difference from all other fermions : ν^c has no gauge charges

$\Rightarrow$ can also write a <u>Majorana</u> <u>mass</u>

$$\mathcal{L} \sim \frac{M}{2} \nu^{c^T} i\sigma_2 \nu^c \equiv \frac{M}{2} \nu^c_i \nu^c_j \epsilon^{ij} \leftarrow \text{spinor indices}$$

Recall, in SM must use Higgs to write fermion masses because of combination of gauge and Lorentz symmetries

$m Q^\dagger Q$ not Lorentz-invariant !

$$\left.\begin{array}{ll} U^c Q = (1,2)_{-1/2} & D^c Q = (1,2)_{1/2} \\ E^c L = (1,2)_{1/2} & \nu^c L = (1,2)_{-1/2} \end{array}\right\} \text{all require Higgs}$$

$U^c U^c = (\bar{3} \times \bar{3}, 1)_{-4/3}$ not gauge invariant

but $\nu^c \nu^c = (1,1)_0$ gauge and Lorentz invariant

ν^c has lepton # $\Rightarrow$ $m \nu^c \nu^c$ breaks Lepton #

a $\mathbb{Z}_2$ parity symmetry remains : $\nu^c \to -\nu^c$
$L \to -L$

<u>choice</u>: • impose L# on theory $\Rightarrow$ ν-masses are Dirac
• do not impose " $\Rightarrow$ longer story ... majorana

Dirac + Majorana masses

1. generation $\quad m\,\nu^c\nu + \frac{M}{2}\,\nu^c\nu^c + \text{h.c.}$

$$= \frac{1}{2}\left(\nu\ \nu^c\right)\begin{pmatrix} 0 & m \\ m & M \end{pmatrix}\begin{pmatrix} \nu \\ \nu^c \end{pmatrix} + \text{h.c.}$$

diagonalize mass matrix

$$m_{H/L} = \frac{M}{2} \pm \sqrt{\frac{M^2}{4} + m^2}$$

cases: • $M=0 \Rightarrow \begin{pmatrix} \nu \\ i\sigma_2\nu^{c*} \end{pmatrix}$ 1 Dirac neutrino, mass m

• $m=0 \Rightarrow$ 2 (Weyl) neutrinos, masses $0, M$

DOF counting

Scalar field $\quad$ real scalar $\phi \quad \longleftrightarrow \quad$ Majorana

complex $\quad \longleftrightarrow \quad$ Dirac

complex ϕ : $\quad m^2 \phi^* \phi \quad$ write $\phi = \dfrac{\phi_1 + i\phi_2}{\sqrt{2}}$

$$= \frac{1}{2} (\phi_1 \; \phi_2) \begin{pmatrix} m^2 & 0 \\ 0 & m^2 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

U(1) symmetry $\quad \phi \to e^{i\alpha}\phi \longleftrightarrow \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \to R(\alpha) \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$

choose $m_1 \neq m_2 \Rightarrow$ symmetry broken ϕ_1, ϕ_2 independent real fields.

Also: ϕ_1 is real $\phi_1 = \phi_1^+ \Rightarrow$ no charge & particle = anti particle

Neutrinos $\quad$ 2 mass eigenstates $m_{H/L} = \dfrac{M}{2} \pm \sqrt{\dfrac{M^2}{4} + m^2}$

case 1. $\quad M \ll m \Rightarrow$ Dirac mass + small correction

approximate Dirac neutrino

case 2. $M \sim m$ interesting mess, but fine-tuned

$$m = \frac{\lambda v}{\sqrt{2}} \sim 0.1\,\text{eV}$$

$$M \sim 0.1\,\text{eV}$$

coincidence if these are the same

case 3: $M \gg m$ expand $m_H \approx M + \frac{m^2}{M} + \mathcal{O}\left(\frac{m^4}{M^3}\right)$

$$m_L \approx -\frac{m^2}{M}$$

can remove $\nu_L \to i\nu_L$

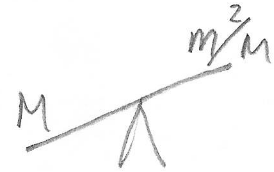
Note this was easy to understand

$$\mu = \begin{pmatrix} 0 & m \\ m & M \end{pmatrix}$$

$$\det\mu = -m^2 = m_1 m_2 \approx M \cdot \left(-\frac{m^2}{M}\right)$$

$$\mathrm{Tr}\,\mu = M = m_1 + m_2 \approx M + \frac{m^2}{M}$$

Interesting limit: $M \to \infty$, $m_L \to 0$ "see saw"



$$m_\nu = 0.1\,\text{eV} \sim \frac{(\lambda v)^2}{2M} \sim \lambda^2 \frac{(100\,\text{GeV})^2}{M} \Rightarrow \begin{cases} \lambda \text{ small} \\ M \text{ large} \end{cases} \text{ or}$$

e.g. $\lambda = \lambda_\tau \sim \frac{m_\tau}{v} \Rightarrow M \approx \frac{m_\tau^2}{m_\nu} \approx \frac{\text{GeV}^2}{0.1\,\text{eV}} = 10^{10}\,\text{GeV}$

"Right-handed ν-mass scale"

3 generations?

$$\lambda_{\nu\, ij}\, \nu^c_i \langle H^T \rangle \begin{pmatrix} \nu \\ e \end{pmatrix}_j + \frac{M_{ij}}{2} \nu^c_i \nu^c_j$$

$\leftarrow$ symmetric

$$\to m_{\text{Dirac}\, ij}\, \nu^c_i \nu_j + \frac{M_{ij}}{2} \nu^c_i \nu^c_j$$

$\Rightarrow$ 6x6 matrix $\frac{1}{2} \begin{pmatrix} \nu^T & \nu^{cT} \end{pmatrix} \begin{pmatrix} 0 & m^T \\ m & M \end{pmatrix} \begin{pmatrix} \nu \\ \nu^c \end{pmatrix}$

2/3/2020

diagonalize in limit $M \gg m$

1 generation case $m_H = M$

$$m_L \cong -\frac{m^2}{M} = -\frac{m\ m}{M}$$

3 generations $\frac{1}{M} \to M^{-1}$

$$m_H = M$$

$$m_L \cong -m^T M^{-1} m$$

diagram: $\nu \times \nu^c \times \nu$

$m^T \ -\frac{1}{M} \ m$