Neutral scalar meson mixing

$B^0 \sim b\bar{d}$ $\qquad \bar{B}^0 \sim \bar{b}d$ $\quad \leftarrow$ same charge, "opposite" flavor

$B_s^0 \sim b\bar{s}$ $\qquad \bar{B}_s^0 \sim \bar{b}s$

$D^0 \sim c\bar{u}$ $\qquad \bar{D}^0 \sim \bar{c}u$

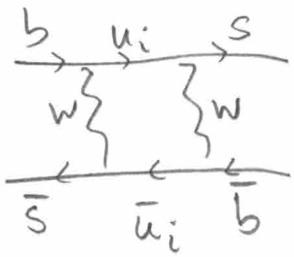
$K^0 \sim s\bar{d}$ $\qquad \bar{K}^0 \sim \bar{s}d$

particle $\leftrightarrow$ antiparticle

flavor is not conserved by the weak interactions $\Rightarrow$ mixing

$\Rightarrow$ flavor eigenstates $(B^0, \bar{B}^0)$ are not mass eigenstates

e.g.



proportional to off-diagonal CKM matrix elements

Mixing can be described by a 2-state QM system $|P\rangle, |\bar{P}\rangle$

meson

anti-m.

general state: $|\psi\rangle = a|P\rangle + b|\bar{P}\rangle$

at $t=0$ have $|\psi(0)\rangle = a(0)\,|p\rangle + b(0)\,|\bar{p}\rangle$

- time evolution by 2 state Hamiltonian

$$\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = e^{-iMt} \begin{pmatrix} a(0) \\ b(0) \end{pmatrix} \quad \text{with} \quad M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}$$

ignoring decays (for now) probability must be conserved

$$\langle \psi | \psi \rangle = \text{const} \iff |a|^2 + |b|^2 = 1$$

$\iff$ M hermitian $\Rightarrow$ $M_{12} = M_{21}^*$, M_{11}, M_{22} real

also CPT : $M_{11} = M_{22}$.

strategy : • diagonalize M to find $\overbrace{\text{mass eigenstates}}^{\text{orthonormal}}$ $|P_H\rangle$, $|P_L\rangle$

eigenvalues M_H, M_L

- time evolution of these states is trivial

$$e^{-iM_{H/L}t}\,|P_{H/L}\rangle$$

explicitly: $$M = \begin{pmatrix} M_{11} & M_{12} \\ M_{12}^* & M_{11} \end{pmatrix}$$

$$= m\,\mathbb{1} + \frac{1}{2}\begin{pmatrix} 0 & \Delta m \\ \Delta m & 0 \end{pmatrix}$$

where we have redefined the relative phase of $|p\rangle$ and $|\bar{p}\rangle$ to make M_{12} real. (and $m \equiv M_{11}$, $\Delta m \equiv 2M_{12}$)

Eigenstates determined by $\begin{pmatrix} & \Delta m \\ \Delta m & \end{pmatrix}$: $|p_{H/L}\rangle = \frac{1}{\sqrt{2}}(|p\rangle \pm |\bar{p}\rangle)$

$$M_{H/L} = m \pm \frac{\Delta m}{2}$$

Say, produced $|p\rangle = \frac{1}{\sqrt{2}}(|p_H\rangle + |p_L\rangle)$ at $t=0$

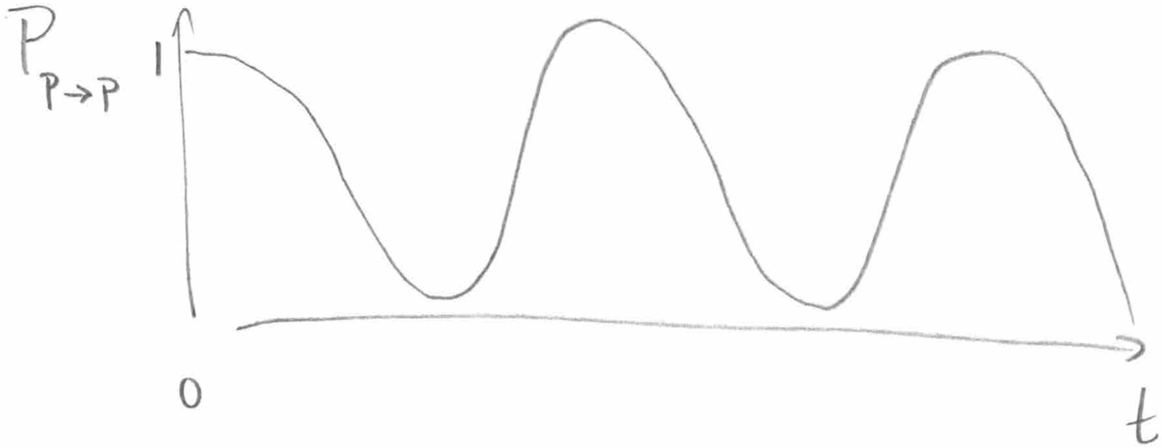
$$\Rightarrow \quad |p(t)\rangle = \frac{1}{\sqrt{2}}\left(e^{-iM_H t}|p_H\rangle + e^{-iM_L t}|p_L\rangle\right)$$

