"A Skeleton Key to Particle Physics"

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Lecture 1 : July 18

In these lectures, I will try to explain the basic structures which underlie the fundamental interactions of particle physics. These structures are known collectively as "the Standard Model", a boring and mundane name for a very beautiful set of concepts. In essence, the Standard Model gives a unified explanation of the three basic interactions of particle physics:

the strong, electromagnetic, and weak interactions

in terms of 3 types of elementary particles:

quarks, leptons, and gauge bosons

The explanation of the diverse properties of these interactions in terms of these simple ingredients is a wonderful story, and I will try to convey that in these lectures.

The basic ingredients of particle physics are quite remote from human experience. So I must also try to convey something else — that these elementary particles are not just constructs but are quite <u>real</u>. Many people find it difficult to grasp that microscopic objects, even those as large as atoms, can be <u>real</u> as pencils and shoes are. For me, there are two aspects to the reality of things. First, these things should have <u>familiar</u>, pre<u>dict</u>able behavior. Everyone believes that electric current is real, even though it is invisible, because the light goes on when you flip the switch. Those of you who work this summer with experimental groups at LEP may get a feel for the predictability of the high-energy phenomena that you observe. A second aspect of reality is that real objects (at least elementary ones) follow <u>simple</u> <u>patterns</u> of <u>behavior</u>. The main focus of this course will be to explain three especially simple patterns in the behavior of the ingredients of the Standard Model and to see how many different phenomena observed in experiments are built up from these patterns. The three patterns I will highlight are

① "current - current coupling" (lectures 1, 3, 5, 7)

② "splitting" (lectures 2, 4)

③ "running of coupling constants" (lecture 8)

By the end of lecture 8 of this series, we will have the whole structure of the Standard Model in front of us. Then I will use the last three lectures of this course to discuss some ideas that go beyond the Standard Model, and relate to further concepts in fundamental physics that you might have a role in investigating. These ideas will also be built up using the three patterns listed above.

This is an elementary course in particle physics, and so I will simplify the full story using some judicious approximations. The most important of these will be that I will treat the elementary particles — quarks, leptons, and gauge bosons — in the limit of very high energy, in particular, at

$$E \gg mc^2$$

I will then ignore the masses of these particles and consider them as elementary entities moving at the speed of light

But, if these particles are truly elementary and structureless and they move at the speed of light, what distinguishing properties could they have? A key property, which will play a central role in these lectures, is *spin*. We know from quantum mechanics that quantum states can have intrinsic angular momentum, spin. A photon has spin = $1 \cdot \hbar$, and the spin wavefunction is just the polarization vector $\vec{\epsilon}$. An electron has intrinsic spin = $\frac{1}{2}\hbar$. For a relativistic particle, it is natural to quantize the spin along the direction of motion. Let $\hat{p}$ be a *unit vector* in the direction of the particle's momentum. Then, for an elementary, high-energy spin-$\frac{1}{2}$ state, we can have

$$\hat{p} \cdot \left(\frac{\hbar \vec{\sigma}}{2}\right) \xi_\pm = \left(\pm \frac{1}{2}\hbar\right) \xi_\pm$$

ξ_+ is the spinor of a right-handed particle 

ξ_- is the spinor of a left-handed particle 

For a photon (or any other elementary massless spin-1 particle) we similarly can find polarization vectors satisfying

$$(\hat{p}\cdot\vec{S})\,\vec{\epsilon}_\pm = (\pm\hbar)\,\vec{\epsilon}_\pm$$

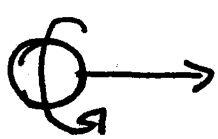
$\vec{\epsilon}_+$ ($\vec{\epsilon}_-$) indicates a photon of right (left-) handed polarized light:

$$\vec{\epsilon}_\pm = \frac{1}{\sqrt{2}}(\hat{x} \pm i\hat{y}) \quad \text{for } \hat{p} = \hat{z}$$

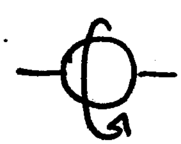
In general, we call

$$\hat{p}\cdot\vec{S} = h \qquad \text{"\underline{helicity}"}$$

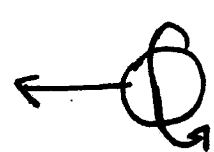
In nonrelativistic quantum mechanics, helicity is an unnatural and unfamiliar property. There is a good reason for this: helicity is not invariant under boosts. If we have a massive particle which is right-handed



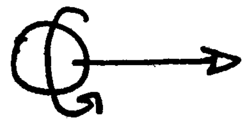
we can run after it and bring it to rest, where it has indefinite helicity.



A further boost makes the particle left-handed:



A massless particle, however, moves at the speed of light. We can never run fast enough to turn a right-handed particle into a left-handed particle.



As we explore the structure of the Standard Model, we will find that this ~~statement~~ is not only correct but profound. The basic ingredients of the Standard Model are particle states of definite helicity. Eventually, when we relax the approximation $E \gg mc^2$ and try to put the masses back, we will find that the mass terms are precisely terms that mix the states of definite helicity.

Before I begin the technical material, there is one more set of concepts that I would like to introduce. In particle physics, we are working in a regime where special relativity and quantum mechanics are playing essential roles. So it is important to point out that the combination of relativity and quantum mechanics leads to some profound implicat-

These follow from the connection between particles and fields. In relativity theory, signals cannot propagate faster than the speed of light. Classically, this requires that signals propagate via fields: We disturb the field at one point, and the disturbance spreads according to its wave equation. When we add quantum mechanics, particles arise in the following way: We consider the field oscillations as harmonic oscillators, the quantum excitations of these oscillators are particles. In this viewpoint, the field is an operator ~~that~~ creates and destroys particles.

$$\text{field} \quad \underset{\sim}{\phi}(x) \quad \longleftrightarrow \quad \text{particle states} \quad |\vec{p}\rangle$$

Since the field is an operator, it can change the quantum numbers of a state (such as charge and angular momentum) by a definite amount.

$$\text{e.g.} \quad \phi(x)\,|A\rangle = \underbrace{|A + (\vec{p})\rangle}$$

w. Q increased by $+1$, J^z increased by $\frac{1}{2}\hbar$.

This has two important implications:
(which I state here without proof)

① Existence of antiparticles :

If $\phi(x)$ creates a particle of charge Q and helicity h, it must also be able to destroy a particle of charge $-Q$ and helicity $-h$. One kind of particle cannot exist without the other. Typically, the particle and antiparticle can annihilate into a state with zero charge.

② Spin - Statistics theorem

For Lorentz invariance and causality, fields which create integer spin must obey

$$\underset{\sim}{\phi}(x)\,\underset{\sim}{\phi}(y) = \underset{\sim}{\phi}(y)\,\underset{\sim}{\phi}(x) \qquad ([\underset{\sim}{\phi}(x),\underset{\sim}{\phi}(y)] = 0$$

when x and y have spacelike separation. This is quite natural; the operator actions should not interfere with one another. However, fields of half-integer spin must obey

$$\underset{\sim}{\phi}(x)\,\underset{\sim}{\phi}(y) = -\,\underset{\sim}{\phi}(y)\,\underset{\sim}{\phi}(x) \qquad (\{\phi(x), \phi(y)\} = 0$$

when x and y have spacelike separation. When we quantize these fields and find particles, the resulting particles obey :

for integer spin, Bose-Einstein statistics

for half-integer spin, Fermi-Dirac statistics

Thus, particles of half-integer spin obey the Pauli exclusion principle, while particles of integer spin prefer to clump together to form macroscopic waves. The value of the spin is the only essential difference between electrons and photons, but it does explain why we look at these particles in very different ways.

With this introduction, we are ready to begin the course proper. To start, we need some technical formulae. One of the things I would like to do in this course is to compare the results of computations in the Standard Model to experimental data. So we will need the basic formulae for performing these computations.

Since we will be working in a setting where relativity and quantum mechanics are essential, I will use units in which

$$c = 1 \qquad \hbar = 1$$

that is, I will measure <u>time</u> in cm and <u>momentum</u> in $(\mathrm{cm})^{-1}$! The 4-vector of position is

$$x^\mu = (t, \vec{x}) \qquad x^2 = t^2 - |\vec{x}|^2 \text{ is the } \underline{\text{interval}}$$

The 4-vector of momentum is

$$p^\mu = (E, \vec{p}) \qquad p^2 = E^2 - |\vec{p}|^2$$
$$= m^2 \quad \text{the mass of the particl}$$

It is most convenient to measure

$$x \quad \text{in} \quad \mathrm{fm} = 10^{-13}\,\mathrm{cm} \quad \text{or} \quad (\mathrm{GeV})^{-1} \qquad (\mathrm{fm} = \text{"Fermi"})$$

$$p \quad \text{in} \quad \mathrm{GeV} = 10^{9}\,\mathrm{eV} \quad \text{or} \quad (\mathrm{fm})^{-1}$$

v is dimensionless, and for us it will usually be ≈ 1.

The conversion factor between units is

$$\hbar c = 0.197 \ \mathrm{GeV\text{-}fm}$$

So, e.g.
$$\frac{1}{m_{\text{proton}}} = 0.21 \ \mathrm{fm}$$

These units take a little getting used to, but it is worth the effort, because they allow many particle properties to be estimated by dimensional analysis.

Most of the data of particle physics comes from scattering experiments:

producing two or more final particles.

I will now give some basic formulae for quantum-mechanical scattering processes. The quantity we wish to compute is the cross-section

$$\sigma = \left(\# \text{ of } \frac{\text{scattering events}}{\text{time}} \right) \Big/ (\text{flux of incoming particles})$$

(units of (cm^2) an effective area)

σ is proportional to the square of a quantum-mechanical amplitude for the scattering process, say

$$p_1 + p_2 \to k_1 + \cdots + k_n$$

Since momentum and energy will be conserved in these processes, I will write these matrix elements as

$$\langle k_1 \cdots k_n | p_1 p_2 \rangle_{\substack{\text{past} \\ \text{future}}}$$

$$= (2\pi)^4 \delta^{(4)}(p_1 + p_2 - \Sigma k_i)\, \mathcal{M}(p_1 p_2 ; k_1 - k_n)$$

We can make $\mathcal{M}$ simpler if we pay some attention to how the particle states $|p\rangle$ are normalized. In ordinary nonrelativistic quantum mechanics, we usually write

completeness of states: overlap:

$$\underset{\sim}{1} = \int \frac{d^3p}{(2\pi)^3} \, |p\rangle\langle p| \qquad\qquad \langle p|p'\rangle = (2\pi)^3 \delta^{(3)}(\vec{p}-\vec{p}\,')$$

This conventional normalization is not relativistically invariant. To find a better form, consider the manifestly relativistically invariant integral

$$\int \frac{d^4p}{(2\pi)^4} \; 2\pi\, \delta(p^2-m^2) \; \underbrace{\Theta(p^0)}_{p^0>0}$$

write $p^2 = (p^0)^2 - |\vec{p}|$, & integrate over p^0:

$$\int dp^0 \; \delta((p^0)^2 - E_p^2) = \frac{1}{2E_p}$$

$$= \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_p} \qquad \text{with} \qquad E_p = \left[|\vec{p}|^2 + m^2\right]^{\frac{1}{2}}$$

Then we choose:

<u>completeness</u>: <u>overlap</u>:

$$\underset{\sim}{1} = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_p} \; |p\rangle\langle p| \qquad\qquad \langle p|p'\rangle = 2E_p\,(2\pi)^3 \delta^{(3)}(\vec{p}-\vec{p}\,')$$

Now the normalizat of $|p\rangle$ is relativistically invariant. $|p\rangle$ has the dimensions of $(\mathrm{GeV})^{-1}$. Then, from the formula on p. 11, $\mathcal{M}$ has the dimensions

$$(\mathrm{GeV})^{-2-n} / (\mathrm{GeV})^{-4} = (\mathrm{GeV})^{2-n}$$

From this $\mathcal{M}$, we calculate the cross-section for the reaction $p_1 + p_2 \to \Sigma k_i$ by

$$\sigma = \frac{1}{2E_1\, 2E_2} \frac{1}{|\vec{v_1} - \vec{v_2}|} \prod_i \int \frac{d^3k_i}{(2\pi)^3} \frac{1}{2E_i} (2\pi)^4 \delta^{(4)}(p_1 + p_2 - \Sigma k_i)$$

$$\cdot \; |\mathcal{M}(p_1 p_2; k_i)|^2$$

The factors $\frac{1}{2E_i}$ come from our normalizat of states; the factor $\frac{1}{|v_1 - v_2|}$ arises when we divide by the flux. The dimension of the formula is

$$(\mathrm{GeV})^{-2} \cdot (\mathrm{GeV})^{2n} (\mathrm{GeV})^{-4} |(\mathrm{GeV})^{2-n}|^2$$

$$= (\mathrm{GeV})^{-2} = (\mathrm{cm})^2 \quad (\text{area!})$$

which is correct. The factor $\frac{1}{2E_1\, 2E_2 |\vec{v_1} - \vec{v_2}|}$ is invariant to longitudinal boosts and Lorentz-contracts correctly when we boost transversely. The remaining factors are relativistically invariant.

