Similarly, whatever the nature of Planck-scale physics, it leads to some effective continuum quantum field theory. This quantum field theory might well be an accurate approximation to the underlying physics already at distances of 100 Planck lengths, corresponding to momenta of 10^{17} GeV. From here to the scale of weak interactions, and from there up to the wavelength of light, and from there to the size of the universe, quantum field theory can be treated as the basic framework for the equations of physics. By recognizing the symmetries of the particular set of field equations that Nature has provided us, we can learn to compute all of the details of physical processes over this whole enormous domain. And, by contemplating the origin of these symmetries, perhaps we will also be able to see through to the next level and unlock the true structure of spacetime.

Appendix

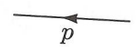
Reference Formulae

This Appendix collects together some of the formulae that are most commonly used in Feynman diagram calculations.

A.1 Feynman Rules

In all theories it is understood that momentum is conserved at each vertex, and that undetermined loop momenta are integrated over: $\int d^4p/(2\pi)^4$. Fermion (including ghost) loops receive an additional factor of (-1), as explained on page 120. Finally, each diagram can potentially have a symmetry factor, as explained on page 93.

ϕ^4 theory: $\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$

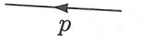
Scalar propagator:  $= \dfrac{i}{p^2 - m^2 + i\epsilon}$ (A.1)

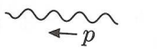
ϕ^4 vertex:  $= -i\lambda$ (A.2)

External scalar: $= 1$ (A.3)

(Counterterm vertices for loop calculations are given on page 325.)

Quantum Electrodynamics: $\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi - \frac{1}{4}(F_{\mu\nu})^2 - e\bar{\psi}\gamma^\mu\psi A_\mu$

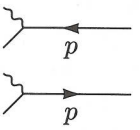
Dirac propagator:  $= \dfrac{i(p\!\!\!/ + m)}{p^2 - m^2 + i\epsilon}$ (A.4)

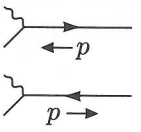
Photon propagator:  $= \dfrac{-ig_{\mu\nu}}{p^2 + i\epsilon}$ (A.5)

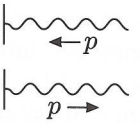
(Feynman gauge; see page 297 for

QED vertex:  $= iQe\gamma^\mu$ (A.6)

($Q = -1$ for an electron)

External fermions:  $= u^s(p)$ (initial), $= \bar{u}^s(p)$ (final) (A.7)

External antifermions:  $= \bar{v}^s(p)$ (initial), $= v^s(p)$ (final) (A.8)

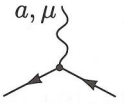
External photons:  $= \epsilon_\mu(p)$ (initial), $= \epsilon^*_\mu(p)$ (final) (A.9)

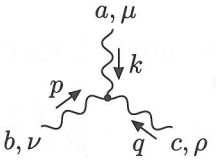
(Counterterm vertices for loop calculations are given on page 332.)

Non-Abelian Gauge Theory:

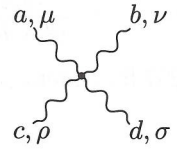
$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi - \tfrac{1}{4}(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)^2 + gA^a_\mu\bar{\psi}\gamma^\mu t^a\psi$$
$$- gf^{abc}(\partial_\mu A^a_\nu)A^{\mu b}A^{\nu c} - \tfrac{1}{4}g^2(f^{eab}A^a_\mu A^b_\nu)(f^{ecd}A^{\mu c}A^{\nu d})$$

The fermion and gauge boson propagators are the same as in QED, times an identity matrix in the gauge group space. Similarly, the polarization of external particles is treated the same as in QED, but each external particle also has an orientation in the group space.