$$= \frac{1}{\sqrt{2}} e^{-imt}\left(e^{-i\frac{\Delta m t}{2}}|p_H\rangle + e^{i\frac{\Delta m}{2}t}|p_L\rangle\right)$$

$\Rightarrow$ Prob to measure $|p\rangle$ at t:

$$|\langle p|p(t)\rangle|^2 = \frac{1}{4}\left|e^{i\frac{\Delta m t}{2}} + e^{-i\frac{\Delta m t}{2}}\right|^2 = \cos^2\left(\frac{\Delta m t}{2}\right)$$

$= \frac{1}{2}(1 + \cos \Delta m t)$ oscillations



Now lets take into account decay. Both $|p\rangle$ and $|\bar{p}\rangle$ can decay leading to a non-unitary Hamiltonian (prob. to remain in 2-state system decays)

Decompose H as

$$H \equiv M - \frac{i}{2} \Gamma$$

$\uparrow$ Hermitian $\uparrow$ M from before

$\uparrow$ Hermitian $\uparrow$ exponential decay

Flavored meson states (e.g. B^0, $\bar{B}^0$)

$$|P\rangle, |\bar{P}\rangle$$

Propagation Hamiltonian:

$$H = M - \frac{i}{2}\Gamma$$

Hermitian

CPT: $H_{11} = H_{22} \equiv H_0$

CP: $\arg M_{12} = \arg \Gamma_{12}$ $\Rightarrow$ make both real at the same time

$$\Rightarrow |P_{H/L}\rangle = \frac{1}{\sqrt{2}}\left(|P\rangle \pm |\bar{P}\rangle\right) \quad \text{orthogonal}$$

Eigenvalues $\lambda_\pm = H_0 \pm \left(M_{12} - \frac{i}{2}\Gamma_{12}\right) = M_{H/L} - \frac{i}{2}\Gamma_{H/L}$

$$m = \frac{M_H + M_L}{2} \qquad \Delta m = M_H - M_L > 0$$

$$\Gamma = \frac{\Gamma_H + \Gamma_L}{2} \qquad \Delta\Gamma = \Gamma_H - \Gamma_L$$

in general phase convention: $\Delta m = 2|M_{12}|$

$$\Delta\Gamma = 2|\Gamma_{12}|$$

$$\Delta \equiv \lambda_+ - \lambda_- = 2\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)$$

time evolution: $e^{-i\lambda_\pm t}$ on $|P_{H/L}\rangle$

note $t = \tau$ proper time (time in frame of particle)

in lab frame have time dilation

example: lifetime in rest frame $\Gamma_0 = \frac{1}{2m}\int dP_{final}|A|^2$

any other frame $\Gamma = \frac{m}{E}\frac{1}{2m}$ " $= \frac{1}{2E}$ "

$\uparrow \sqrt{1-v^2/c^2}$

assume slow, ignore time dilation.

$t=0 \quad |P\rangle = \frac{1}{\sqrt{2}}\left(|P_H\rangle + |P_L\rangle\right)$

$$|p(t)\rangle = \frac{1}{\sqrt{2}}\left(e^{-i\lambda_+ t}|p_H\rangle + e^{-i\lambda_- t}|p_L\rangle\right)$$

$$= \underset{\substack{\text{universal}\\ \text{phase}}}{\underbrace{e^{-imt}}}\;\underset{\substack{\text{universal}\\ \text{decay}}}{\underbrace{e^{-\frac{\Gamma}{2}t}}}\;\underbrace{\left(\frac{e^{-i\frac{\Delta}{2}t}}{\sqrt{2}}|p_H\rangle + \frac{e^{i\frac{\Delta}{2}t}}{\sqrt{2}}|p_L\rangle\right)}_{C_+(t)|p\rangle - C_-(t)|\bar{p}\rangle}$$

$$C_\pm = \frac{1}{2}\left(e^{-i\frac{\Delta m}{2}t - \frac{\Delta\Gamma}{4}t} \pm e^{i\frac{\Delta m}{2}t + \frac{\Delta\Gamma}{4}t}\right)$$

prob to find $|p\rangle$ at time t:

$$P(t) = |\langle p|p(t)\rangle|^2 = e^{-\Gamma t}|C_+|^2 = e^{-\Gamma t}\,\frac{\cos\Delta m t + \cosh\frac{\Delta\Gamma}{2}t}{2}$$

$$\bar{P}(t) = |\langle\bar{p}|p(t)\rangle|^2 = \qquad C_- = e^{-\Gamma t}\,\frac{-\cos\Delta m t + \cosh\frac{\Delta\Gamma}{2}t}{2}$$

limits? • $\Delta m = \Delta\Gamma = 0$ no mixing $\bar{P} = 0$ $P = e^{-\Gamma t}$