The integral

$$\int d\Pi_n = \prod_{i=1}^{n} \int \frac{d^3k_i}{(2\pi)^3} \frac{1}{2E_i} (2\pi)^4 \delta^{(4)}(p_1+p_2-\Sigma k_i)$$

is called "phase space". For the case $n=2$, phase space is dimensionless, so we should expect a simple formula. Since the integral is relativistically invariant, we can compute it in any frame. Choose, then, the frame where

$$p_1+p_2 = (P, \vec{0}) \qquad \text{(center of mass frame) (CM)} \quad P = E_{cm}$$

In this frame (for $n=2$)

$$\vec{k}_1 = -\vec{k}_2 \quad \Rightarrow \quad |\vec{k}_1| = |\vec{k}_2|$$

We can integrate over d^3k_2 using the δ-function to find

$$\int d\Pi_2 = \int \frac{d^3k_1}{(2\pi)^3} \frac{1}{2k_1^0} \frac{1}{2k_2^0} (2\pi)\delta(P-k_1^0-k_2^0)$$

$$= \int \frac{dk_1\, k_1^2\, d\cos\theta\, d\phi}{(2\pi)^2\; 4\, k_1^0 k_2^0} \delta(P-(k_1^2+m_1^2)^{1/2}-(k_1^2+m_2^2)^{1/2})$$

For $m_1=m_2=0$ we find just $\int dk_1\, \delta(P-2k_1) = \frac{1}{2}$

Then

$$\int d\Pi_2 = \frac{1}{(2\pi)^2} \frac{1}{2} \frac{1}{4} 2\pi \int_{-1}^{1} d\cos\theta$$

$$= \frac{1}{16\pi} \int_{-1}^{1} d\cos\theta$$

The general formula is

$$\int d\Pi_2 = \frac{1}{16\pi}\left(\frac{2k_1}{E_{cm}}\right)\int_{-1}^{1} d\cos\theta$$

where k_1 is the particle momentum in the CM frame.

Now consider $p_1+p_2 \to k_1+k_2$, ignoring all masses. $|v_1-v_2| = 2$ in the CM frame, and $2E_1 = 2E_2 = E_{cm}$. Our formula for the cross-section reduces to

$$\sigma = \frac{1}{32\pi\, E_{cm}^2}\int_{-1}^{1} d\cos\theta\; |\mathcal{M}|^2$$

where $\mathcal{M}$ is a relativistically invariant amplitude. This is a very simple starting point for our physical predictions.

Finally, let me illustrate the use of this formula for one basic process which involves only electromagnetism. Among the basic spin-$\frac{1}{2}$ particles of the Standard Model are three with electric charge -1 [or $(-e)$], called

e^- (electron), μ^- (muon), τ^- (tau)

Each has a massless neutral partner, called a neutrino: ν_e, ν_μ, ν_τ. The masses of e μ τ are very

different: $m(e) = 0.51$ MeV, $m(\mu) = 106$ MeV, $m(\tau) = 1780.$ MeV.
However, the couplings of these particles to the interactions of the standard model are identical. I will show a considerable amount of evidence for this in the course of these lectures. The particles

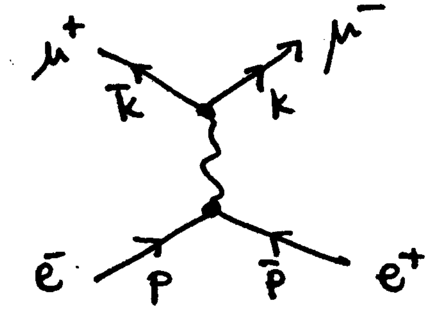
$$\begin{matrix} \nu_e & \nu_\mu & \nu_\tau \\ e^- & \mu^- & \tau^- \end{matrix} \qquad \text{(\& their antiparticles)}$$

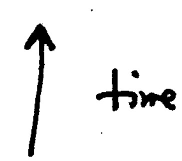
are collectively called "leptons".

The process I would like to consider is

$$e^- + e^+ \rightarrow \mu^- + \mu^+$$

The electron e^- can annihilate with its antiparticle e^+ (the positron) into a quantum of electromagnetic excitation. This in turn can re-materialize into a pair of $e^- e^+$ or into a pair of other leptons. The process is represented by the Feynman diagram:





$p, \bar{p}, k, \bar{k}$ are the moment

which shows its space time evolution.

The amplitude for this process depends on the helicities of the initial and final particles. For definiteness, consider

$$e^-_R + e^+_L \to \mu^-_R + \mu^+_L$$

The e^+_L is the antiparticle of the e^-_R, so once we have chosen e^-_R the positron must be left-handed. (This statement is true only in the limit of zero electron mass.) Similarly, once we consider production of a μ^-_R, it must be accompanied by a μ^+_L. The process

$$e^-_R e^+_L \to \mu^-_L \mu^+_R$$

is also possible, and we will consider it later.

The annihilation of $e^- e^+$ into electromagnetic excitation should have an amplitude proportional to the electric charge of the electron. Conventionally, e has strange units, but, in SI units, the combination

$$\frac{e^2}{\hbar c} = \alpha \qquad \text{"the fine structure constant"}$$

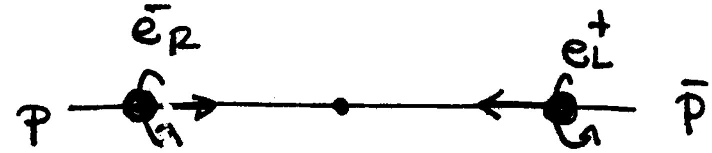
is dimensionless and has the value $\alpha = \frac{1}{137}$.

Particle physicists usually use a slightly different convention in which

$$\frac{e^2}{4\pi} = \alpha = \frac{1}{137}. \qquad \text{with } \hbar = c = 1.$$

With this convention, the amplitude for our process is proportional to $\underline{e}^2$.

Now look at the initial state

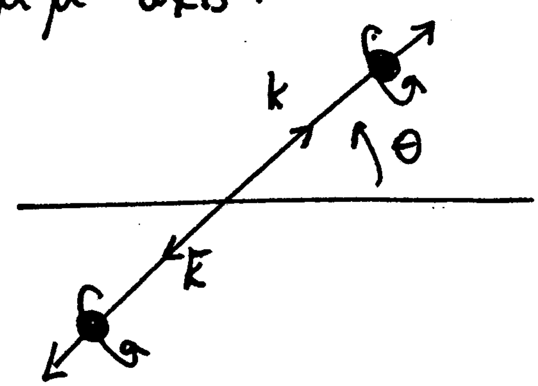


this has $J^z = +1 \cdot \hbar$. That is quite natural, because the electromagnetic quantum into which $e^-_R e^+_L$ will annihilate must have a polarization vector. To give $J^z = +1$, this should be

$$\vec{\epsilon}_+(p) = \frac{1}{\sqrt{2}}(1, i, 0)$$

as on p.5.

Similarly, the final state $\mu^-_R \mu^+_L$ must be created from an electromagnetic quantum that has $J^i = +1$ along the outgoing $\mu^-\mu^+$ axis:



Rotating the above by θ, we find

$$\vec{\epsilon}_+(k) = \frac{1}{\sqrt{2}}(\cos\theta, i, -\sin\theta)$$

Now I claim that the amplitude for the process shown on p.16 is proportional to $e^2 \cdot (\vec{\epsilon}_+(k))^* \cdot \vec{\epsilon}_+(p)$.
Indeed, an explicit computation using the machinery of quantum field theory gives

$$\mathcal{M}(e^-_R(p) + e^+_L(\bar{p}) \to \mu^-_R(k) + \mu^+_L(\bar{k}))$$

$$= 2e^2 \quad \vec{\epsilon}_+^{\,*}(k) \cdot \vec{\epsilon}_+(p)$$

$$= e^2 \quad (1+\cos\theta)$$

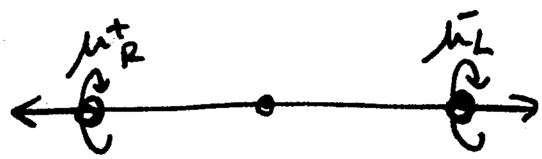
Inserting this into the formula on p. 15, we find

$$\frac{d\sigma}{d\cos\theta}(e^-_R e^+_L \to \mu^-_R \mu^+_L) = \frac{e^4}{32\pi E_{cm}^2}(1+\cos\theta)^2 = \frac{\pi\alpha^2}{2E_{cm}^2}(1+\cos\theta)^2$$

What about the related process

$$e^-_R e^+_L \to \mu^-_L \mu^+_R \qquad ?$$

For this process, the final state has the form



This has $J^z = -1$ about the $\mu^-\mu^+$ axis. This $\mu^-\mu^+$ pair must come from an electromagnetic state with polarization vector $\vec{\epsilon}_-(k) = \frac{1}{\sqrt{2}}(\cos\theta, -i, -\sin\theta)$.

Similarly, for $e^-_L e^+_R$, change $\vec{\epsilon}_+(p)$ to $\vec{\epsilon}_-(p) = \frac{1}{\sqrt{2}}(1,-i,0)$.

The four possibilities give four nonvanishing cross-sections

$$\frac{d\sigma}{d\cos\theta}(e^-_R e^+_L \to \mu^-_R \mu^+_L) = \frac{d\sigma}{d\cos\theta}(e^-_L e^+_R \to \mu^-_L \mu^+_R)$$

$$= \frac{\pi\alpha^2}{2E_{cm}^2}(1+\cos\theta)^2$$

$$\frac{d\sigma}{d\cos\theta}(e^-_R e^+_L \to \mu^-_L \mu^+_R) = \frac{d\sigma}{d\cos\theta}(e^-_L e^+_R \to \mu^-_R \mu^+_L)$$

$$= \frac{\pi\alpha^2}{2E_{cm}^2}(1-\cos\theta)^2$$

The cross sections involving $e^-_L e^+_L$, $e^-_R e^+_R$, $\mu^-_L \mu^+_L$, $\mu^-_R \mu^+_R$ vanish in the limit $m/E \to 0$.

If we work with unpolarized beams, we must average over the initial polarizations and sum over the final polarizations. With this prescription

$$\frac{d\sigma}{d\cos\theta} = \frac{1}{4}\cdot 2\cdot \frac{\pi\alpha^2}{2E_{cm}^2}\left[(1+\cos\theta)^2 + (1-\cos\theta)^2\right]$$

so that, for unpolarized beams,

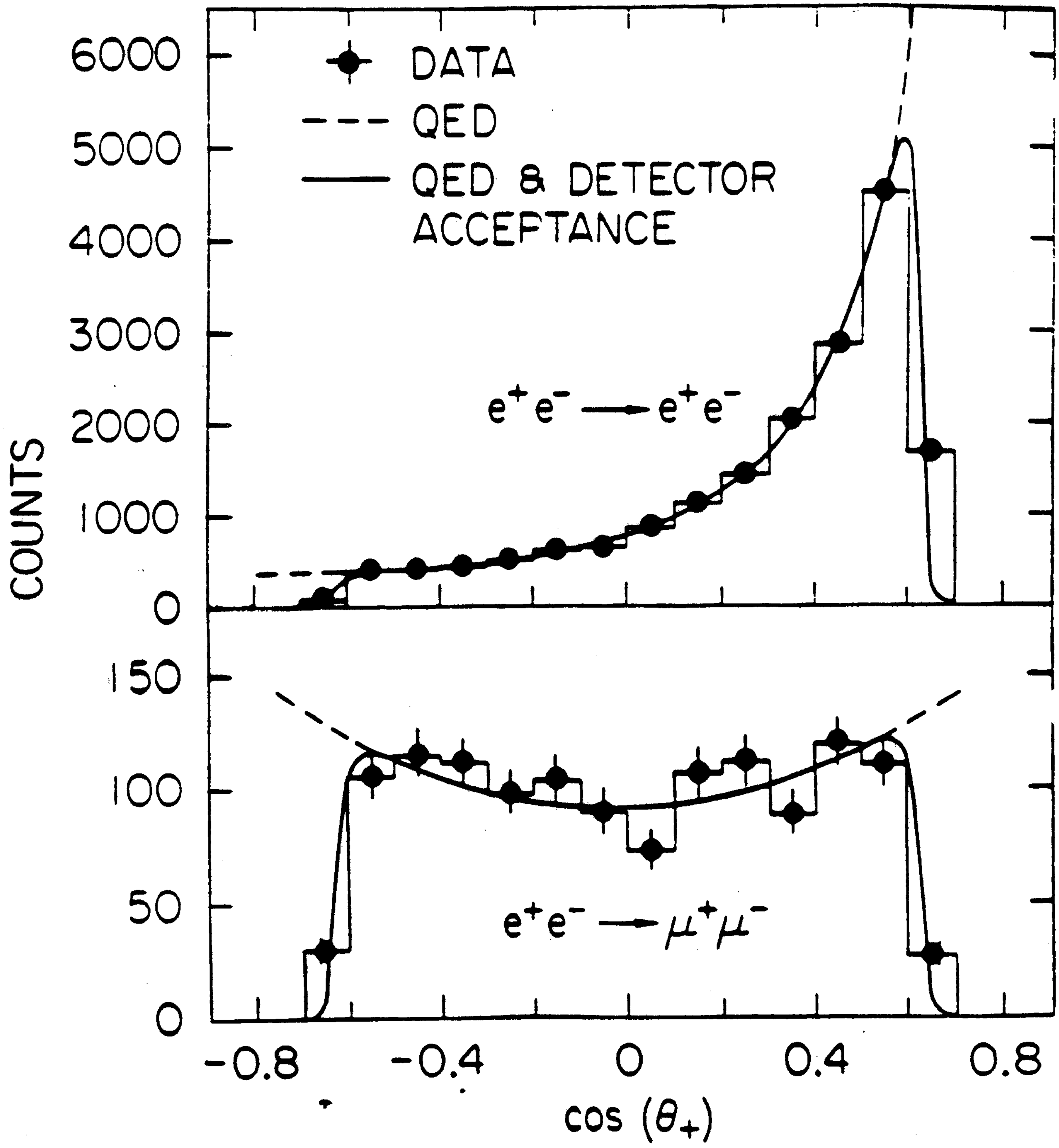
$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2E_{cm}^2}\left[1+\cos^2\theta\right]$$

$$\sigma = \int_{-1}^{1} d\cos\theta \; \frac{d\sigma}{d\cos\theta} = \frac{4\pi\alpha^2}{3E_{cm}^2}$$

We can compare these formulae to experimental data. In Fig. 1, I show the differential cross section for $e^+e^- \to \mu^+\mu^-$ at $E_{cm} = 4.8$ GeV, as measured in 1975 by the Mark 1 experiment at the e^+e^- storage ring SPEAR. The magnitude is right, though the statistics are not impressive. In Fig. 2, I show measurements of $d\sigma/d\cos\theta$ for $e^+e^- \to \mu^+\mu^-$ and for $e^+e^- \to \tau^+\tau^-$ at $E_{cm} = 35$ GeV, made by the JADE experiment at the PETRA storage ring. This is a high-statistics measurement, and the shape is very clean. This shape is quite close to the prediction above, which is indicated by the dotted line. The small discrepancy is due to the effects of the weak interactions; I will explain this in Lecture 5.

