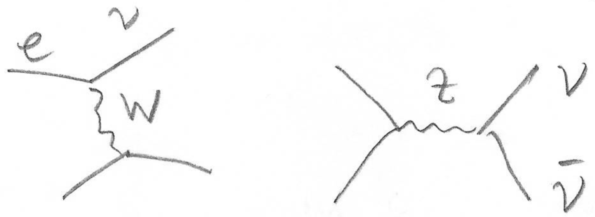
Cosmology:

big bang, SM particles in chemical & thermal equilibrium at high temperatures



this includes the neutrinos.

for $T \gg$ mass, have $n \sim T^3$, $\rho \sim T^4$

$T \sim$ MeV is coincidentally where neutrinos decouple because

$\Gamma \sim \frac{T^5}{M_W^4}$ becomes too small for neutrinos to scatter.

$\Rightarrow$ from then on neutrinos are free particles and their number is conserved

$T \sim$ MeV is also when He forms from free p, p, n, n

"Nucleosynthesis" (BBN)

to get prediction of $\frac{n_{He}}{n_H}$ correct must have

$m_\nu \ll$ MeV,

at BBN also have $n_\nu \sim T^3 \sim n_\gamma$

at later times the number densities of photons and neutrinos dilute like $1/a^3$ ← scale factor

$$\Rightarrow \quad n_\nu^{\text{today}} \sim n_\gamma^{\text{today}}$$

if neutrinos have masses $> 10^{-3}$ eV then they are non-relativistic today and their energy density is

$$\rho_\nu^{\text{today}} \sim m_\nu \, n_\gamma^{\text{today}}$$

of for all 3 neutrino species

$$\rho_\nu^{\text{today}} = \sum_i m_i \, n_\gamma^{\text{today}}$$

for the energy density in matter we have $\rho_m(T\sim eV) \sim \rho_\gamma(T\sim eV)$

$$\Rightarrow \rho_m^{\text{today}} \sim eV \, n_\gamma^{\text{today}} \qquad \qquad \simeq eV \, n_\gamma$$

demanding that not all matter is neutrinos we have

$$\rho_\nu^{\text{today}} < \rho_m^{\text{today}} \quad \Leftrightarrow \quad \sum m_i \lesssim eV \, .$$

this can be refined by studying the impact of neutrino masses on the CMB

Planck 2018 (1807.06209) find

$$\sum m_i \lesssim 0.2\, eV$$

Oscillations   weak interactions produce flavor eigenstates

e.g. $\pi^+ \to \mu^+ \nu_\mu$

these are linear combinations of the mass eigenstates

$$|\nu_\alpha\rangle = \sum_i V^*_{\alpha i} |\nu_i\rangle$$

↑ flavor basis   ↑ mass basis

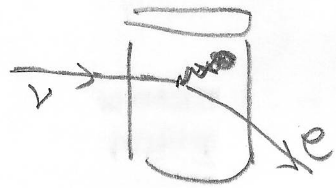
assuming for simplicity that these are plane waves with definite momentum the states evolve in time with e^{-iHt}

$$\Rightarrow |\nu_\alpha(t)\rangle = \sum_i V^*_{\alpha i} e^{-iE_i t} |\nu_i\rangle$$

with $E_i = \sqrt{p^2 + m_i^2}$

after propagating for some distance $L = ct$

we measure the flavor of the state with another weak interaction

e.g.  in a detector

$\Rightarrow$ We measure $|\langle \nu_\beta | \nu_\alpha(t) \rangle|^2 = \left| \sum_i e^{-iE_i t} V^*_{\alpha i} V_{\beta i} \right|^2$

$$= \sum_{i,j} e^{-i(E_i - E_j)t} V^*_{\alpha i} V_{\beta i} V_{\alpha j} V^*_{\beta j}$$

Example: 2 flavors 1,2 CP preserved for 2 flavors (see HW)

$$\Rightarrow V = \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix} = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix}$$

$$P_{1\to 2} = \sum_{i,j=1}^{2} e^{-i(E_i - E_j)L} V_{1i} V_{2i} V_{1j} V_{2j}$$

$$= \overset{i=j=1}{s_\theta^2 c_\theta^2} + \overset{i=j=2}{s_\theta^2 c_\theta^2} - \overset{i=1\ j=2}{s_\theta^2 c_\theta^2} \left(e^{-i(E_1-E_2)L} + e^{i(E_1-E_2)L} \right)$$

$$= 2 s_\theta^2 c_\theta^2 \left(1 - \cos (E_1 - E_2) L\right)$$

$$= \sin^2 2\theta \ \sin^2\left(\frac{(E_1 - E_2)}{2} L\right)$$

now expand $E_i = \sqrt{p^2 + m_i^2} \approx p + \frac{m_i^2}{2p} + \mathcal{O}(m_i^4)$

$$\Rightarrow E_1 - E_2 = \frac{m_1^2 - m_2^2}{2p} \equiv \frac{\Delta m^2}{2E}$$

$$\Rightarrow P_{1\to 2} = \sin^2 2\theta_{12} \ \sin^2\left(\frac{\Delta m_{12}^2}{4E} L\right)$$

and $P_{1\to 1} = 1 - P_{1\to 2} = 1 - \sin^2 2\theta \ \sin^2 \frac{\Delta m^2}{4E} L$

↑ survival probability

limits: $L \ll \frac{4E}{\Delta m^2} \equiv L_{osc} \Rightarrow P_{1\to 2} \propto \left(\frac{L}{L_{osc}}\right)^2$

$L = 0 \Rightarrow P_{1\to 2} = 0$ (obviously)

$L \gg L_{osc} \Rightarrow P_{1\to 2} = \sin^2 2\theta_{12} \cdot \frac{1}{2}$

↖ oscillations averaged.