Flavor in the Standard Model

charge conjugate fields:

Ex: complex scalar field ϕ $\qquad \mathcal{L} \sim (\partial_\mu \phi)^* (\partial^\mu \phi) - m^2 \phi^* \phi + \ldots$

U(1) symmetry: $\phi \to e^{i\theta Q} \phi$

θ transformation parameter

Q charge of ϕ

consider $\phi^c \equiv \phi^*$, the charge conjugate field

$$U(1): \quad \phi^c \equiv \phi^* \longrightarrow (e^{i\theta Q} \phi)^* = e^{i\theta(-Q)} \phi^c$$

$\Rightarrow \phi^c$ has the opposite charge, $-Q$.

$$\mathcal{L} \sim (\partial_\mu \phi^c)^* \partial^\mu \phi^c - m^2 \phi^{c*} \phi^c + \ldots$$

Theories with ϕ & ϕ^c are equivalent.

Note that the anti-particles of ϕ are the particles of ϕ^c and vice-versa.

Ex: Right-handed (chirality) fermion spinor ψ_R

$$\mathcal{L} \sim \psi_R^\dagger \, i\sigma^\mu D_\mu \psi_R$$

Symmetry: $\psi_R \to e^{i\theta Q}\psi_R$

$$D_\mu = \partial_\mu - igQA_\mu \quad \text{when acting on } \psi_R$$

complex conjugate field ψ_R^* has charge $-Q$

but transforms incorrectly under rotations:

$$\psi_R \to e^{i\alpha^a \frac{\sigma^a}{2}}\psi_R$$

$$\psi_R^* \to e^{-i\alpha^a \frac{\sigma^{a*}}{2}}\psi_R^*$$

so rotations about the three coordinates become messed up:

$$\sigma^1 \to -\sigma^1$$
$$\sigma^2 \to -\sigma^{2*} = \sigma^2$$
$$\sigma^3 \to -\sigma^3$$

consider instead $\psi^c \equiv i\sigma_2 \psi_R^*$

$\uparrow$ in spinor space

transforms correctly: $\psi^c \to i\sigma_2 e^{-i\alpha^a \frac{\sigma^{a*}}{2}}\psi_R^* = e^{i\alpha^a \frac{\sigma^a}{2}} i\sigma_2\psi_R^*$

$$= \quad " \quad \psi^c$$

can show that ψ^c transforms opposite to ψ_R under boosts $\Rightarrow$ ψ^c is a left-handed spinor, ψ^c_L

lesson: charge conjugate fermion spinors have opposite charge and opposite chirality.

Can use this to rewrite all the right-chirality fields in SM in terms of left-chirality fields.

Note also $(D_\mu \psi_R)^* = (\partial_\mu - ig(-Q)A_\mu)\psi_R^* \equiv D_\mu \psi_R$

$\Rightarrow$ covariant derivative "knows" to reverse the sign of the charge when acting on conjugate field.

Check kinetic term: $\psi_R^\dagger i\sigma^\mu D_\mu \psi_R$

substitute: $\psi_R = i\sigma_2 \psi^{c*}$ transpose fermi statistics acts on field on left

$$\psi^{cT} -i\sigma_2 \, i\sigma^\mu D_\mu \, i\sigma_2 \psi^{c*} = - \psi^{c\dagger}(i\sigma_2)^T (i\sigma^\mu \overleftarrow{D}_\mu)^T (-i\sigma_2)^T \psi^c$$

integrate by parts $\overrightarrow{=}$ $+\psi^{c\dagger}(-i\sigma_2)\, i\sigma^{\mu T} \overrightarrow{D}_\mu \, i\sigma_2 \psi^c = \psi^{c\dagger} i\bar{\sigma}^\mu D_\mu \psi^c$

using $\sigma_2 \sigma^{iT} \sigma_2 = -\sigma^i$ and $\sigma^\mu = (1, \sigma^i)$, $\bar{\sigma}^\mu = (1, -\sigma^i)$.

$\Rightarrow$ proper left-handed kinetic term.

$\Rightarrow$ can use charge conjugate fields also for fermions.

<u>fermions in SM</u> $Q \; U^c \; D^c \; L \; E^c$ all left-handed

charges:

	Q	U^c	D^c	L	E^c
$SU(3)$	3	3^*	3^*	-	-
$SU(2)$	2	-	-	2	-
$U(1)$	$1/6$	$-2/3$	$1/3$	$1/2$	1

kinetic terms: $Q^\dagger i\bar{\sigma}^\mu D_\mu Q + U^{c\dagger} i\bar{\sigma}^\mu D_\mu U^c + \ldots$