Finally, I would like to note that, if we restore the mass of the final-state lepton, the above formula for the total cross-section becomes

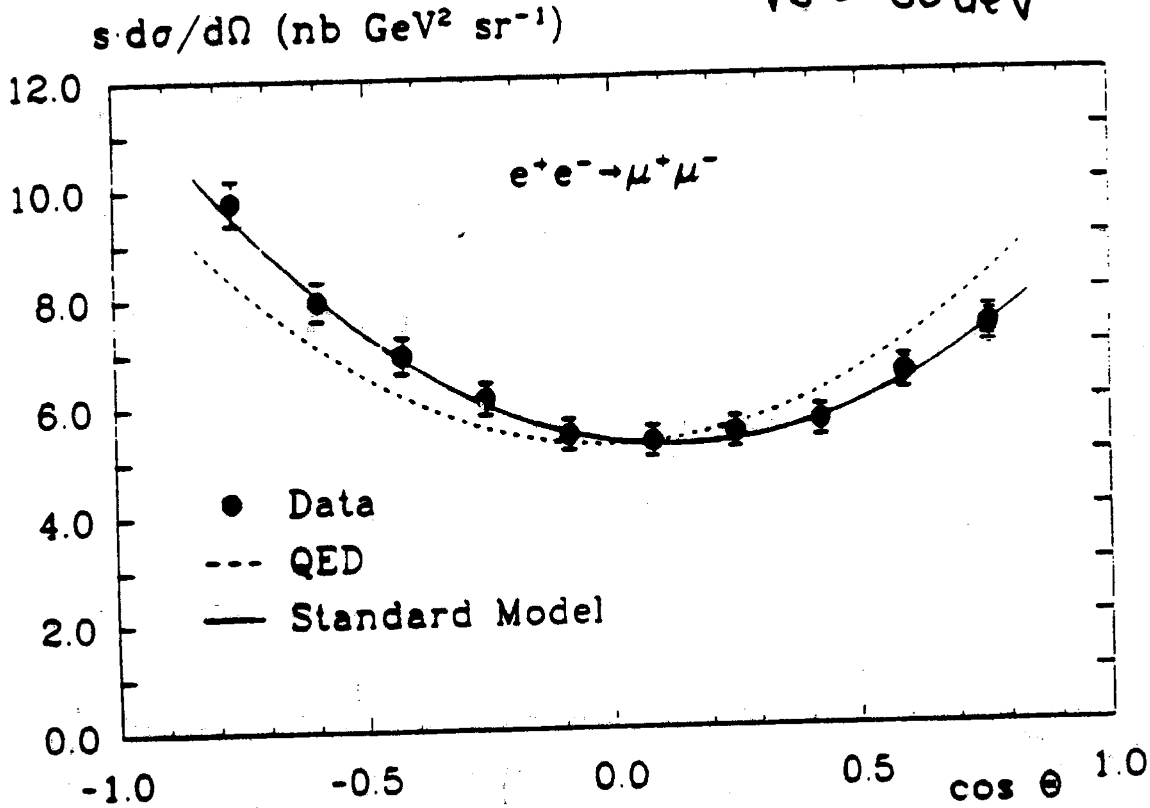
$\sqrt{s} = 4.8$ GeV

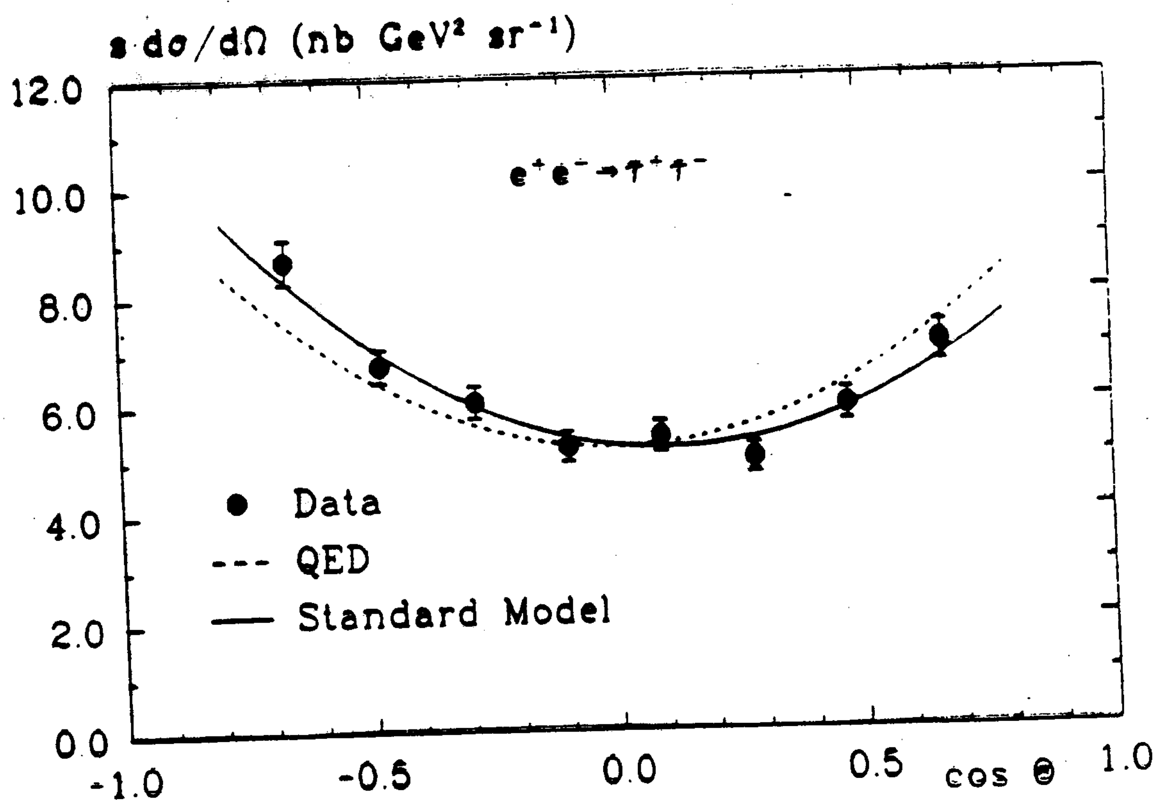


J.E. Augustin et al (Mark 1)

Phys. Rev. Lett. 34, 233 (1975)

$\sqrt{s} = 35$ GeV





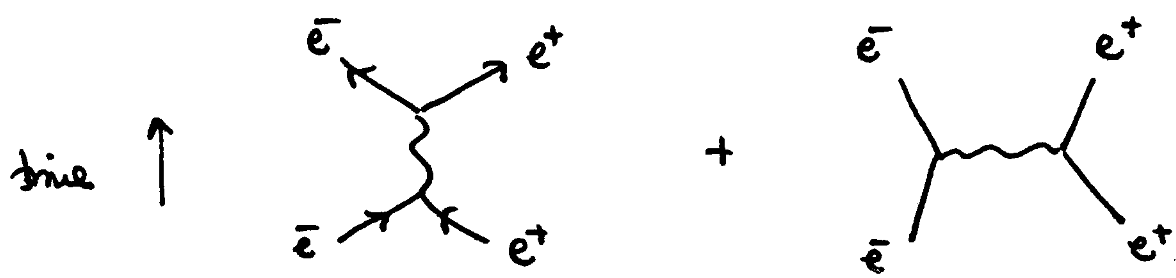
S. Hegner et al (JADE)

Z Phys. C 46, 547 (1990)

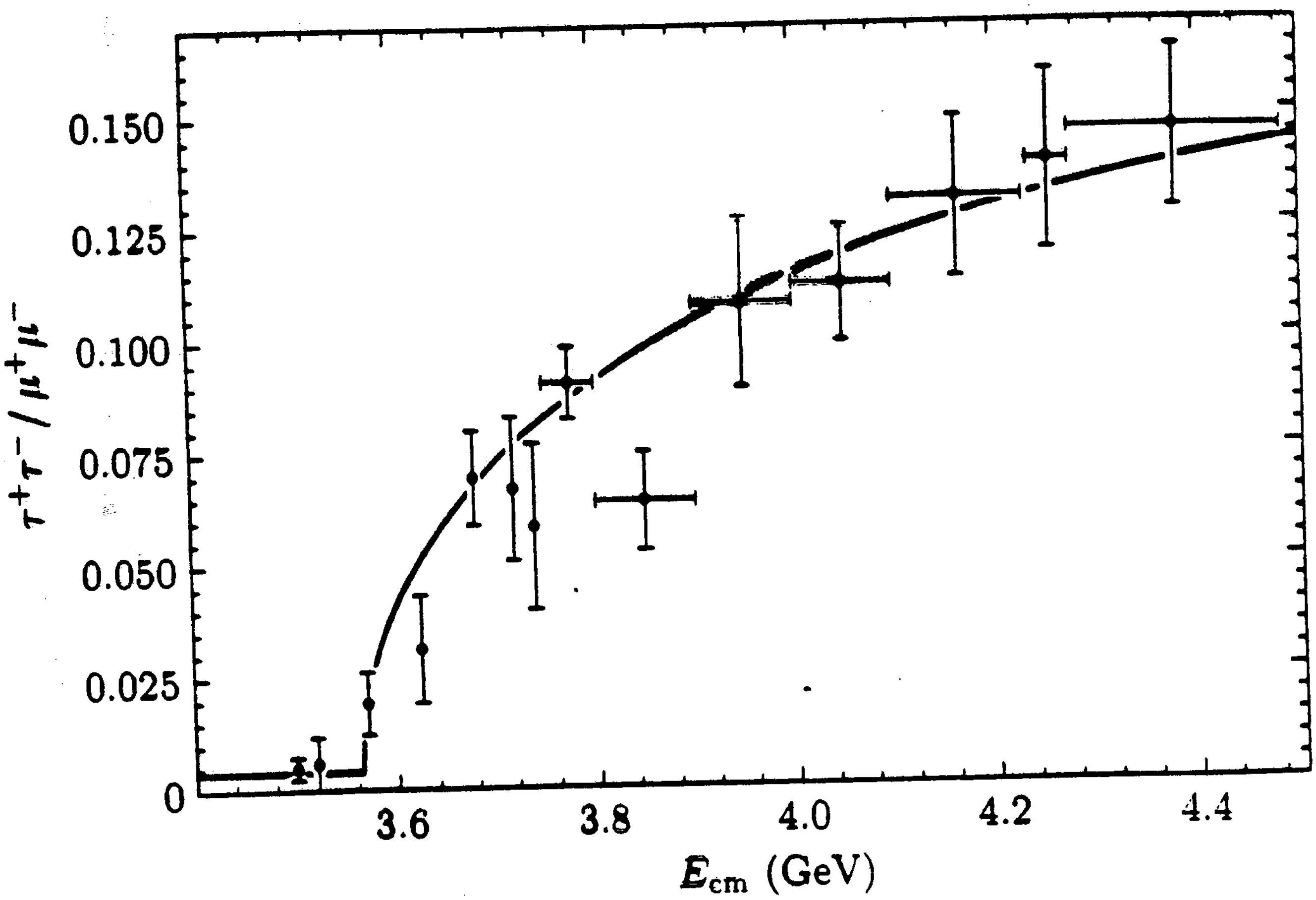
$$\sigma = \frac{4\pi\alpha^2}{3E_{cm}^2}\left(1+\frac{2m^2}{E_{cm}^2}\right)\left(1-\frac{4m^2}{E_{cm}^2}\right)^{1/2}$$

In Fig 3, this formula is compared to the cross section for $e^+e^- \to \tau^+\tau^-$ near $E_{cm} \doteq 2m_\tau$, as measured by the DELCO experiment at SPEAR. The formula captures the shape of the data nicely.