Fermion vertex:  $= ig\gamma^\mu t^a$ (A.10)

3-boson vertex: 

$$= \begin{aligned} & gf^{abc}\big[g^{\mu\nu}(k-p)^\rho \\ & + g^{\nu\rho}(p-q)^\mu \\ & + g^{\rho\mu}(q-k)^\nu\big] \end{aligned} \quad \text{(A.11)}$$

4-boson vertex: 

$$= \begin{aligned} & -ig^2\big[f^{abe}f^{cde}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}) \\ & + f^{ace}f^{bde}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\rho}) \\ & + f^{ade}f^{bce}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma})\big] \end{aligned} \quad \text{(A.12)}$$

$= iQe\gamma^\mu$ (A.6)

$= -1$ for an electron)

$= u^s(p)$ (initial)
$= \bar{u}^s(p)$ (final) (A.7)

$= \bar{v}^s(p)$ (initial)
$= v^s(p)$ (final) (A.8)

$= \epsilon_\mu(p)$ (initial)
$= \epsilon^*_\mu(p)$ (final) (A.9)

iven on page 332.)

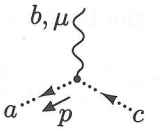
$gA^a_\mu\bar{\psi}\gamma^\mu t^a\psi$
$A^a_\mu A^b_\nu)(f^{ecd}A^{\mu c}A^{\nu d})$

the same as in QED, times
imilarly, the polarization of
), but each external particle

(A.10)

$k-p)^\rho$
$^\rho(p-q)^\mu$ (A.11)
$^\mu(q-k)^\nu]$

$^{cde}(g^{\mu\rho}g^{\nu\sigma}-g^{\mu\sigma}g^{\nu\rho})$
$^{bde}(g^{\mu\nu}g^{\rho\sigma}-g^{\mu\sigma}g^{\nu\rho})$ (A.12)
$^{bce}(g^{\mu\nu}g^{\rho\sigma}-g^{\mu\rho}g^{\nu\sigma})]$

Ghost vertex: 

$= -gf^{abc}p^\mu$ (A.13)

Ghost propagator: $a \cdots\cdots \overset{p}{\blacktriangleleft} \cdots\cdots b = \dfrac{i\delta^{ab}}{p^2+i\epsilon}$ (A.14)

(Counterterm vertices for loop calculations are given on pages 528 and 532.)

Other theories. Feynman rules for other theories can be found on the following pages:

Yukawa theory	page 118
Scalar QED	page 312
Linear sigma model	page 353
Electroweak theory	pages 716, 743, 753

A.2 Polarizations of External Particles

The spinors $u^s(p)$ and $v^s(p)$ obey the Dirac equation in the form

$$\begin{aligned} 0 &= (\not{p}-m)u^s(p) = \bar{u}^s(p)(\not{p}-m) \\ &= (\not{p}+m)v^s(p) = \bar{v}^s(p)(\not{p}+m), \end{aligned} \tag{A.15}$$

where $\not{p} = \gamma^\mu p_\mu$. The Dirac matrices obey the anticommutation relations

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}. \tag{A.16}$$

We use a chiral basis,

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \qquad \gamma^5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{A.17}$$

where

$$\sigma^\mu = (1, \boldsymbol{\sigma}), \qquad \bar{\sigma}^\mu = (1, -\boldsymbol{\sigma}). \tag{A.18}$$

In this basis the normalized Dirac spinors can be written

$$u^s(p) = \begin{pmatrix} \sqrt{p\cdot\sigma}\,\xi^s \\ \sqrt{p\cdot\bar{\sigma}}\,\xi^s \end{pmatrix}, \qquad v^s(p) = \begin{pmatrix} \sqrt{p\cdot\sigma}\,\eta^s \\ -\sqrt{p\cdot\bar{\sigma}}\,\eta^s \end{pmatrix}, \tag{A.19}$$

where ξ and η are two-component spinors normalized to unity. In the high-energy limit these expressions simplify to

$$u(p) \approx \sqrt{2E}\begin{pmatrix} \frac{1}{2}(1-\hat{p}\cdot\boldsymbol{\sigma})\xi^s \\ \frac{1}{2}(1+\hat{p}\cdot\boldsymbol{\sigma})\xi^s \end{pmatrix}, \qquad v(p) \approx \sqrt{2E}\begin{pmatrix} \frac{1}{2}(1-\hat{p}\cdot\boldsymbol{\sigma})\eta^s \\ -\frac{1}{2}(1+\hat{p}\cdot\boldsymbol{\sigma})\eta^s \end{pmatrix}. \tag{A.20}$$

Using the standard basis for the Pauli matrices,

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{A.21}$$

we have, for example, $\xi^s = \binom{1}{0}$ for spin up in the z direction, and $\xi^s = \binom{0}{1}$ for spin down in the z direction. For antifermions the physical spin is opposite to that of the spinor: $\eta^s = \binom{1}{0}$ corresponds to spin *down* in the z direction, and so on.

