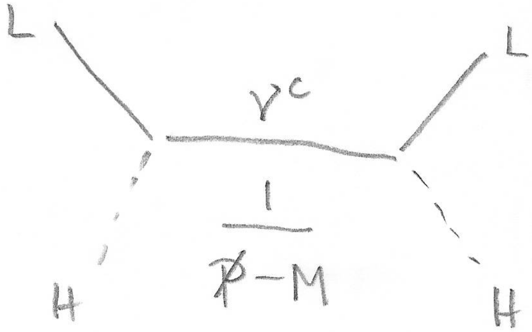
Effective Field theory perspective

$M \gg \text{TeV} \Rightarrow$ integrate out ν^c

$$\mathcal{L} = -\frac{M}{2}\nu^c\nu^c - \lambda_\nu \nu^c (HL)$$

SU(2)weak indices contracted



at energies $E \ll M$, approximate $\frac{1}{\not{p}-M} \approx \frac{-1}{M}$

the effect of this interaction is reproduced by the effective Lagrangian

$$\mathcal{L}_{eff} = -\frac{1}{2}\lambda_\nu (HL)\frac{-1}{M}\lambda_\nu(HL) = \frac{1}{2}\lambda_\nu \frac{1}{M}\lambda_\nu (HL)^2$$

$$H = \frac{v}{\sqrt{2}}\begin{pmatrix}1\\0\end{pmatrix} \Rightarrow \frac{1}{2} m_D \frac{1}{M} m_D \nu\nu \Rightarrow m_\nu = \frac{m_D^2}{M}$$

Higher dimensional ~~operators~~ (couplings)

Mass dimensions of fields follow from kinetic terms

$[\,.\,]$ ~ mass dimension notation

e.g. $[M]=1$, $[1]=0$, $[E]=[p]=1$, $[x]=-1$, $[dx]=-1$

$[\text{Action}]=0 \qquad \text{Action} = \int d^4x\, \mathcal{L}$

$$\Rightarrow [\mathcal{L}]=4$$

$$\mathcal{L} \sim (\partial_\mu \phi)^2 + (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + \psi^\dagger \sigma \cdot \partial \psi + \ldots$$

$$\Rightarrow [\phi]=1 \qquad \Rightarrow [A_\mu]=1 \qquad \Rightarrow [\psi]=3/2$$

<u>interactions</u>: $[\phi^4]=4 \Rightarrow \underset{\substack{\uparrow \\ \text{dimensionless}}}{\lambda}\phi^4$

$$D_\mu = \underset{\substack{\nwarrow \\ 1}}{\partial_\mu} - i\underset{\substack{\uparrow \\ 0}}{g}\underset{\substack{\nwarrow \\ 1}}{A_\mu}, \qquad \underset{\substack{\uparrow \\ 0}}{\lambda^E}\, \underbrace{E^c \tilde{H} L}_{4}$$

$\Rightarrow$ SM couplings are all dimensionless!

couplings with more powers of fields have $\frac{1}{\text{mass}}$ couplings.

Unique "dimension 5" operator in SM is $[(LH)^2]=5$.

__Note__: we arrived at $(HL)^2$ coupling from "integrating out" ν^c, but we could have just not had ν^c from the start and written

$$\mathcal{L} = -\frac{\lambda_{eff}}{2\Lambda}(HL)^2$$

$$\overset{\langle H \rangle}{\Rightarrow} \quad m_\nu = \lambda_{eff}\frac{v^2}{2\Lambda}$$

This is the simplest way of adding neutrino masses in the SM (no ν^c), just write $(HL)^2$ coupling.

$\Rightarrow$ __2 equally good paradigms in SM__

① $U(1)_L$ preserved

$$\lambda_\nu \nu^c HL$$

$$\Rightarrow m_\nu = \lambda_\nu \frac{v}{\sqrt{2}}$$

② $U(1)_L$ broken to $\mathbb{Z}_{2L}$

$$-\frac{\lambda}{2\Lambda}(LH)^2$$

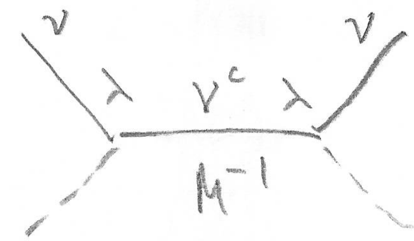
$$\Rightarrow m_\nu = \frac{\lambda v^2}{2\Lambda}$$