This theory of e^+e^- annihilation to $\mu^+\mu^-$ is a piece of a larger theory — the quantum electrodynamics of leptons, — which is well tested experimentally. For other processes, the formulae for $\mathcal{M}$ are more complicated. For example, the process $e^+e^- \to e^+e^-$ ("Bhabha scattering") involves two quantum mechanical amplitudes



which interfere with one another. (The second process, Coulomb scattering, leads to the peak at $\cos\theta = 1$ in the top graph of Fig. 1.) However, for all of these processes, the direct quantum mechanical generalization



W. Bacino et al (DELCO)

Phys. Rev. Lett. 41, 13 (1978)

$m_\tau = 1782^{+2}_{-7}$ MeV

(best current value: $1777.00^{+.30}_{-.27}$)

of electrodynamics gives correct predictions

In the next few lectures, I will work from this understanding of quantum electrodynamics to develop pictures of the other fundamental interactions.

Lecture 2: July 21

In the previous lecture, I discussed the processes of e^+e^- annihilation into lepton pairs. The process $e^+e^- \to \mu^+\mu^-$, for example, involves only the electromagnetic interactions; for this process, we found the cross-section

$$\frac{d\sigma}{d\cos\theta}(e^+e^- \to \mu^+\mu^-) = \frac{\pi\alpha^2}{2E_{cm}^2}(1+\cos^2\theta)$$

The electromagnetic excitation resulting from e^+e^- annihilation can also create strongly interacting particles, and in fact this is a very effective tool to study the strong interactions. In this lecture, I will explain what we have learned about the strong interactions in this way. Mainly, in this lecture, we will be looking at the data. I will explain some basic features of the data, and for other features I will introduce concepts that I will account for later in the course.

There are many types of strongly-interacting particles. These particles are collectively called "hadrons". Bosonic hadrons are "mesons", examples are the $\pi^+\pi^0\pi^-$ (spin 0) and the $\rho^+\rho^0\rho^-$ (spin 1). Fermionic hadrons are called "baryons" (or anti-baryons), examples are the proton and neutron (spin 1/2). The discovery in the 1950's and 1960's of huge numbers of hadrons led to the idea that these particles are not elementary but rather are built out of more elementary constituents. These constituents, which were thought to have spin-1/2, are the <u>quarks</u>.

A constituent model does account nicely for the spectrum of light hadrons. Let's postulate two types of quark, u and d. There are four particle-antiparticle bound states

$$u\bar{d} \qquad u\bar{u} \qquad d\bar{d} \qquad d\bar{u}$$

If quarks have spin 1/2, these states can be either in combinations of total spin 0 or total spin 1. If u and d differ by 1 unit of electric charge, this accounts for the light <u>mesons</u>

$$\pi^+ \ \pi^0 \ \pi^- \quad \eta^0 \qquad \text{spin } 0$$
$$\rho^+ \ \rho^0 \ \rho^- \quad \omega^0 \qquad \text{spin } 1$$

To make <u>baryons</u> we need an odd number of quarks. The right choice is 3 quarks in a totally <u>symmetric</u> state. With 2 "flavor" values u, d and 2 spin values, there are 4 states per quark and so

$$\frac{4.5.6}{3!} = 20 \quad \text{symmetric combinations}$$

These have the quantum numbers

uuu	uud	udd	ddd	spin $3/2$	$4 \cdot 4 = 16$
	uud	udd		spin $1/2$	$2 \cdot 2 = 4$
					20

accounts for the light baryons

Δ^{++}	Δ^{+}	Δ^{0}	Δ^{-}	spin $3/2$
	p	n		spin $1/2$

if the electric charges of (u, d) are $(+2/3, -1/3)$ respectively. Gell-Mann and Zweig, who proposed this scheme, added a third quark s and successfully explained all of the hadrons known at that time. Today we have evidence for 6 quarks, of which the last 3 are rather heavy:

	u	d	s	c	b	t
Q	$2/3$	$-1/3$	$-1/3$	$2/3$	$-1/3$	$2/3$
mass	(few MeV)		(100 MeV)	(1.2 GeV)	(4.2 GeV)	(170 GeV)

The physics of coupling quarks into hadrons is straightforward quantum mechanics except for one unusual feature. Spin-$\frac{1}{2}$ particles are supposed to be <u>fermions</u> and thus obey the Pauli exclusion principle. So why do we put 3 quarks in a totally symmetric rather than an antisymmetric state? Han + Nambu postulated that quarks have an additional, "visible" quantum number called "color", and that the combination principle for baryons should be that the three quarks are antisymmetric in color. Then quarks should come in 3 colors – "red", "green", and "blue". If quarks are antisymmetrized in color, they should be symmetrized in all other quantum numbers to have an overall antisymmetric wavefunction.

All this makes sense (I hope) but it is very abstract. Can we make it more pictorial? This is what the study of e^+e^- annihilation to hadrons will provide.

The simplest process of e^+e^- annihilation to hadrons is

$$e^+_R e^-_L \to \pi^+\pi^-$$

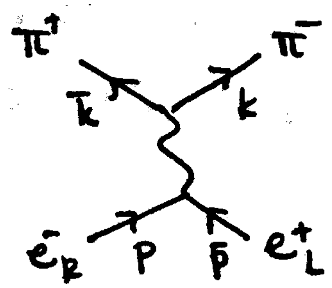
We can compute this in quantum electrodynamics (QED) assuming that the π^+ and π^- are elementary particles of spin 0. From the previous lecture, we expect

$$\mathcal{M} \sim e^2 \quad \vec{\epsilon}_f^{\,*} \cdot \vec{\epsilon}_+(p)$$

where $\vec{\epsilon}_f$ is the polarization vector of the electromagnetic state from which π^+ and π^- materialize. But since π^+ and π^- have spin 0, the only available vector is

$$\vec{\epsilon}_f \sim \frac{\vec{k} - \vec{\bar{k}}}{E_{cm}}$$

in fact, a QED computation gives



$$\mathcal{M} = e^2 \, \frac{\overrightarrow{k - \bar{k}}}{E_{cm}} \cdot \vec{\epsilon}_+(p)$$

$$|\mathcal{M}|^2 = e^2 \sin^2\Theta$$

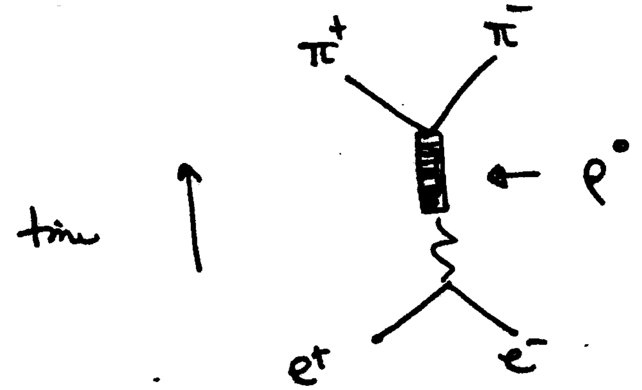
then

$$\frac{d\sigma}{d\cos\Theta}(e^-_R e^+_L \to \pi^+\pi^-) = \frac{\pi\alpha^2}{2 E_{cm}^2} \sin^2\Theta$$

For $e^-_L e^+_R$ we find the same result. For unpolarized beams

$$\frac{d\sigma}{d\cos\Theta}(e^+e^- \to \pi^+\pi^-) = \frac{\pi\alpha^2}{4 E_{cm}^2} \sin^2\Theta$$

The $\sin^2\Theta$ angular distribution is correct (this is a general quantum-mechanical result for $e^+e^- \to$ two spin zero particles) but the energy dependence is all wrong. The data for $e^+e^- \to \pi^+\pi^-$ and $e^+e^- \to$ (all hadrons) as a function of E_{cm} is shown in Fig. 4. There is a huge resonance at $E_{cm} \sim 800$ MeV, due to the process



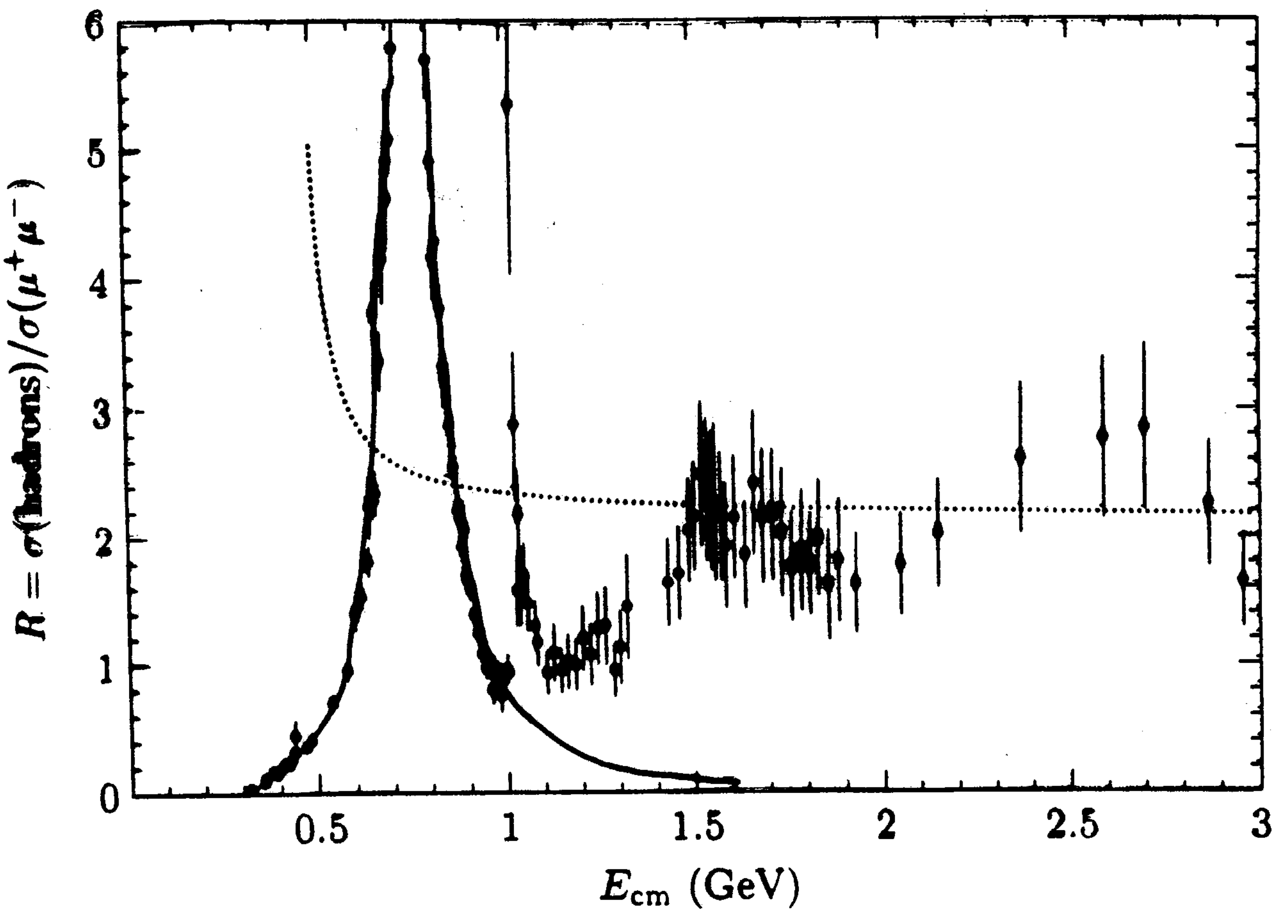
$$e^+e^- \to \rho^0 \xrightarrow{\text{decay}} \pi^+\pi^-$$

and then the cross section becomes very small. The cross-section for $e^+e^- \to$ (any hadrons), on the other hand, does not become tiny; rather

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

approaches a value of order 1. As the energy increases, typical final states contain more and more pions and other hadrons. This is shown in Fig. 5.