• $\Delta\Gamma = 0$ equal lifetime

$$P = e^{-\Gamma t} \; \frac{1 + \cos \Delta m t}{2} = \cos^2(\tfrac{1}{2}\Delta m t)\, e^{-\Gamma t}$$

$$\bar{P} \qquad 1 - \qquad \sin^2(\quad) \;\; ''$$

oscillations with $\Delta m t$

$\Rightarrow$ measure P & $\bar{P}(t)$ $\Rightarrow$ Δm

$\uparrow$

extract V_{CKM}

e.g. $\Delta m_B \sim 3 \cdot 10^{-13}$ GeV $\Rightarrow L_{osc} \sim$ mm

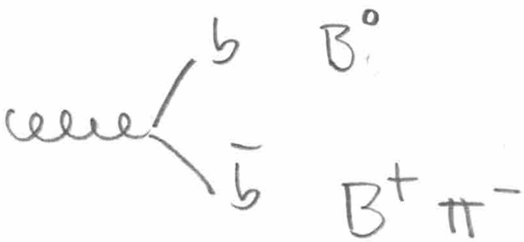
$c\tau_B = c\frac{1}{\Gamma_B} \sim 0.5$ mm measurable

Probability for $|p\rangle$ at t

$$P_{p\to p}(t) = \left|\langle p | p(t)\rangle\right|^2 = e^{-\Gamma t}\,\frac{\cos(\Delta m\, t) + \cosh\left(\frac{\Delta\Gamma}{2} t\right)}{2}$$

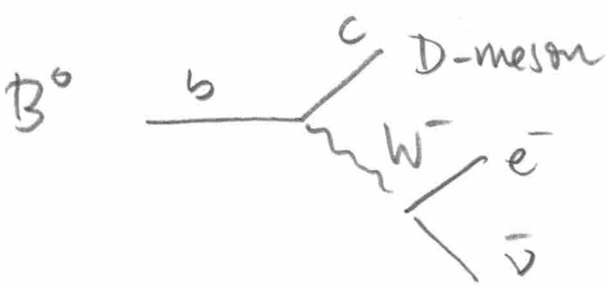
how do experiments tell p from $\bar{p}$?

<u>production</u>



if one side hadronizes into a B^+ (which must contain a $\bar{b}$) that determines that the other B-meson is a B^0.

<u>decay</u>



$\bar{B}^0 \longrightarrow D\, e^+ \nu$ $\Rightarrow$ charge of lepton tags B flavor time scales? def $x \equiv \frac{\Delta m}{\Gamma}$ $y \equiv \frac{\Delta \Gamma}{2\Gamma}$

$x \ll 1$ $\Rightarrow$ slow oscillations (compare w. decay)

$\Rightarrow$ decay before oscillations $\Rightarrow$ look for rare "wrong-sign" decays

D-mesons $x, y \sim 10^{-2}$

here getting an upper bound on x is always possible.

$x \gg 1$ rapid oscillations

requires good spatial resolution in detector to see oscillation. Otherwise just observe average

$$P = \bar{P} \sim \frac{e^{-\Gamma}}{2}$$

can get a lower bound on x, B_s: $x \sim 10$

$x \simeq 1$ easiest, can see oscillation

K, B_d both have $x \sim 1$ (lucky!)

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note on computation of Δm, $\Delta\Gamma$