In computing unpolarized cross sections one encounters the polarization sums

$$\sum_s u^s(p)\bar{u}^s(p) = \not{p} + m, \qquad \sum_s v^s(p)\bar{v}^s(p) = \not{p} - m. \tag{A.22}$$

For polarized cross sections one can either resort to the explicit formulae (A.19) or insert the projection matrices

$$\left(\frac{1+\gamma^5}{2}\right), \qquad \left(\frac{1-\gamma^5}{2}\right), \tag{A.23}$$

which project onto right- and left-handed spinors, respectively. Again, for antifermions, the helicity of the spinor is opposite to the physical helicity of the particle.

Many other identities involving Dirac spinors and matrices can be found in Chapter 3.

Polarization vectors for photons and other gauge bosons are conventionally normalized to unity. For massless bosons the polarization must be transverse:

$$\epsilon^\mu = (0, \boldsymbol{\epsilon}), \qquad \text{where } \mathbf{p} \cdot \boldsymbol{\epsilon} = 0. \tag{A.24}$$

If $\mathbf{p}$ is in the $+z$ direction, the polarization vectors are

$$\epsilon^\mu = \frac{1}{\sqrt{2}}(0, 1, i, 0), \qquad \epsilon^\mu = \frac{1}{\sqrt{2}}(0, 1, -i, 0), \tag{A.25}$$

for right- and left-handed helicities, respectively.

In computing unpolarized cross sections involving photons, one can replace

$$\sum_{\text{polarizations}} \epsilon_\mu^* \epsilon_\nu \longrightarrow -g_{\mu\nu}, \tag{A.26}$$

by virtue of the Ward identity. In the case of massless non-Abelian gauge bosons, one must also sum over the emission of ghosts, as discussed in Section 16.3. In the massive case, one must in addition include the emission of Goldstone bosons, as discussed in Section 21.1.

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$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{A.21}$$

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In computing unpolarized cross sections one encounters the polarization sums

$$\sum_s u^s(p)\bar{u}^s(p) = \not{p} + m, \qquad \sum_s v^s(p)\bar{v}^s(p) = \not{p} - m. \tag{A.22}$$

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A.3 Numerator Algebra

Traces of γ matrices can be evaluated as follows:

$$\begin{aligned} \mathrm{tr}(\mathbf{1}) &= 4 \\ \mathrm{tr}(\text{any odd \# of } \gamma\text{'s}) &= 0 \\ \mathrm{tr}(\gamma^\mu\gamma^\nu) &= 4g^{\mu\nu} \\ \mathrm{tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) &= 4(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho}) \\ \mathrm{tr}(\gamma^5) &= 0 \\ \mathrm{tr}(\gamma^\mu\gamma^\nu\gamma^5) &= 0 \\ \mathrm{tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^5) &= -4i\epsilon^{\mu\nu\rho\sigma} \end{aligned} \tag{A.27}$$

Another identity allows one to reverse the order of γ matrices inside a trace:

$$\mathrm{tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\cdots) = \mathrm{tr}(\cdots\gamma^\sigma\gamma^\rho\gamma^\nu\gamma^\mu). \tag{A.28}$$

Contractions of γ matrices with each other simplify to:

$$\begin{aligned} \gamma^\mu\gamma_\mu &= 4 \\ \gamma^\mu\gamma^\nu\gamma_\mu &= -2\gamma^\nu \\ \gamma^\mu\gamma^\nu\gamma^\rho\gamma_\mu &= 4g^{\nu\rho} \\ \gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_\mu &= -2\gamma^\sigma\gamma^\rho\gamma^\nu \end{aligned} \tag{A.29}$$