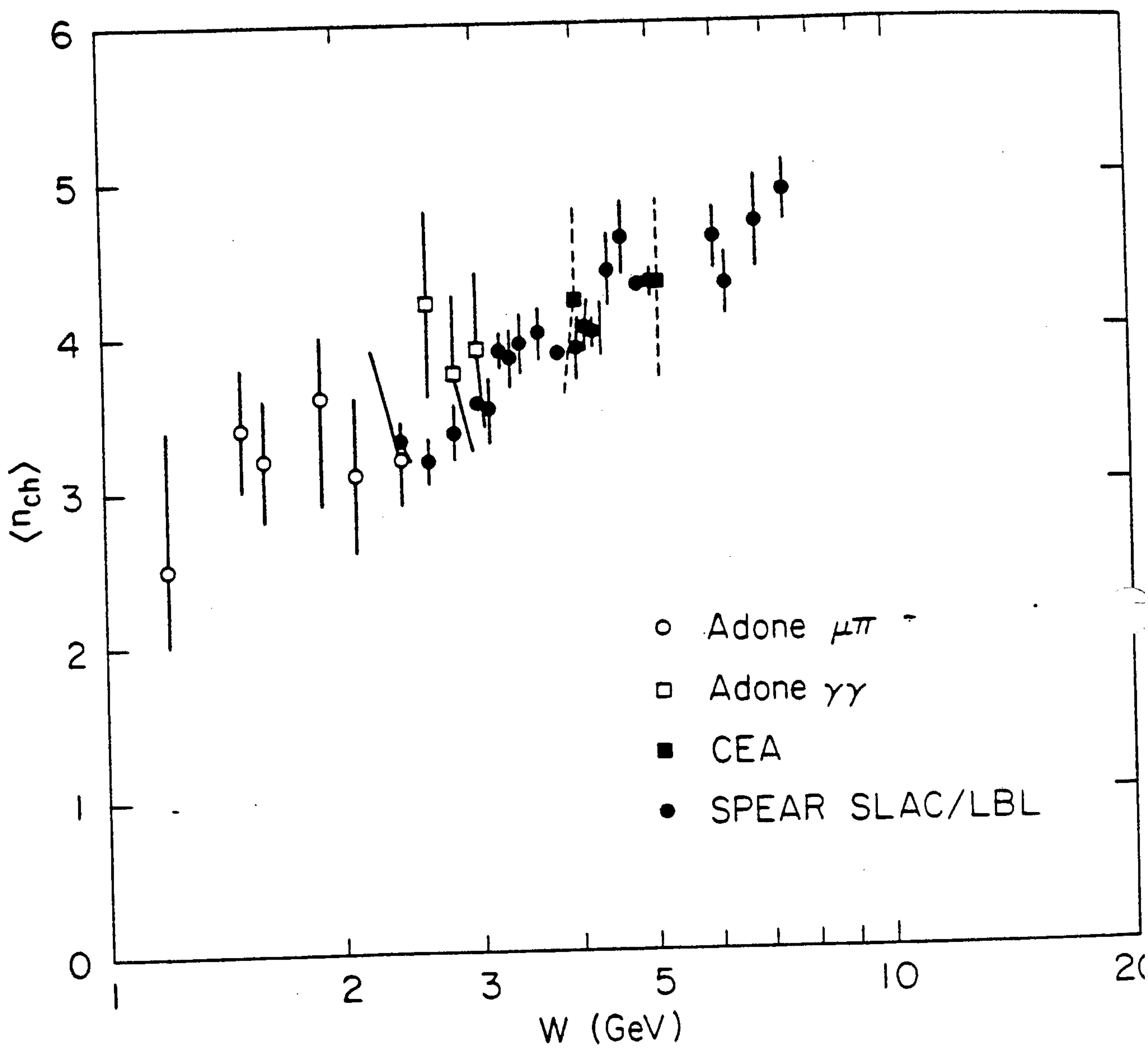
In principle, the e^+e^- annihilation events could look like an electromagnetic hot spot boiling off pions isotropically. If you look at the events, however, you see something quite different. In the typical events, the particles are collimated along some axis, forming



— $e^+e^- \to \pi^+\pi^-$

data compiled by M. Swartz

Figure 5



data compilation of

R. Schwitters + K. Strauch

Ann. Rev Nucl. Sci. 26, 89 (1976)

two "jets" of hadrons (mainly pions). Fig. 6 shows a two-jet event from the JADE experiment at PETRA; Fig. 7 shows a two-jet event from the SLD experiment at SLAC ($E_{cm} = 91$ GeV).

Given a two-jet event, it is interesting to find the "best" axis and discuss the orientation of this axis and the statistical distribution of observed hadrons about the axis. There are many ways to find the axis; one is to construct

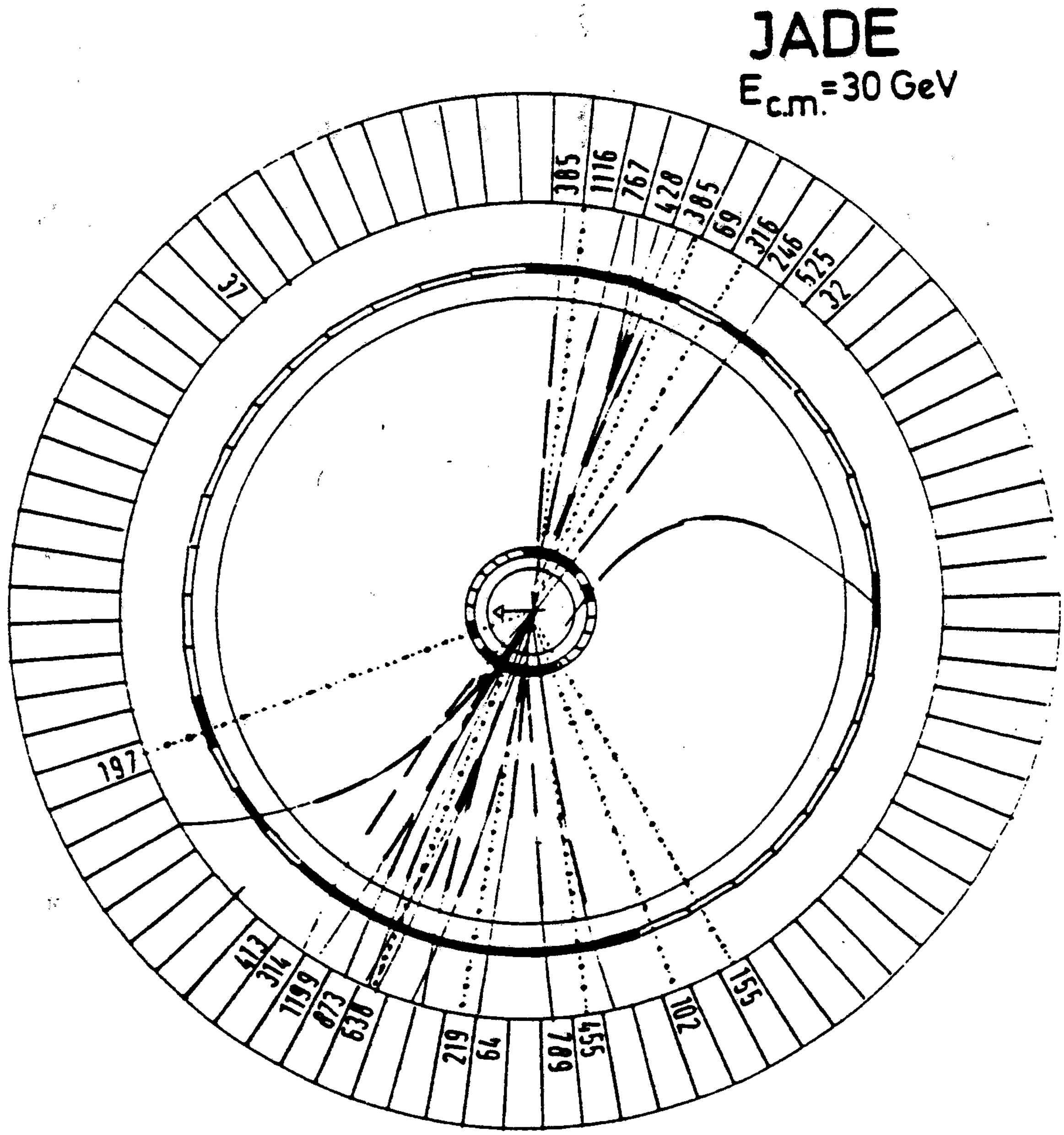
$$T(\hat{n}) = \sum_i \frac{|\hat{n}\cdot\vec{p}_i|}{|\vec{p}_i|}$$

where $\hat{n}$ is a unit vector and $\vec{p}_i$ are the produced particle momenta. Maximize this with respect to $\hat{n}$. Then T is the "thrust" of the event and $\hat{n}$ is the "thrust axis". Let $p_{i\perp}$ be the component of $\vec{p}_i$ normal to the plane defined by $\hat{n}$ and the initial e^+e^- axis. Then

$$A = \sum_i \left|\frac{p_{i\perp}}{p_i}\right|$$

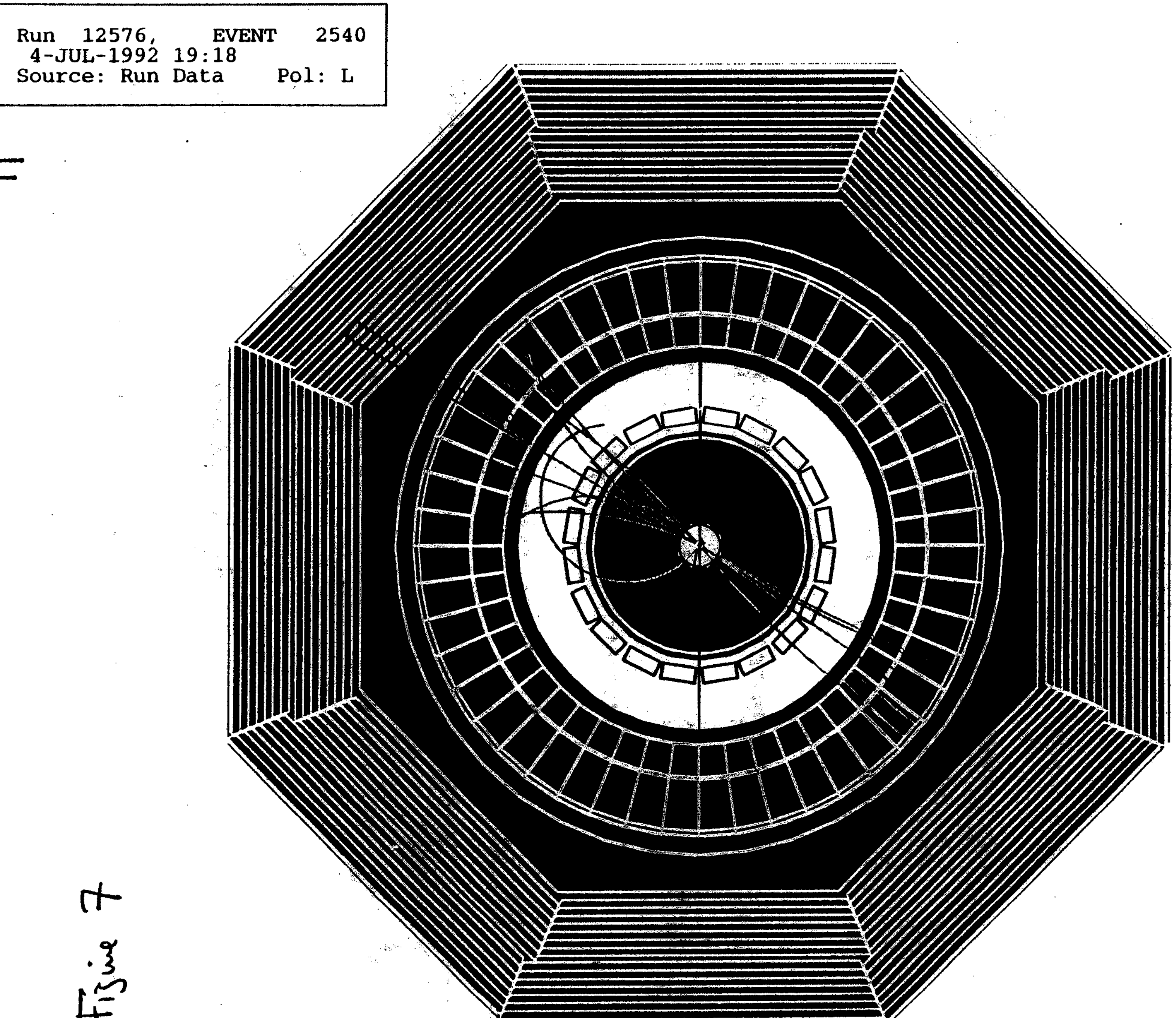
is the "acoplanarity". Fig. 8 shows the acoplanarity

Figure 6



from S. Orito,
in Proc. of the 1979 Lepton-Photon Conference

Figure 7



from
SLD experiment
Web pages.

distribution of $e^+e^- \to$ hadrons events at $E_{cm} = 22, 34, 44$ GeV, from the JADE experiment at PETRA. Most of the events have very small acoplanarity, and this is increasingly true as the energy increases.