3 copies of each for the 3 generations

e.g. $Q^\dagger_i \, i\bar{\sigma}^\mu D_\mu Q_i \qquad i \in 1,2,3$

has a $U(3)$ flavor symmetry $Q \to \mathcal{U} Q$ unitary 3x3 matrix

$\Rightarrow$ kinetic terms for Q, U^c, D^c, L, E^c have 5 independent $U(3)$ symmetries $\Rightarrow U(3)^5$

fermion masses: recall Dirac mass

$$-m\psi_R^\dagger\psi_L + h.c. = -m\psi^{cT}{-i\sigma_2}\psi_L + h.c.$$

$$= m\psi^{cT} i\sigma_2\psi_L + h.c.$$

$$\equiv m\psi^c\psi + h.c.$$

both left-handed
contracted with $i\sigma_2$

3 generations: $\Rightarrow$ $m_{ij}\psi^c_i\psi_j$. m_{ij} arbitrary complex 3x3

e.g. up-type quarks in SM:

$m^u_{ij}u^c_iu_j$ from $\lambda^u_{ij}u^c_i\langle H\rangle^TQ_j$

$\langle H\rangle = \frac{v}{\sqrt{2}}\begin{pmatrix}1\\0\end{pmatrix}$

$Q_j = \begin{pmatrix}u\\D\end{pmatrix}_j$

$$\Rightarrow m^u_{ij} = \lambda^u_{ij}\frac{v}{\sqrt{2}}$$

down... $\lambda^D_{ij}D^c_i\langle\tilde{H}\rangle^TQ_j \rightarrow m^D_{ij} = \lambda^D_{ij}\frac{v}{\sqrt{2}}$

where $\tilde{H}\equiv i\sigma_2 H^*$

in $SU(2)_{weak}$-space

physical masses? diagonalize m by redefining fermion fields

matrix notation $$D^{cT} m^D D = \underbrace{D^{cT} V_{D^c}^\dagger}_{\equiv D^{c\prime T}} \overbrace{\begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix}}^{m^D} \underbrace{V_D D}_{\equiv D'}$$

where we use that a complex matrix can be diagonalized with 2 unitary matrices.

so we redefine: $D' \equiv V_D D$

$D^{c\prime} \equiv V_{D^c}^* D^c$

but these are $U(3)$ transformations $\Rightarrow$ kinetic terms invariant.

aside:
an arbitrary complex matrix is diagonalized by bi-unitary transformation $U_L M U_R^\dagger = \text{diag}$.

how to find U_L, U_R? $U_L M M^\dagger U_L^\dagger = \text{diag}^2$

$U_R M^\dagger M U_R^\dagger = \text{diag}^2$

$MM^\dagger$ and $M^\dagger M$ are hermitian, $\Rightarrow$ can be diagonalized by single unitary transformation.

problem: D lives inside $Q = \begin{pmatrix} u \\ D \end{pmatrix}$ $\Rightarrow$ rotation changes relative basis between u, D

$$\begin{pmatrix} u \\ D \end{pmatrix}^{+} i\bar\sigma^\mu \left(\partial_\mu - i g_3 G_\mu - i \frac{g_2}{\sqrt{2}} \begin{pmatrix} \frac{W_{3\mu}}{\sqrt{2}} & W_\mu^{+} \\ W_\mu^{-} & \frac{W_{3\mu}}{\sqrt{2}} \end{pmatrix} - i g_Y \frac{1}{6} B_\mu \right) \begin{pmatrix} u \\ D \end{pmatrix}$$

$$\rightarrow -\frac{g}{\sqrt{2}} u^{+} i\bar\sigma^\mu W_\mu^{+} V_D^{+} D' \quad + h.c.$$

all other terms are invariant.

diagonalize up-mass: $u^{cT} m^u u = u^{c\prime T} \begin{pmatrix} m^u & & \\ & m^c & \\ & & m^t \end{pmatrix} u'$

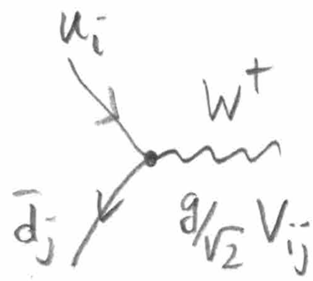
$$u' \equiv V_u u$$

$\Rightarrow$ W coupling $-\frac{g}{\sqrt{2}} u'^{+} i\bar\sigma^\mu W_\mu^{+} \underbrace{V_u V_D^{+}}_{\equiv V_{CKM}} D' \quad + h.c.$

or with explicit flavor indices (dropping primes on fields)

$$\frac{g}{\sqrt{2}} V_{CKM}^{ij} u_i^{+} i\bar\sigma^\mu W_\mu^{+} D_j$$

flavor changing charged current



note that Z, γ couplings involve

$V_u V_u^\dagger = \mathbb{1} = V_D V_D^\dagger \Rightarrow$ no flavor changing neutral currents in SM!

(at tree level).

$V_{CKM}^\dagger V_{CKM} \equiv \mathbb{1} \Rightarrow$ 9 parameters: 3 angles, 6 phases

can still redefine quark field phases $u_i \to e^{i\alpha_i} u_i$
$d_i \to e^{i\beta_i} d_i$

$\Rightarrow$ can remove 5 phases from V_{CKM}

only 5 (and not 6) because overall phase $u, d \to e^{i\alpha} u, d$ does nothing to V_{CKM}.

$\Rightarrow$ in simplest basis for quarks have V_{CKM} with 3 angles, 1 phase.