(or the more elaborate $\lambda \nu^c HL + \frac{M}{2}\nu^c\nu^c$)

lepton # not conserved

$\Rightarrow$ can have $nn \to p^+e^-p^+e^-$

3 generations



$$m_\nu = - m^T M^{-1} m$$

or leaving out ν^c:

$$- \frac{\lambda_{ij} (H L_i)(H L_j)}{2\Lambda} \Rightarrow m_{\nu_{ij}} = \frac{\lambda_{ij} v^2}{2\Lambda}$$

in either case m_ν is not diagonal, it is symmetric, complex

$\Rightarrow$ can be written as $m_\nu = V_\nu^T \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix} V_\nu$

redefine $\nu' = V_\nu \nu \quad \Rightarrow V_{PMNS} = V_\nu V_E^\dagger$

as in the Dirac case.

back to single generation, with ν^c, $M \gg m_D$

$$(\nu\ \nu^c)\begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix}\begin{pmatrix} \nu \\ \nu^c \end{pmatrix}$$

ν has weak interactions, "active"

ν^c has no " " , "sterile"

$$\nu = \cos\theta\ \nu_L + \sin\theta\ \nu_H$$

$$\nu^c = -\sin\theta\ \nu_L + \cos\theta\ \nu_H$$

ν_L, ν_H mass eigenstates

$$\theta \sim \frac{m_D}{M}$$

W couplings:

$$\frac{g}{\sqrt{2}}\, \nu^\dagger \bar{\sigma}^\mu W_\mu^+ E^- \rightarrow \frac{g}{\sqrt{2}} \cos\theta\ \nu_L^\dagger \bar{\sigma}^\mu W_\mu^+ E^-$$

$\Rightarrow$ W-couplings are suppressed by $\cos\theta$ due to active-sterile mixing

e.g. μ^- decay 

$\sim (g\cos\theta_\mu)^2 (g\cos\theta_e)^2$

experiment: $\delta g \lesssim 10^{-4} \Rightarrow \theta \lesssim 10^{-2}$

Neutrino mass review:

A. Lepton # preserved L

requires "right-handed" neutrino $\lambda^\nu \nu^c H L \rightarrow m^\nu \nu^c \nu$

$\uparrow$ 10^{-10} — Dirac mass

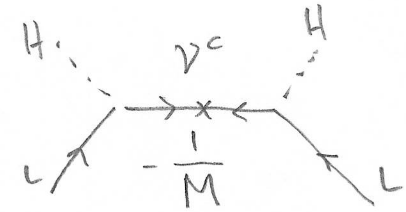
B. Lepton # violated $\cancel{L}$

can write $(HL^i)_\alpha \epsilon^{\alpha\beta} (HL^j)_\beta \dfrac{k_{ij}}{\Lambda} \longrightarrow \dfrac{m^\nu_{ij}}{2} \nu_i \nu_j$ — Majorana mass

$\uparrow$ spinor space; k_{ij}: 3×3 flavor

this is a dimension 5 operator $\Rightarrow$ New Physics at scale $\lesssim \Lambda$

e.g.



$\sim -\frac{1}{2}\left(\lambda_\nu^T \frac{1}{M} \lambda_\nu\right)_{ij} (HL)^i (HL)^j$

"right-handed" neutrino with large Majorana mass $\Rightarrow \dfrac{k_\nu}{\Lambda} \equiv -\lambda_\nu^T M^{-1} \lambda_\nu$

example $\lambda_\nu \sim \lambda_\tau \sim 10^{-2} \Rightarrow M \sim 10^{11}$ GeV

3 light neutrinos

3 super-heavy "

assume super-heavy ν's irrelevant $M \gg \text{TeV}$

majorana mass $\frac{1}{2} m_\nu^{ij} \nu^i \nu^j$

diagonalize $U_\nu^T m_\nu U_\nu = \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix}$

$$\Rightarrow \frac{1}{2} \nu^T m_\nu \nu = \frac{1}{2} \underbrace{\nu^T U_\nu^{\dagger T}}_{\nu'^T} \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix} \underbrace{U_\nu^\dagger \nu}_{\nu'}$$

$$\Rightarrow \nu = U_\nu \nu'$$

plug into W-couplings: $e^\dagger \bar{\sigma}^\mu W_\mu \nu \to e'^\dagger \underbrace{U_e^\dagger U_\nu}_{\equiv \tilde{V}_{PMNS}} \bar{\sigma}^\mu W_\mu \nu'$

parameters in $\tilde{V}$?

			real	imag
total parameters	λ^e	18	9	9
	K^ν	12	6	6
− symmetries (all broken)	$(U(3))^2$	9+9	3+3	6+6
physical parameters		12	9	3
			6 masses 3 angles	3 phases !

extra phases $\overset{\text{relative to Dirac case}}{}$ because cannot redefine phases of ν^i without making m^ν_i complex.

$$\tilde{V}_{PMNS} = V_{PMNS}\, P \leftarrow \begin{pmatrix} e^{i\eta_1} & & \\ & e^{i\eta_2} & \\ & & 1 \end{pmatrix}$$

$\uparrow$ 3 angles, ϕ like CKM

hard to observe

experiments?