Now we can ask, which way does the jet axis point? Fig. 9 shows a preliminary result at about 30 GeV, from the TASSO experiment at PETRA. Fig. 10 shows a high-statistics measurement by the ALEPH experiment at LEP (91 GeV). (using a slightly different definition of the jet axis). The distribution is

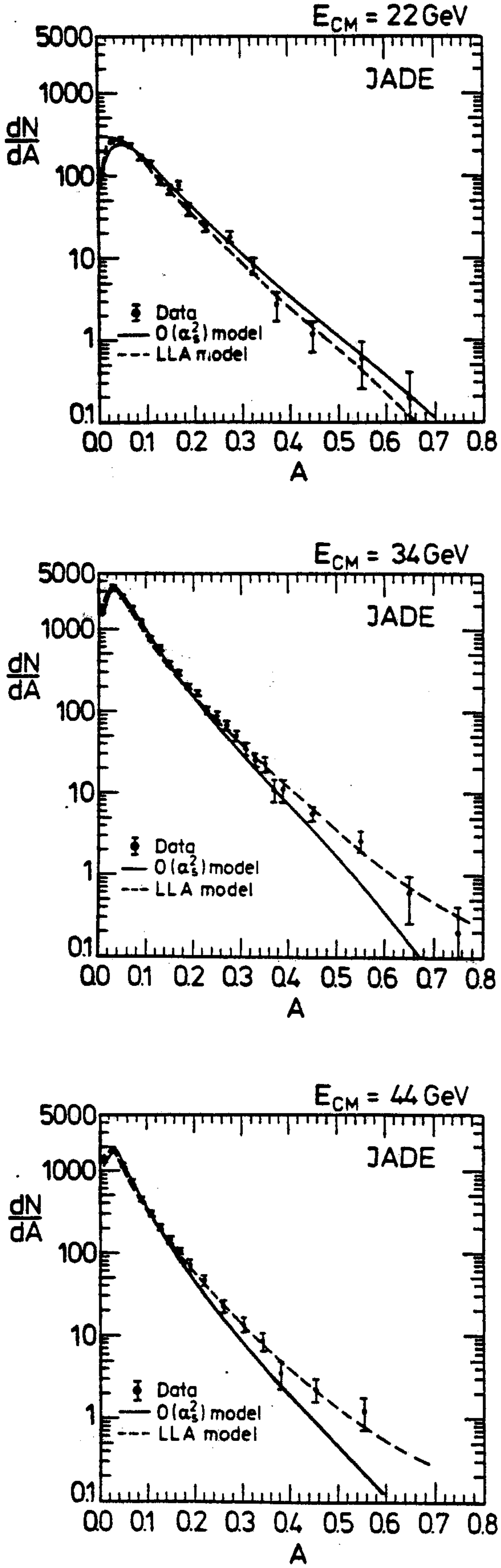
$$\frac{d\sigma}{d\cos\theta} \sim (1+\cos^2\theta) \qquad !$$

All of the particles that come out are <u>bosons</u>, mainly pions. But this distribution is characteristic of

$e^+e^- \to$ massless spin-$\frac{1}{2}$ fermions.

Apparently, we are producing $e^+e^- \to q\bar{q}$, with a later stage in which the quarks produce jets of pions.

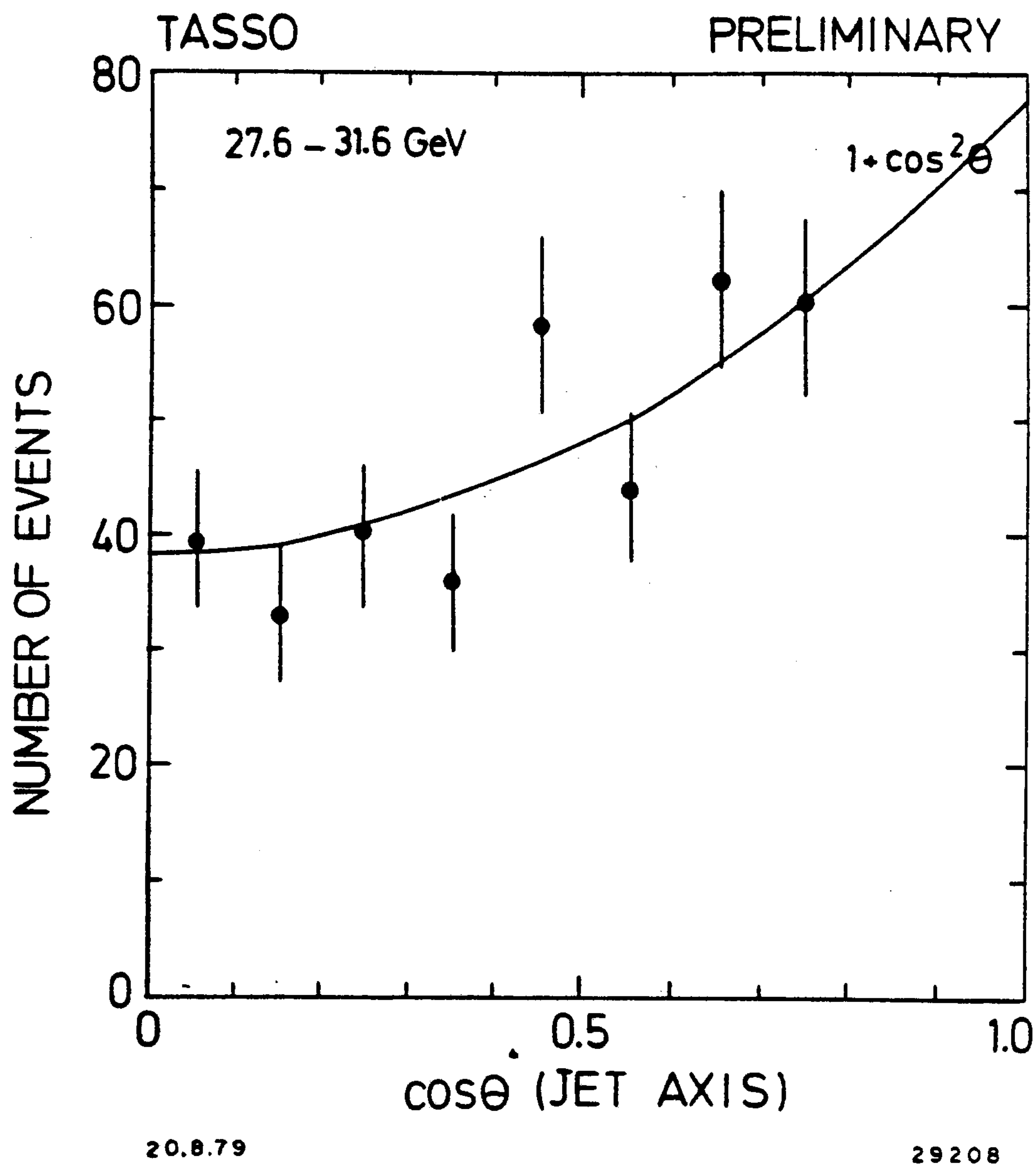
Figure 8



W. Bartel et al. (JADE)

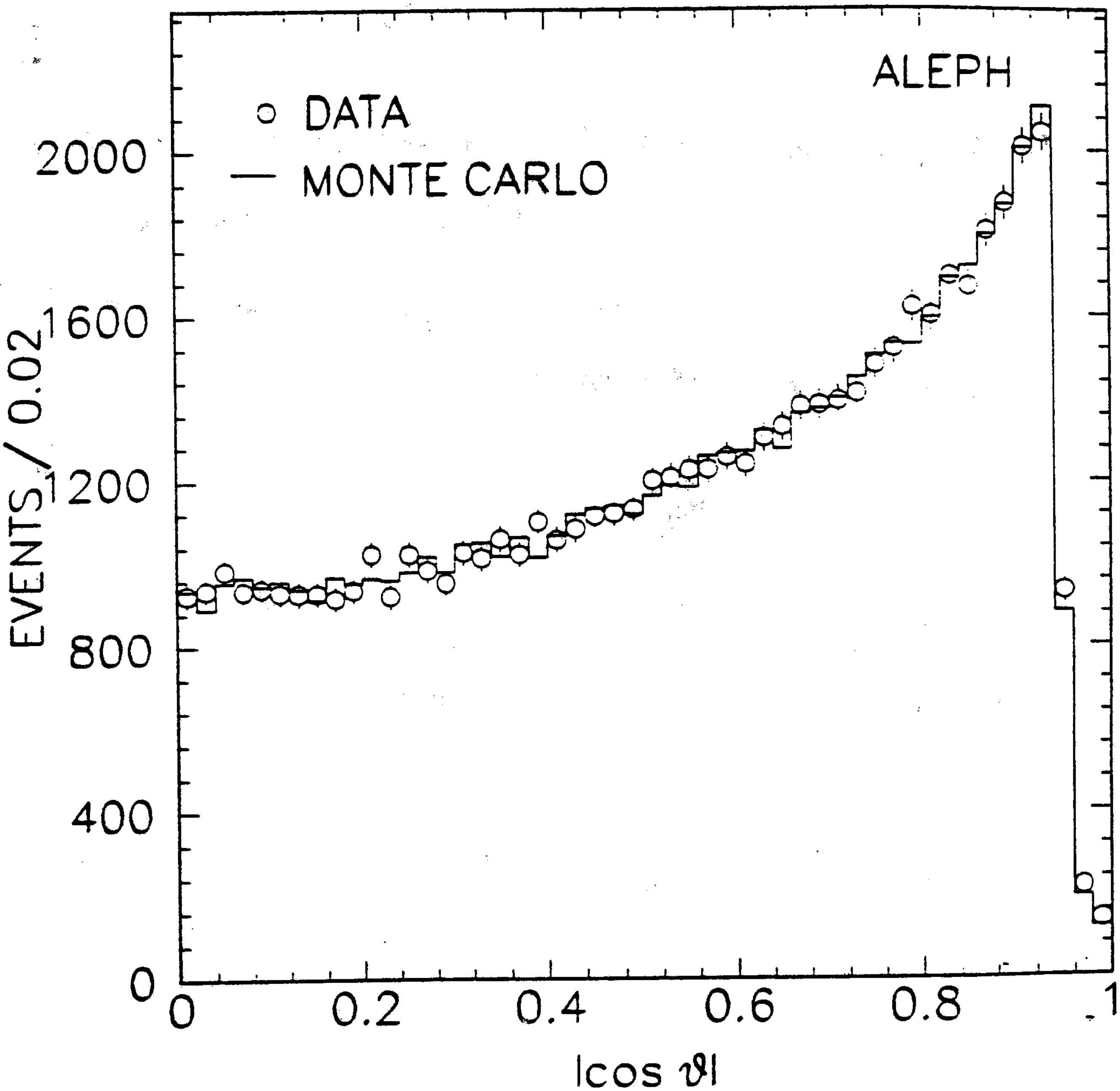
Z Phys. C 33, 23 (1986)

Figure 9



from G. Wolf, in Proc. of the 1979 Lepton-Photon Conference

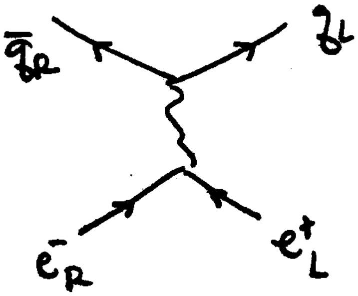
Figure 10



D. Decamp (ALEPH)
Z. Phys. C48, 365 (1990)

We ought to be able to compute the cross-section for

$e^+e^- \to q\bar{q}$



As long as we can take the quarks to be massless, everything should be as for $e^+e^- \to \mu^+\mu^-$, except that the amplitude for producing a $q\bar{q}$ pair from the electromagnetic quantum should be proportional to

$$(Q_f e)$$

where Q_f is the quark charge $Q_u = \frac{2}{3}$, $Q_d = -\frac{1}{3}$, etc.

For simplicity, consider the ratio on p. 6

$$\left\langle R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \sum_f Q_f^2 \right\rangle$$

With u, d, s : $\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 = \frac{2}{3}$

Look back at Fig. 4; this is much too small a value.

But, Han + Nambu told us that quarks come in 3 colors. Each color state can be separately pair-produced. With color, the formula on p. 15 becomes:

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 3 \cdot \sum_f Q_f^2 \qquad \left[\begin{array}{l}\text{independent}\\ \text{of } E_{cm}\ !\end{array}\right]$$

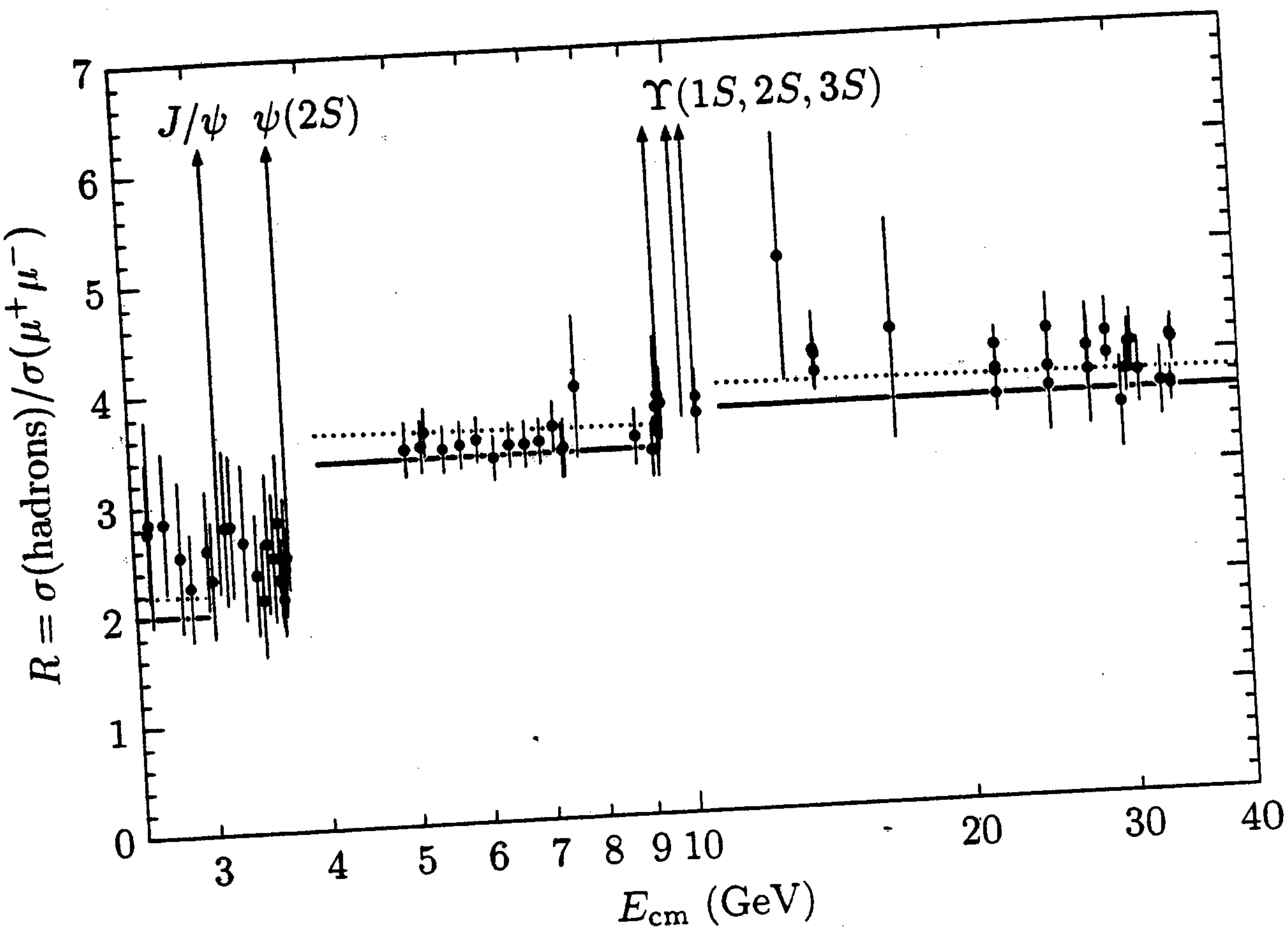
$$= \begin{cases} 2 & u+d+s \\ 3\frac{1}{3} & u+d+s+c \\ 3\frac{2}{3} & u+d+s+c+b \end{cases}$$

The sum should be taken over those quarks whose masses can be ignored at the given value of E_{cm}. The data up to 40 GeV is shown in Fig. 11. The prediction above is the solid line. (A more sophisticated prediction in the Standard Model is the dotted line; I'll discuss the different in lecture 8.)