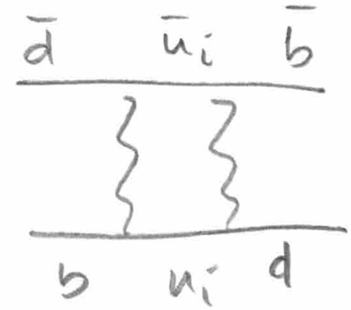
$$e^{-i(M - \frac{i}{2}\Gamma)t}$$

$$\overbrace{M_{11} = M_{22}}^{CPT} \approx M_{P,\bar{P}}$$

$$\Gamma_{11} = \Gamma_{22} \approx \Gamma_{\text{free decay of } b}$$

$$M_{12} = \frac{1}{2m_B} \langle B \,|\, H_{weak} \,|\, \bar{B} \rangle$$

$\nwarrow \propto \bar{b} d \bar{b} d$



$$\text{coeff of operator} = \frac{g^4}{M_W^2} \sum_{ij} V_{id}^* V_{ib} V_{jd}^* V_{jb} \, F(x_i, x_j)$$

$$x_i = \frac{m_i^2}{m_W^2} \qquad i = u, c, t$$

1. GIM: vanishes when either x_i or $x_j = 0$

2. in B mesons $F(x_t, x_t)$ term dominates

3. D mesons internal quark down-type

$$\Rightarrow x \sim \frac{m_{b,s}^2}{m_W^2} \text{ small}$$

4. matrix elements come from lattice QCD $\langle B | d\bar{b} d\bar{b} | \bar{B} \rangle$

<u>Discrete symmetries</u>

Parity P: $\vec{x} \to -\vec{x}$, $\vec{p} \to -\vec{p}$, $\vec{s} \to \vec{s}$ $(\vec{r} \times \vec{p})$

$\Rightarrow$ helicity changes 

parity relates L, R chiralities $L \overset{P}{\leftrightarrow} R$

$\Rightarrow$ weak interactions violate parity  but not 

QED, QCD preserve parity.

note: intrinsic parity of particle state can be + or −

e.g. parity changes L ⟷ R but could also introduce sign ±

$$|\psi_L\rangle \xrightarrow{P} \pm|\psi_R\rangle$$

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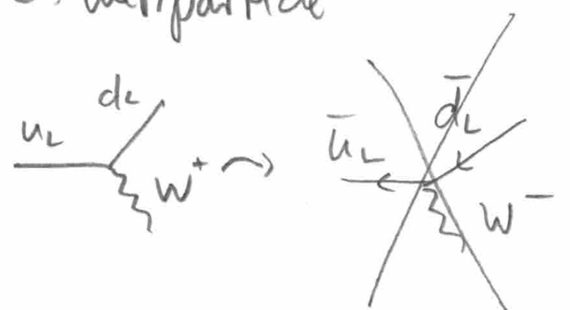
charge conjugation $\mathbf{C}$: particle ⟷ antiparticle

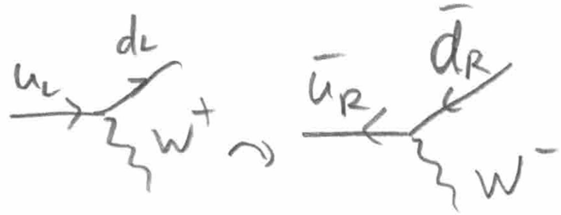
weak interactions also violate



does not exist

QED, QCD preserve C.

CP

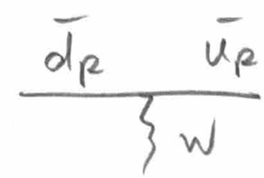


OK!

Feynman rule at vertex V_{ud} versus V_{ud}^*

⇒ if we can find a basis for V_{CKM} such that $V_{ud} = V_{ud}^*$ then CP is manifestly preserved (in other basis too but it's not obvious)

CPT: also changes initial $\leftrightarrow$ final states



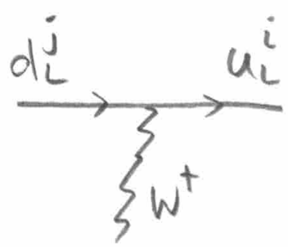
Can show that all local, unitary, Lorentz-invariant QFTs have CPT symmetry (CPT theorem).

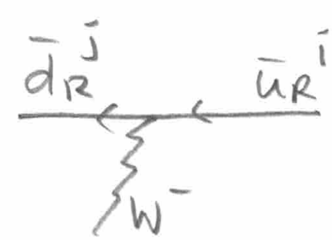
$\Rightarrow \quad m_p = m_{\bar{p}}$

- Field operator $\psi \sim \begin{Bmatrix} a \\ b^\dagger \end{Bmatrix}$ destroys particles / creates anti-particles

  $\Rightarrow$ same Feynman rules for $\psi|p\rangle$ vs. $\langle\bar{p}|\psi$

  (CPT conjugate processes)

- 



$u_i^\dagger \, i\bar{\sigma}\cdot W^+ d_j V_{ij}$ $\qquad$ $d_j^\dagger \, i\bar{\sigma}\cdot W^- u_i V_{ij}^*$

if V real $\Rightarrow$ same Feynman rule

V complex $\Rightarrow$ c.c. Feynman rule, this can matter in processes with <u>interference</u>.