar{A}_f| \Rightarrow$ no direct $\cancel{CP}$

if B and $\bar{B}$ mix then we can have interference

$$\begin{tikzcd} B \arrow[rr, bend left, "A_f"] \arrow[r, bend right, "mix"'] & \bar{B} \arrow[r, "\bar{A}_f"'] & f \end{tikzcd}$$

$$A_f^{\cancel{CP}}(t) \equiv \frac{\Gamma(B(t) \to f) - \Gamma(\bar{B}(t) \to f)}{\text{''} \quad + \quad \text{''}}$$

here $B(t)$ is a state that was B at $t=0$ and is a mix due to oscillations at later times.

f for example $\overset{c\bar{c}}{\psi} \, \overset{\frac{\bar{s}d + \bar{d}s}{\sqrt{2}}}{K_s}$

compute A_f^{CP} in 2 state formalism:

$H = M - \frac{i}{2}\Gamma$ eigenvectors $\begin{pmatrix} p \\ q \end{pmatrix}$ with $|p|^2 + |q|^2 = 1$

satisfy $H\begin{pmatrix} p \\ q \end{pmatrix} = \lambda \begin{pmatrix} p \\ q \end{pmatrix} \Rightarrow \frac{p^2}{q^2} = \frac{M_{12} - \frac{i}{2}\Gamma_{12}}{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}$

time evolution of states:

$$|B(t)\rangle = g_+(t)|B\rangle - \frac{q}{p} g_-(t)|\bar{B}\rangle$$

$$|\bar{B}(t)\rangle = g_+(t)|\bar{B}\rangle - \frac{p}{q} g_-(t)|B\rangle$$

$$g_\pm = \frac{1}{2}\left(e^{-im_H t - \frac{1}{2}\Gamma_H t} \pm e^{-im_L t - \frac{1}{2}\Gamma_L t}\right)$$

$$\Gamma(B \to f) \propto |\langle f | B(t)\rangle|^2$$

$= |A_f|^2 e^{-\Gamma t}\Big[\left(\cosh \frac{\Delta\Gamma}{2}t + \cos \Delta m t\right)$ ← prob to find B at time t

$+ |\lambda_f|^2 \left(\cosh \frac{\Delta\Gamma}{2}t - \cos \Delta m t\right)$ ← prob of $\bar{B}$

$- 2\,\mathrm{Re}\left[\lambda_f \left(\sinh \frac{\Delta\Gamma}{2}t + i \sin \Delta m t\right)\right]\Big]$ ← interference where $\lambda_f = \frac{\bar{A}_f}{A_f} \frac{q}{p}$

$\Gamma(\bar{B} \to f) \propto$ same with $\lambda_f \to \frac{1}{\lambda_f}$

general $A_f^{\not{CP}}(t)$ is messy ... but in $\underline{B \text{ system } \Delta\Gamma \ll \Delta m}$

simplify $\Rightarrow$ $\cosh \approx 1$ $\quad \sinh \approx 0$ $\quad |q/p| \approx 1$

also $|\bar{A}_f| \approx |A_f| \Rightarrow \lambda_f = e^{2i\theta} = \cos 2\theta + i \sin 2\theta$

$$\Rightarrow \Gamma(B \to f) \propto |A_f|^2 e^{-\Gamma t} \left[1 + \cancel{\cos \Delta m t} + 1 - \cancel{\cos \Delta m t} + 2 \sin 2\theta \sin \Delta m t \right]$$

$$= \quad '' \quad 2(1 + \sin 2\theta \sin \Delta m t)$$

$\Gamma(\bar{B} \to f)$ same with $\theta \to -\theta$

$$\Rightarrow \boxed{A_f^{\not{CP}}(t) = \sin 2\theta \sin \Delta m t}$$ CP violation due to mixing

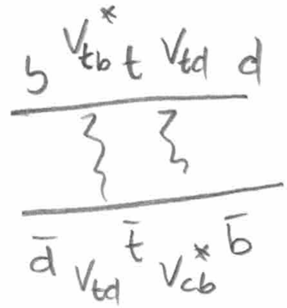
measures $2\theta = \arg \lambda_f$

note that here θ is CP odd "weak" phase
$\Delta m t$ is CP even "strong" phase

$\arg(\lambda_f)$ depends on the process.