(These identities apply in four dimensions only; see the following section.) Contractions of the ϵ symbol can also be simplified:

$$\begin{aligned} \epsilon^{\alpha\beta\gamma\delta}\epsilon_{\alpha\beta\gamma\delta} &= -24 \\ \epsilon^{\alpha\beta\gamma\mu}\epsilon_{\alpha\beta\gamma\nu} &= -6\delta^\mu{}_\nu \\ \epsilon^{\alpha\beta\mu\nu}\epsilon_{\alpha\beta\rho\sigma} &= -2\left(\delta^\mu{}_\rho\delta^\nu{}_\sigma - \delta^\mu{}_\sigma\delta^\nu{}_\rho\right) \end{aligned} \tag{A.30}$$

In some calculations, it is useful to rearrange products of fermion bilinears by means of *Fierz identities*. Let $u_1, \ldots, u_4$ be Dirac spinors, and let $u_{iL} = \frac{1}{2}(1-\gamma^5)u_i$ be the left-handed projection. Then the most important Fierz rearrangement formula is

$$(\bar{u}_{1L}\gamma^\mu u_{2L})(\bar{u}_{3L}\gamma_\mu u_{4L}) = -(\bar{u}_{1L}\gamma^\mu u_{4L})(\bar{u}_{3L}\gamma_\mu u_{2L}). \tag{A.31}$$

Additional formulae can be generated by the use of the following identities for the 2×2 blocks of Dirac matrices:

$$(\sigma^\mu)_{\alpha\beta}(\sigma_\mu)_{\gamma\delta} = 2\epsilon_{\alpha\gamma}\epsilon_{\beta\delta}; \qquad (\bar{\sigma}^\mu)_{\alpha\beta}(\bar{\sigma}_\mu)_{\gamma\delta} = 2\epsilon_{\alpha\gamma}\epsilon_{\beta\delta}. \tag{A.32}$$

In non-Abelian gauge theories, the Feynman rules involve gauge group matrices t^a that form a representation r of a Lie algebra G. The symbol G also denotes the adjoint representation of the algebra. The matrices t^a obey

near $x = -n$. Here γ is the Euler-Mascheroni constant, $\gamma \approx 0.5772$. The following combination of terms often appears in calculations:

$$\frac{\Gamma(2-\frac{d}{2})}{(4\pi)^{d/2}}\left(\frac{1}{\Delta}\right)^{2-\frac{d}{2}} = \frac{1}{(4\pi)^2}\left(\frac{2}{\epsilon} - \log\Delta - \gamma + \log(4\pi) + \mathcal{O}(\epsilon)\right), \tag{A.52}$$

with $\epsilon = 4 - d$.

Notice that Δ is positive if it is a combination of masses and *spacelike* momentum invariants. If Δ contains timelike momenta, it may become negative. Then these integrals acquire imaginary parts, which give the discontinuities of S-matrix elements. To compute the S-matrix in a physical region, choose the correct branch of the function by the prescription

$$\left(\frac{1}{\Delta}\right)^{n-\frac{d}{2}} \to \left(\frac{1}{\Delta - i\epsilon}\right)^{n-\frac{d}{2}}, \tag{A.53}$$

where $-i\epsilon$ (not to be confused with ϵ in the previous paragraph!) gives a tiny negative imaginary part.

Traces in Eq. (A.27) that do not involve γ^5 are independent of dimensionality. However, since

$$g^{\mu\nu}g_{\mu\nu} = \delta^\mu{}_\mu = d \tag{A.54}$$

in d dimensions, the contraction identities (A.29) are modified:

$$\begin{aligned}
\gamma^\mu\gamma_\mu &= d \\
\gamma^\mu\gamma^\nu\gamma_\mu &= -(d-2)\gamma^\nu \\
\gamma^\mu\gamma^\nu\gamma^\rho\gamma_\mu &= 4g^{\nu\rho} - (4-d)\gamma^\nu\gamma^\rho \\
\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_\mu &= -2\gamma^\sigma\gamma^\rho\gamma^\nu + (4-d)\gamma^\nu\gamma^\rho\gamma^\sigma
\end{aligned} \tag{A.55}$$