The points of discontinuity in Fig. 11 are the energies at which a new quark appears: $E_{cm} \sim 2m_q$. The shape of the cross section for $e^+e^- \to$ hadrons in this transition region is interesting; I'll discuss it in a moment.

There is still something very mysterious going on here. The basic process we are observing is $e^+e^- \to q\bar{q}$, but the quarks do not come out. Instead we see pions,

Figure 11

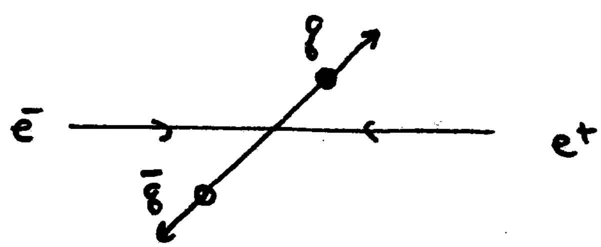


data compiled by M. Swartz.

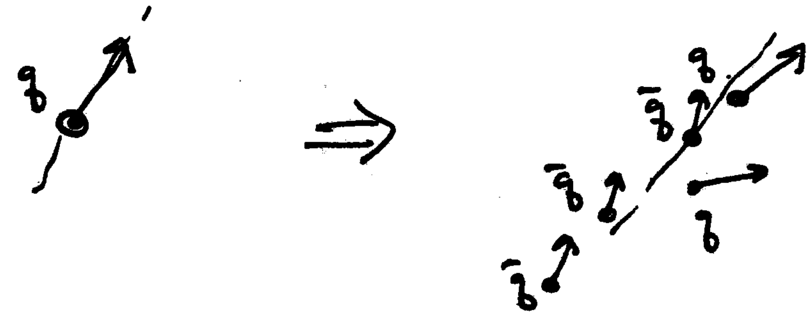
bound state of quarks. Why?

Let me give you a picture which addresses this question. Eventually, we will have the task of explaining how this picture follows from the Standard Model. I will discuss that in lecture 8.

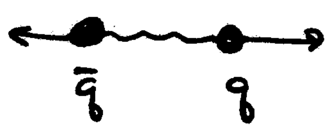
The process $e^+e^- \to \mu^+\mu^-$ or $e^+e^- \to q\bar{q}$ is a process with large momentum transfer : $\Delta p \sim E_{cm}$. Let's assume that the forces which bind quarks together only allow small momentum transfers. We might say that electromagnetic interactions are "hard" while strong interactions are "soft". Then, after we produce the $q\bar{q}$ state



the strong interactions can take over and produce more $q\bar{q}$ pairs. However, since these interactions are soft, they do not change the overall momentum flow in the event but only rearrange the momentum and divide it up

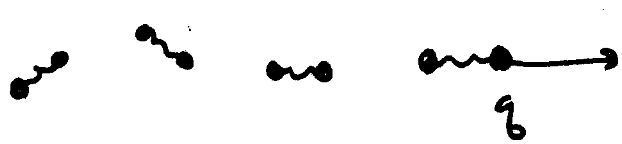


A nice picture of this process is to imagine that there are "strings" connecting the q's and $\bar q$'s



If we stretch the string, we must increase its energy. Eventually, it is more energetically favorable to break the string and produce another $q\bar q$ pair

The process continues until there are no more stretched strings; then we have a system of $q\bar q$ bound states moving roughly colinearly, a "jet"



A prediction of this picture is that the particle production at one end of the string should be independent of what is at the other end. Thus we expect that the probability to produce a pion of momentum p_π from a quark of momentum P should depend only on the relative boost from P to p and should be independent of E_{cm}.

Let

$$z = \frac{p_\pi}{P} \qquad 0 < z < 1 \text{ if the } \pi \text{ is in the jet.}$$

"Feynman scaling variable"

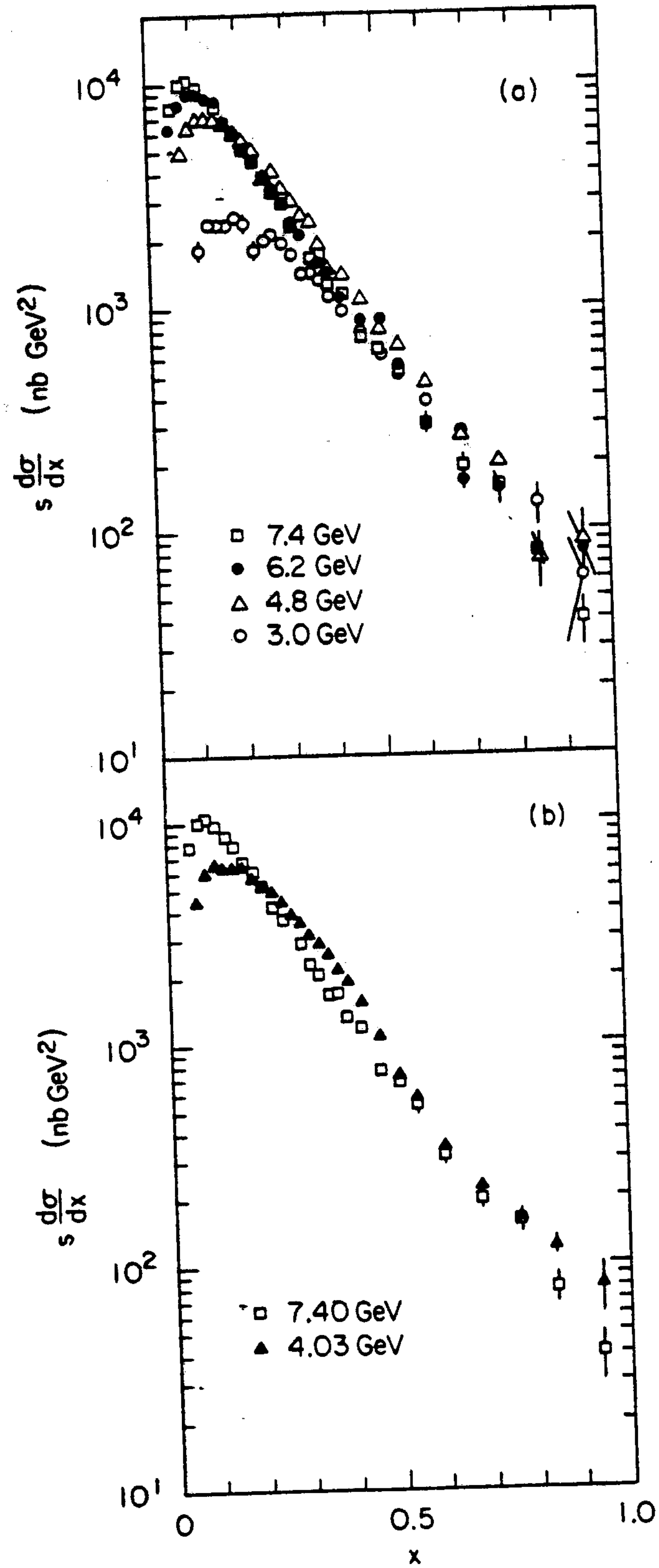
Then I claim:

Prob of producing a pion π^+ w. $p_\pi = zP$ from a quark q w. mom P

$$= f_{\pi^+ \leftarrow q}(z)$$

a universal function. The functions $f_{h\leftarrow q}(z)$ are called "fragmentation functions". Fig. 12 shows fragmentation functions derived from e^+e^- data of the Mark I experiment at SPEAR. They are at least roughly

Fig. 12



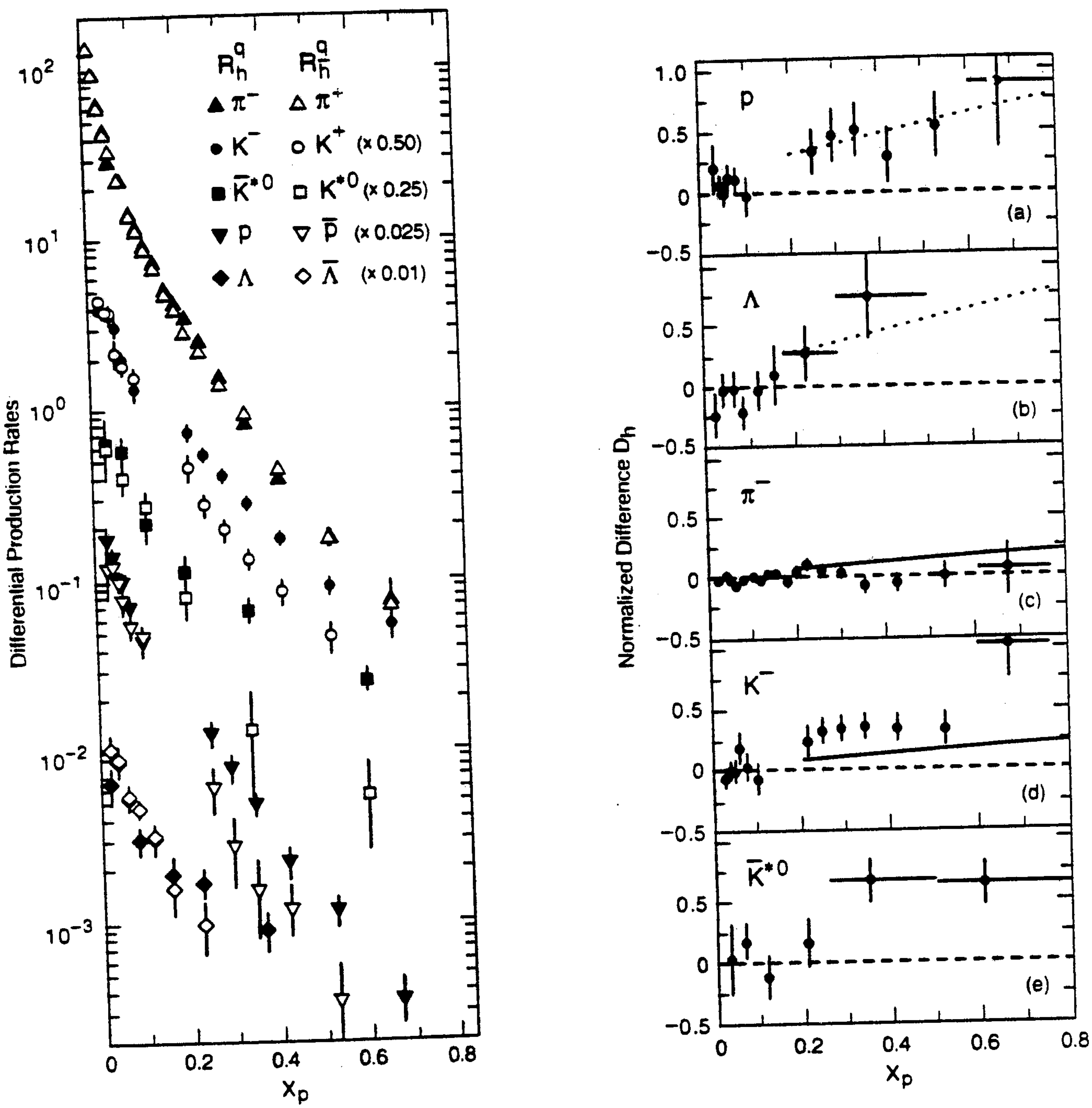
J.L. Siegrist et al (Mark 1) Phys. Rev. D26, 969 (1980)

$$x = z = \frac{\hat{n}\cdot\vec{p}_\pi}{(\frac{1}{2}E_{cm})}$$ where $\hat{n}$ is the jet axi.

independent of E_{cm}. Fig. 13 shows some raw data from the SLD experiment which makes use of a technique that I will describe in Lecture 7 to tell the quark jet from the antiquark jet. This shows the "leading particle effect": the fastest particle in the jet tends to contain the quark from which the jet was formed. For example, an s-quark jet should make fast K^- $(s\bar{u})$ and Λ (sud) particles more than their antiparticles, preferentially at large z.

