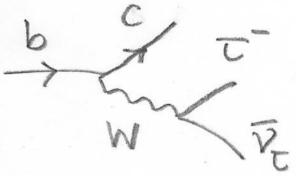
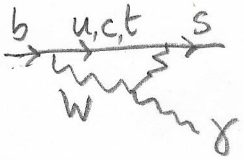
How to measure CKM elements?

1. direct
2. indirect

e.g. V_{cb} from



$V_{tb}V_{ts}^*$ from

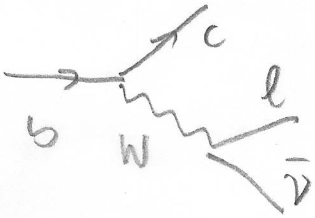


Direct example: $B \to X_c\, \ell\bar\nu$

b-meson (B)

any hadron(s) with a c-quark (X_c)

charged lepton (ℓ)



$$A \propto \underbrace{\langle \ell\bar\nu X_c | c_L^\dagger \bar\sigma^\mu b_L\, \ell_L^\dagger \bar\sigma_\mu \nu_L | B\rangle}\ \frac{g_w^2/2}{-M_W^2}\, V_{cb}$$

$$\approx \underbrace{\langle \ell\bar\nu | \ell^\dagger \bar\sigma_\mu \nu |}_{\bar u_e \gamma_\mu P_L v_\nu} \rangle\langle X_c | c^\dagger \bar\sigma^\mu b | B\rangle$$

$\uparrow$ destroys free b-quark but here b is bound in B-meson.

Binding energy ~ Λ_{QCD}

$m_b \gg \Lambda_{QCD}$

to first approximation b-quark is free

systematic expansion about $\frac{\Lambda_{QCD}}{m_b} \ll 1$ limit

"Heavy Quark Effective Theory (HQET)"

leading term $\langle X_c | c^\dagger \bar{\sigma}^\mu b | B \rangle \simeq \langle c | c^\dagger \bar{\sigma}^\mu b | b \rangle = \bar{u} \gamma^\mu P_L u$

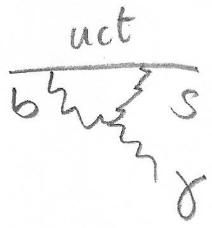
$\Rightarrow \Gamma_{B \to X_c \ell \bar{\nu}} \propto |V_{cb}|^2 \cdot$ known stuff (like μ-decay)

$\Rightarrow$ extract $|V_{cb}|$.

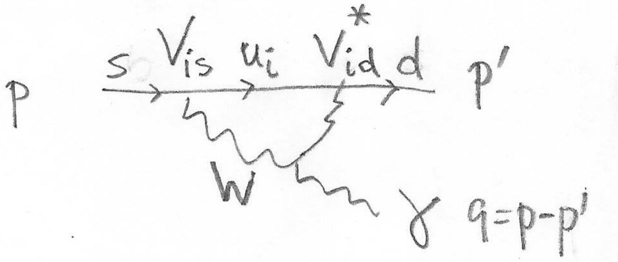
note : $\propto \frac{m_b^5}{m_W^4} \leftarrow$ what value for m_b ?

pole mass ?
$\overline{MS}$ mass?
B-meson mass?

Indirect example: a FCNC: $b \to s\gamma$



$B \to X_s \gamma$



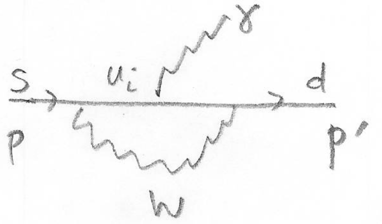
$K^- \to \pi^- \gamma$

$$\Rightarrow A \sim \sum_i V_{id}^* V_{is} = \underbrace{\left[V^\dagger V\right]}_{\mathbb{1}}{}_{ds} = 0$$

forgot that u_i-quark propagator has i-dependence

$$\sum_i V_{id}^* \frac{\not p + m_i}{p^2 - m_i^2} V_{is} \Rightarrow \left[V^\dagger M V\right]_{ds} \neq 0$$

need to estimate the loop diagram



$$A \sim g_W^2 e \sum_i V_{is} V_{id}^* \; \bar u_{p'} \gamma^\mu P_L \frac{\not p' + \not l + m_i}{(p'+l)^2 - m_i^2} \gamma^\alpha \frac{\not p + \not l + m_i}{(p+l)^2 - m_i^2} \gamma^\nu P_L u_p$$

$$\int \frac{d^4 l}{(2\pi)^4} \qquad \frac{g_{\mu\nu}}{l^2 - m_W^2} \epsilon_\alpha$$

imagine 3rd gen not known (70s), charm not discovered
but anticipated

⇒ $m_i \ll m_W$

$$V_{CKM} \sim \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix}$$

expand for small m_i: $\mathcal{O}(m_i^0)$ i-independent propagators

$$\frac{\not p + \not l}{(p+l)^2}$$

$$\mathcal{A}^0 \sim V^\dagger V\big|_{ds} = 0 \quad \text{but no } \frac{1}{M_W^2}$$

$\mathcal{O}(m_i^{odd})$ P_L (odd # of γ-matrices) $P_L = 0$

$$\mathcal{O}(m_i^2) \quad \sum_i m_i^2 V_{is} V_{id}^* \; \bar u_{p'} \gamma^\alpha P_L u_p \int d^4 l \, \frac{1}{\underbrace{(l+p')^2 (l+p)^2 \, l^2 - M_W^2}_{}}$$

$\frac{1}{M_W^2}$ by dim'l analysis

if $p, p' \ll M_W$.

⇒ 

$$e\frac{g^2}{16\pi^2}\left[\underbrace{V_{us}V_{ud}^*}_{c_\theta s_\theta}\overset{\text{small}}{\cancel{\frac{m_u^2}{m_W^2}}} + \underbrace{V_{cs}V_{cd}^*}_{c_\theta s_\theta}\frac{m_c^2}{m_W^2} + \underset{\text{small}}{\cancel{V_{ts}V_{td}^*\frac{m_t^2}{m_W^2}}}\right]$$

(and not anticipated)

$$\Rightarrow A_{K\to\pi\gamma} \propto e\frac{g^2}{16\pi^2} c_\theta s_\theta \frac{m_c^2}{m_W^2} \sim 10^{-7}$$

[naive estimate: $e\frac{g^2}{16\pi^2} c_\theta s_\theta \sim 10^{-3}$ $\Rightarrow \Gamma_{naive}$ 10^8 times too large.]

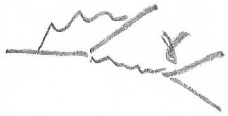
$\Rightarrow \Gamma_{K\to\pi\gamma} \propto m_c^4$ $\Longrightarrow$ from known decay width $K\to\pi\gamma$ an upper bound on m_c predicted $m_c < 2\,\text{GeV}$.

Glashow-Iliopoulos Maiani GIM

GIM mechanism $\sum_j V_{ij} V_{kj}^* f\left(\frac{m_j}{M}\right)$

must be proportional to small quark masses m_j.

note: $K^- \to \pi^- e^+ e^-$ FCNC $Br \sim 3\cdot 10^{-7}$



$\left(\frac{e^2}{16\pi^2}\right)^2$

$K^- \to \pi^0 e^- \bar\nu$ FCCC $Br \sim 3\cdot 10^{-2}$