A.5 Cross Sections and Decay Rates

Once the squared matrix element for a scattering process is known, the differential cross section is given by

$$\begin{aligned}
d\sigma = &\frac{1}{2E_{\mathcal{A}}2E_{\mathcal{B}}\,|v_{\mathcal{A}} - v_{\mathcal{B}}|}\left(\prod_f \frac{d^3p_f}{(2\pi)^3}\frac{1}{2E_f}\right) \\
&\times \left|\mathcal{M}(p_{\mathcal{A}}, p_{\mathcal{B}} \to \{p_f\})\right|^2 (2\pi)^4\delta^{(4)}(p_{\mathcal{A}} + p_{\mathcal{B}} - \textstyle\sum p_f).
\end{aligned} \tag{A.56}$$

The differential decay rate of an unstable particle to a given final state is

$$d\Gamma = \frac{1}{2m_{\mathcal{A}}}\left(\prod_f \frac{d^3p_f}{(2\pi)^3}\frac{1}{2E_f}\right)\left|\mathcal{M}(m_{\mathcal{A}} \to \{p_f\})\right|^2 (2\pi)^4\delta^{(4)}(p_{\mathcal{A}} - \textstyle\sum p_f). \tag{A.57}$$

For the special case of a two-particle final state, the Lorentz-invariant phase space takes the simple form

$$\left(\prod_f \int \frac{d^3p_f}{(2\pi)^3}\frac{1}{2E_f}\right)(2\pi)^4\delta^{(4)}(\textstyle\sum p_i - \sum p_f) = \displaystyle\int \frac{d\Omega_{\rm cm}}{4\pi}\,\frac{1}{8\pi}\left(\frac{2|\mathbf{p}|}{E_{\rm cm}}\right), \tag{A.58}$$

near $x = -n$. Here γ is the Euler-Mascheroni constant, $\gamma \approx 0.5772$. The following combination of terms often appears in calculations:

$$\frac{\Gamma(2-\frac{d}{2})}{(4\pi)^{d/2}}\left(\frac{1}{\Delta}\right)^{2-\frac{d}{2}} = \frac{1}{(4\pi)^2}\left(\frac{2}{\epsilon} - \log\Delta - \gamma + \log(4\pi) + \mathcal{O}(\epsilon)\right), \tag{A.52}$$

with $\epsilon = 4 - d$.

Notice that Δ is positive if it is a combination of masses and *spacelike* momentum invariants. If Δ contains timelike momenta, it may become negative. Then these integrals acquire imaginary parts, which give the discontinuities of S-matrix elements. To compute the S-matrix in a physical region, choose the correct branch of the function by the prescription

$$\left(\frac{1}{\Delta}\right)^{n-\frac{d}{2}} \to \left(\frac{1}{\Delta - i\epsilon}\right)^{n-\frac{d}{2}}, \tag{A.53}$$

where $-i\epsilon$ (not to be confused with ϵ in the previous paragraph!) gives a tiny negative imaginary part.

Traces in Eq. (A.27) that do not involve γ^5 are independent of dimensionality. However, since

$$g^{\mu\nu}g_{\mu\nu} = \delta^\mu{}_\mu = d \tag{A.54}$$

in d dimensions, the contraction identities (A.29) are modified:

$$\begin{aligned}
\gamma^\mu\gamma_\mu &= d \\
\gamma^\mu\gamma^\nu\gamma_\mu &= -(d-2)\gamma^\nu \\
\gamma^\mu\gamma^\nu\gamma^\rho\gamma_\mu &= 4g^{\nu\rho} - (4-d)\gamma^\nu\gamma^\rho \\
\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_\mu &= -2\gamma^\sigma\gamma^\rho\gamma^\nu + (4-d)\gamma^\nu\gamma^\rho\gamma^\sigma
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&\times \left|\mathcal{M}(p_{\mathcal{A}}, p_{\mathcal{B}} \to \{p_f\})\right|^2 (2\pi)^4\delta^{(4)}(p_{\mathcal{A}} + p_{\mathcal{B}} - \textstyle\sum p_f).
\end{aligned} \tag{A.56}$$