$\pi^- \to \mu^- \bar{\nu}$ $\quad |\vec{P}_\mu| = \frac{1}{2m_\pi}\left(m_\pi^2 - m_\mu^2 - m_\nu^2 \frac{m_\pi^2 + m_\mu^2}{m_\pi^2 - m_\mu^2} + \mathcal{O}(m_\nu^4)\right)$

mass difference of initial to final $Q \approx m_\pi - m_\mu \approx 30$ MeV

sensitivity to m_ν^2 proportional to $\frac{m_\pi^2 + m_\mu^2}{m_\pi^2 - m_\mu^2} \approx \frac{2m_\pi}{Q}$

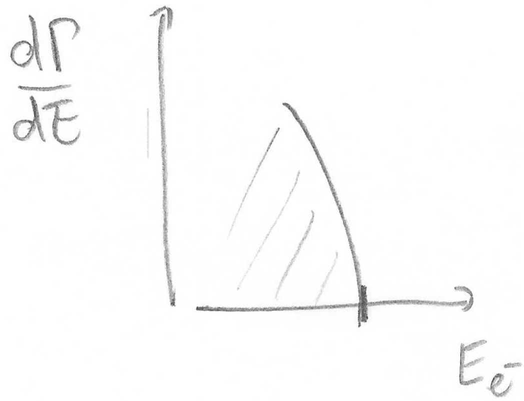
bound: $m_{\nu_\mu} \lesssim 190$ keV

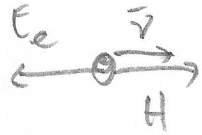
$\tau \to \nu_\tau + 5\pi$ $\quad m_{\nu_\tau} \lesssim 18.2$ MeV

${}^3H \to {}^3He + e + \bar{\nu}$ $\quad m_{\nu_e} \lesssim 1.1$ eV

Tritium $\quad Q \approx 18.6$ eV

look for end point of e^- spectrum

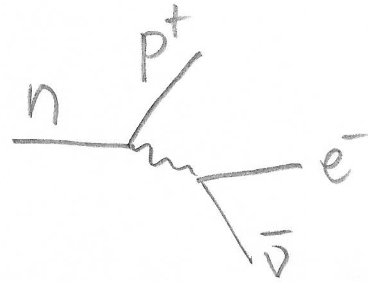




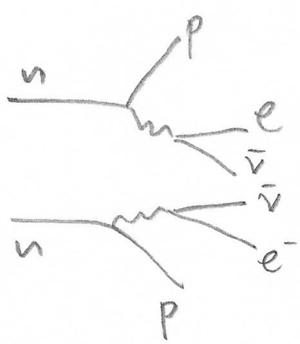
KATRIN experiment, target ~0.2 eV sensitivity

ν-less double β decay: $nn \to ppee$

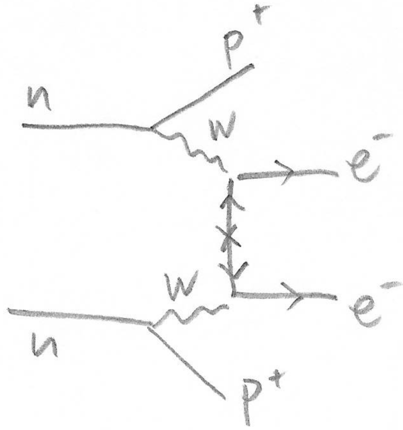
β decay



double β:



neutrino less $\beta\beta$



only exists when $\not{L}$

ee-component

proportional to majorana mass m_ν^{ee} in amplitude

e.g. ${}^{76}_{32}Ge \to {}^{76}_{34}Se + 2e^-$

$$m_\nu^{ee} \lesssim 0.34\,eV$$

significant nuclear physics uncertainty

m_ν^{ee} in charged l mass basis :

$$(\nu_e\ \nu_\mu\ \nu_\tau)\,(m_\nu)\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}$$

transform to neutrino mass basis $\tilde{V}^*_{PMNS}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix} = \begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}$

$$\Rightarrow (\nu_1\ \nu_2\ \nu_3)\,\underbrace{\tilde{V}^\dagger (m_\nu)\,\tilde{V}^*}_{\begin{pmatrix}m_1 & & \\ & m_2 & \\ & & m_3\end{pmatrix}}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

$$\Rightarrow m_\nu^{ee} = \left[\tilde{V}\begin{pmatrix}m_1 & & \\ & m_2 & \\ & & m_3\end{pmatrix}\tilde{V}^T\right]^{ee} = \sum_i m_i \tilde{V}_{ei}^{2}$$

contains the majorana phases η_1, η_2

future SNO+ (eg. arXiv 1904.01418)

^{130}Te double β decay $Q \sim 2.5$ MeV

$m_{ee} \lesssim 0.04 \sim 0.1$ target