"Golden mode" $B_d \to \psi K_s$

$$\lambda_f = \frac{q}{p}\frac{\bar{A}_f}{A_f}$$



have $$\frac{q^2}{p^2} \approx \frac{M_{12}}{M_{12}^*} " \frac{(V_{tb}^* V_{td})^2}{\text{c.c.}}$$

also $A_{\psi K} \propto V_{cb}^* V_{cs}$

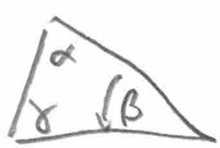
$$\Rightarrow \arg\lambda_f = \arg\left(\frac{q}{p}\frac{\bar{A}_f}{A_f}\right) = 2\arg\left(\frac{V_{tb}V_{td}}{V_{cb}^* V_{cs}}\right)$$

but this was not correct as the final state is K_s, not K

$$|K_s\rangle \propto |K\rangle - \frac{q_K}{p_K}|\bar{K}\rangle = |K\rangle + \frac{V_{cd}^* V_{cs}}{V_{cd} V_{cs}^*}|\bar{K}\rangle$$

$$\Rightarrow \arg\left(\frac{q_B}{p_B}\frac{\bar{A}_{\psi K_s}}{A_{\psi K_s}}\right) = 2\arg\left(\frac{V_{tb}^* V_{td}}{V_{cb}^* \cancel{V_{cs}}} V_{cd}^* \cancel{V_{cs}}\right)$$

$$\Rightarrow \theta = \arg\left(\frac{V_{tb}^* V_{td}}{V_{cb}^* V_{cd}}\right) = \beta$$



fit to amplitude in oscillations of $A^{CP}_{\psi K_S}(t) = \sin 2\beta \sin \Delta m t$

world average (BaBar/Belle/LHCb 2019) from pdg.

$$\sin 2\beta = 0.691 \pm 0.017$$

PLOT of CKM triangle from pdg.

comments on "strong" phase:

- $e^{-iM_H t}$ vs $e^{-iM_L t}$ $\to \sin(\Delta m t)$
- QCD phases in $\bar{A}_f / A_f$ cancel out

$\Rightarrow$ precision measurement of $\sin 2\beta$ possible.

This was all assuming SM physics, the goal here was to measure the SM parameters and secretly hope that one finds an inconsistency so that NP is required.

NP contributes to higher-dimensional operators, e.g. dimension 6 which are also generated from weak interactions

example $K\bar{K}$ mixing (strongest bound on scale of NP)

in SM:

s c d

$\bar{d}$ $\bar{c}$ $\bar{s}$

$$\propto \frac{(V_{cs}^* V_{cd})^2}{m_W^4} \frac{m_c^4}{m_c^2} \frac{g^4}{16\pi^2}$$

GIM

IR dominated loop

[

$$\int \frac{d^4\ell \; m_c^4}{(\ell^2+m_c^2)^3} \approx m_c^2$$]

$$\sim (0.2)^2 (10^{-2})^4 \frac{(0.5)^2}{100} \text{ GeV}^{-2}$$

$$\sim 10^{-12} \text{ GeV}^{-2} \sim \frac{1}{(1000 \text{ TeV})^2}$$

$\Rightarrow$ weak interaction box diagram gives effective operator $\frac{(\bar{s}\gamma^\mu P_L d)^2}{\Lambda^2_{SM}}$ with $\Lambda_{SM} \approx 1000$ TeV

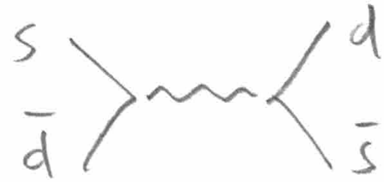
experiment measures $\underbrace{\frac{C_{K\bar{K}}}{\Lambda^2}}_{\text{"wilson coefficient"}} \langle \bar{K} | (\bar{s}\gamma^\mu P_L d)^2 | K \rangle$

lattice QCD

measured $K\bar{K}$ mixing agrees, Δm_K to 10% precision

$\Rightarrow$ NP contributions to same operator must be small

example: FCNC Z'



$\sim \frac{g'^2}{M^2_{Z'}}$

$\Rightarrow M_{Z'} > 1000$ TeV

with CP violation the bound is even stronger 10^4 TeV

$\Rightarrow$ NP @ TeV scale must preserve flavor to good precision.