The differential decay rate of an unstable particle to a given final state is

$$d\Gamma = \frac{1}{2m_{\mathcal{A}}}\left(\prod_f \frac{d^3p_f}{(2\pi)^3}\frac{1}{2E_f}\right)\left|\mathcal{M}(m_{\mathcal{A}} \to \{p_f\})\right|^2 (2\pi)^4\delta^{(4)}(p_{\mathcal{A}} - \textstyle\sum p_f). \tag{A.57}$$

For the special case of a two-particle final state, the Lorentz-invariant phase space takes the simple form

$$\left(\prod \int \frac{d^3p_f}{\quad}\frac{1}{\quad}\right)(2\pi)^4\delta^{(4)}(\textstyle\sum p_i - \sum p_f) = \int \frac{d\Omega_{\rm cm}}{\quad}\frac{1}{\quad}\left(\frac{2|\mathbf{p}|}{\quad}\right) \tag{A.58}$$

where $|\mathbf{p}|$ is the magnitude of the 3-momentum of either particle in the center-of-mass frame.

A.6 Physical Constants and Conversion Factors

Precisely known physical constants:

$$\begin{aligned}
c &= 2.998 \times 10^{10}\ \text{cm/s} \\
\hbar &= 6.582 \times 10^{-22}\ \text{MeV s} \\
e &= -1.602 \times 10^{-19}\ \text{C} \\
\alpha &= \frac{e^2}{4\pi\hbar c} = \frac{1}{137.04} = 0.00730 \\
\frac{G_F}{(\hbar c)^3} &= 1.166 \times 10^{-5}\ \text{GeV}^{-2}
\end{aligned}$$

The values of the strong and weak interaction coupling constants depend on the conventions used to define them, as explained in Sections 17.6 and 21.3. For the purpose of estimation, however, one can use the following values:

$$\begin{aligned}
\alpha_s(10\ \text{GeV}) &= 0.18 \\
\alpha_s(m_Z) &= 0.12 \\
\sin^2\theta_w &= 0.23
\end{aligned}$$

Particle masses (times c^2):

e:	0.5110 MeV	p:	938.3 MeV
μ:	105.6 MeV	n:	939.6 MeV
τ:	1777 MeV	$\pi^\pm$:	139.6 MeV
$W^\pm$:	80.2 GeV	π^0:	135.0 MeV
Z^0:	91.19 GeV		

Useful combinations:

Bohr radius:	$a_0 = \dfrac{\hbar}{\alpha m_e c} = 5.292 \times 10^{-9}$ cm
electron Compton wavelength:	$\lambda\!\!\!^{-} = \dfrac{\hbar}{m_e c} = 3.862 \times 10^{-11}$ cm
classical electron radius:	$r_e = \dfrac{\alpha\hbar}{m_e c} = 2.818 \times 10^{-13}$ cm
Thomson cross section:	$\sigma_T = \dfrac{8\pi r_e^2}{3} = 0.6652$ barn
annihilation cross sect...	$\dfrac{4\pi\alpha^2}{\quad} = 86.8$ nbarn

Conversion factors:

$$
\begin{aligned}
(1\ \text{GeV})/c^2 &= 1.783 \times 10^{-24}\ \text{g} \\
(1\ \text{GeV})^{-1}(\hbar c) &= 0.1973 \times 10^{-13}\ \text{cm} = 0.1973\ \text{fm}; \\
(1\ \text{GeV})^{-2}(\hbar c)^2 &= 0.3894 \times 10^{-27}\ \text{cm}^2 = 0.3894\ \text{mbarn} \\
1\ \text{barn} &= 10^{-24}\ \text{cm}^2 \\
(1\ \text{volt/meter})(e\hbar c) &= 1.973 \times 10^{-25}\ \text{GeV}^2 \\
(1\ \text{tesla})(e\hbar c^2) &= 5.916 \times 10^{-17}\ \text{GeV}^2
\end{aligned}
$$

A complete, up-to-date tabulation of the fundamental constants and the properties of elementary particles is given in the *Review of Particle Properties*, which can be found in a recent issue of either *Physical Review* **D** or *Physics Letters* **B**. The most recent *Review* as of this writing is published in *Physical Review* **D50**, 1173 (1994).