The idea that the strong interactions are "soft" is very familiar to people who study other types of elementary particle reactions, such as proton-proton scattering. There, even at very high energies ($E_{cm} > 50$ GeV or more) the typical pions produced have $p_\perp \lesssim 1$ GeV relative to the collision axis. Fig. 14 shows the distribution of produced π^0's in transverse momentum in pp scattering at the CERN ISR. Note that the x-axis is linear but the y-axis is a many-decade

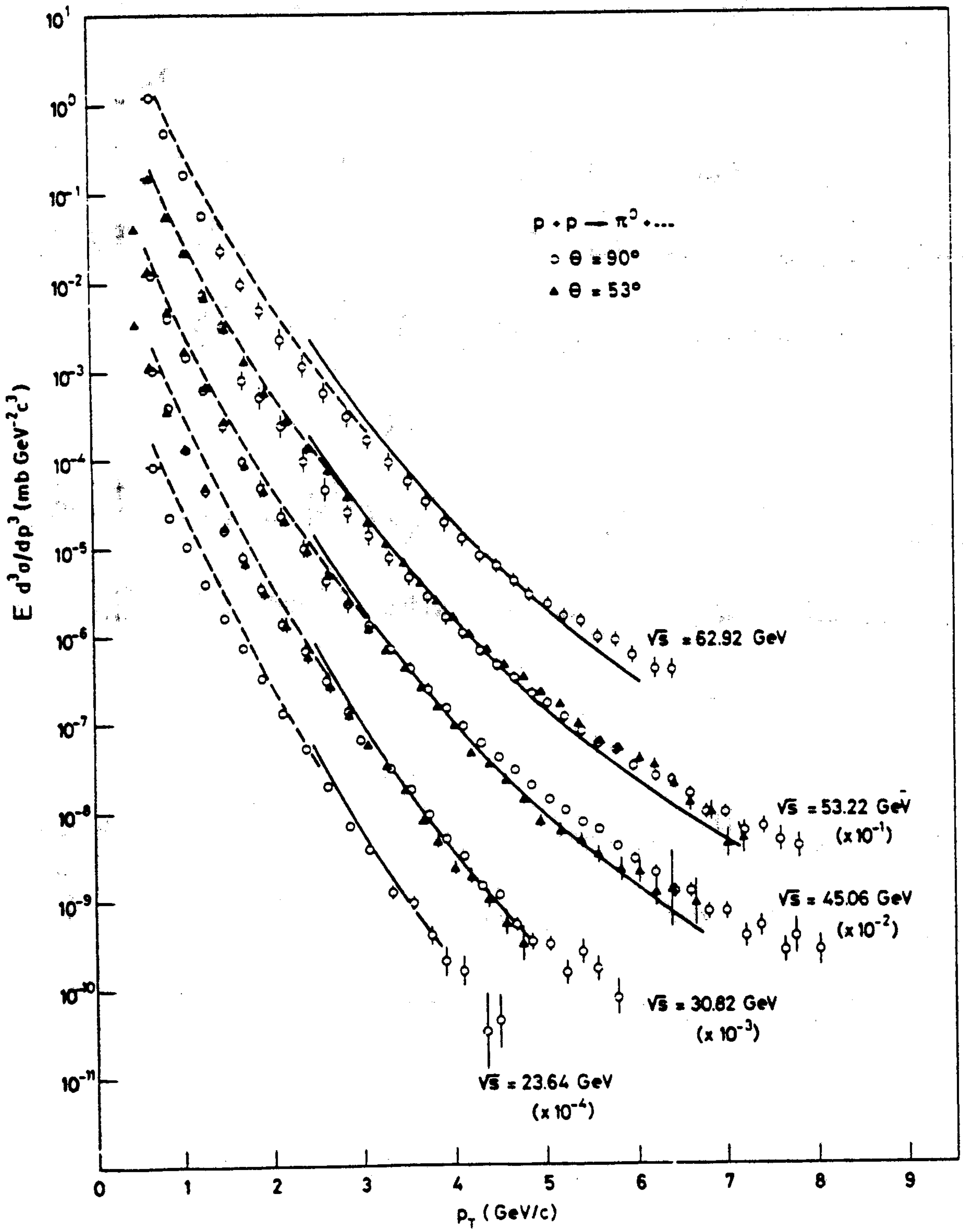
Fig. 13



K. Abe et al (SLD)

Phys. Rev. Lett. 78, 3442 (1997)

Figure 14



K. Eggert et al Nucl. Phys. B98, 49 (1975) logarithmic scale! Fig. 15 shows the same phenomenon at higher energy at the SPS $p\bar{p}$ collider.

And yet, at very high energy, one does observe jets in pp and $p\bar{p}$ collisions. Fig. 16 shows the energy distribution over the sphere for an event observed by the UA2 experiment at the SPS collider. The $p\bar{p}$ experiments refer to high transverse momentum particles by

$$E_{Ti} = E_i \sin\theta$$

rather than $|p_{\perp i}|$, but these quantities are almost the same. Events with <u>hard</u> collisions have very high

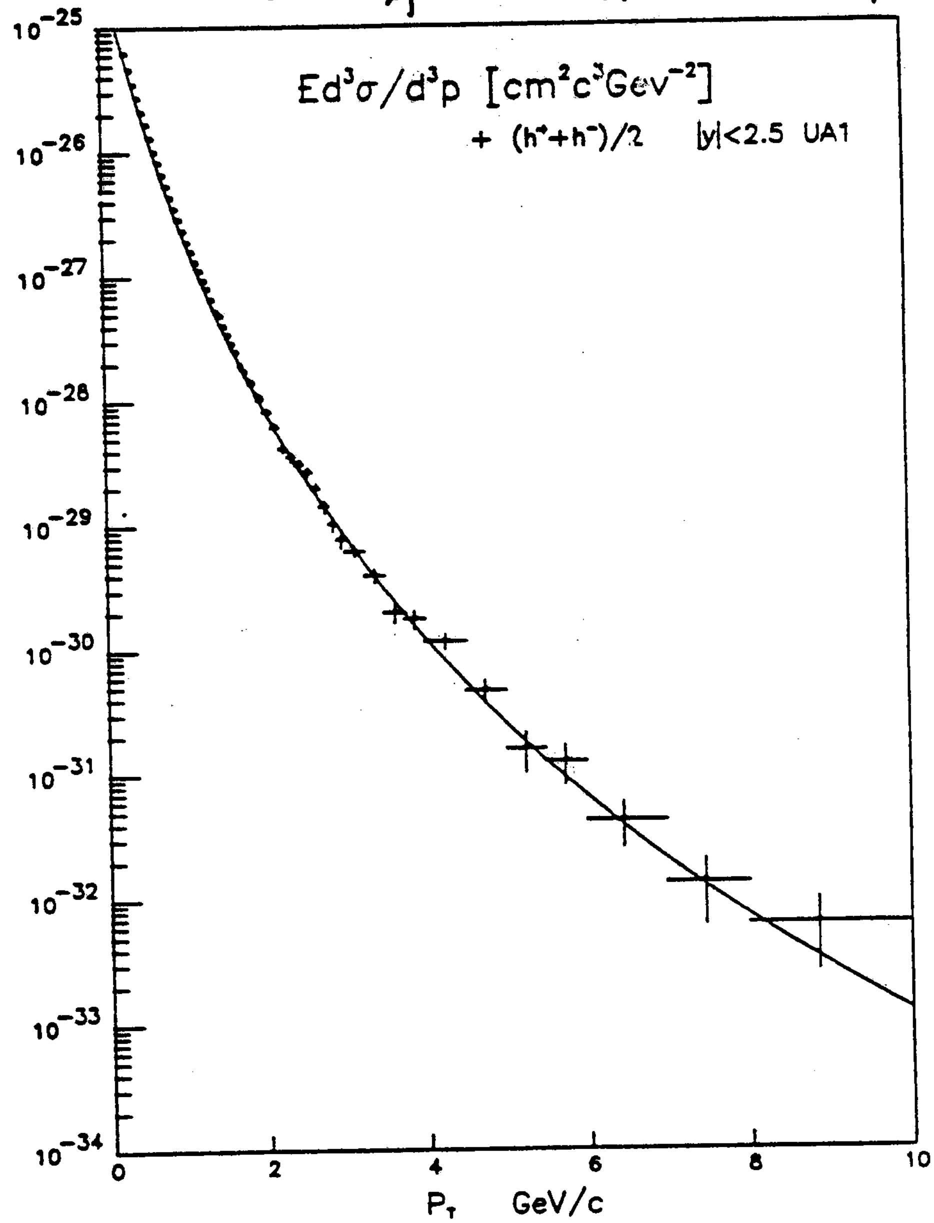
$$E_T = \sum_i E_{Ti} .$$

Fig. 17 shows the fraction of the total E_T which is contained in two cones around jet axes, as a function of the total E_T. The hardest collisions are the most two-jet-like.

It is very tempting to think of these two-jet events as resulting from $q\bar{q}$ hard collisions. To test this idea, boost to the two-jet center

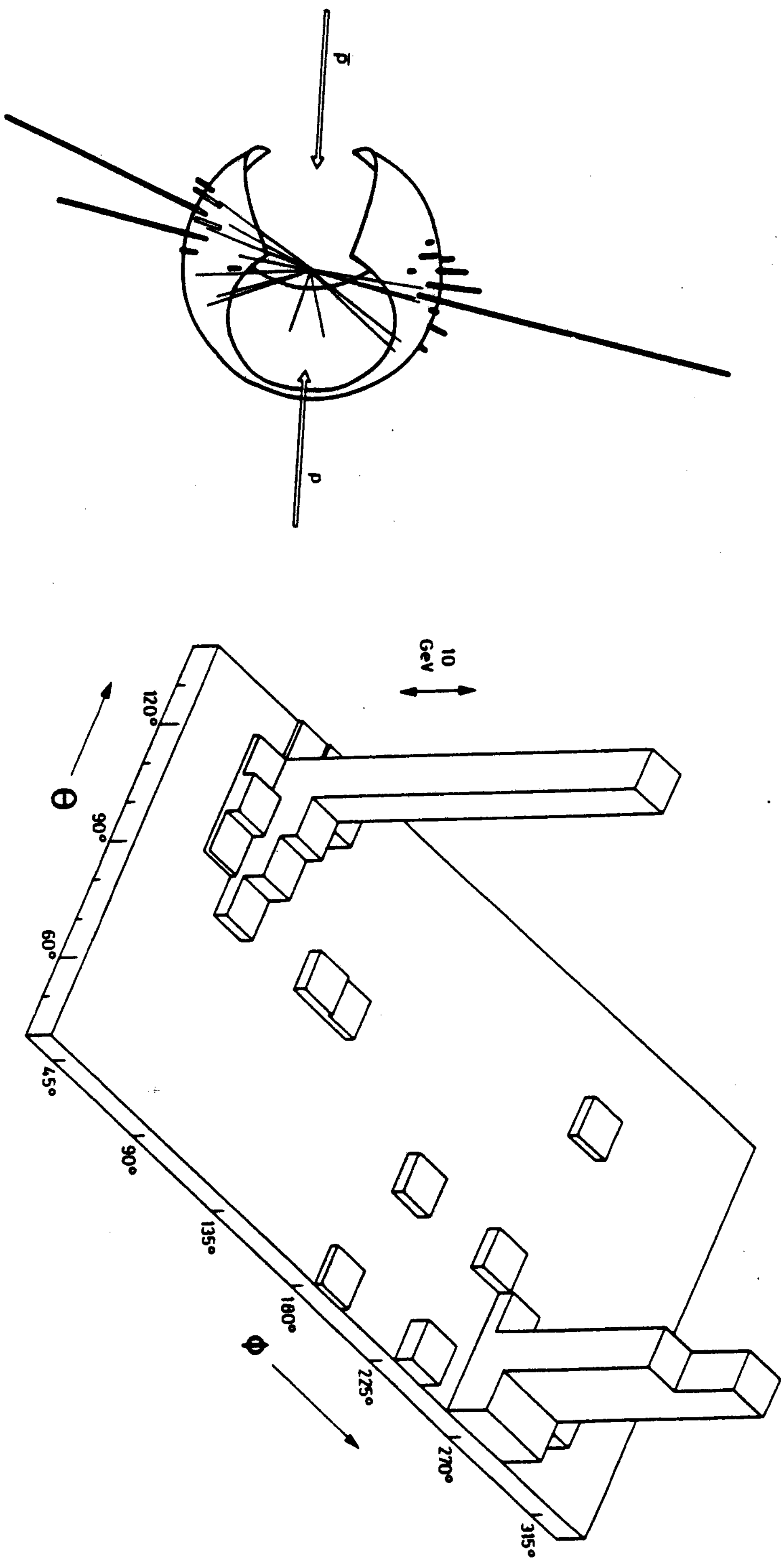
Figure 15

UA1 experiment at E_{cm} = 546 GeV



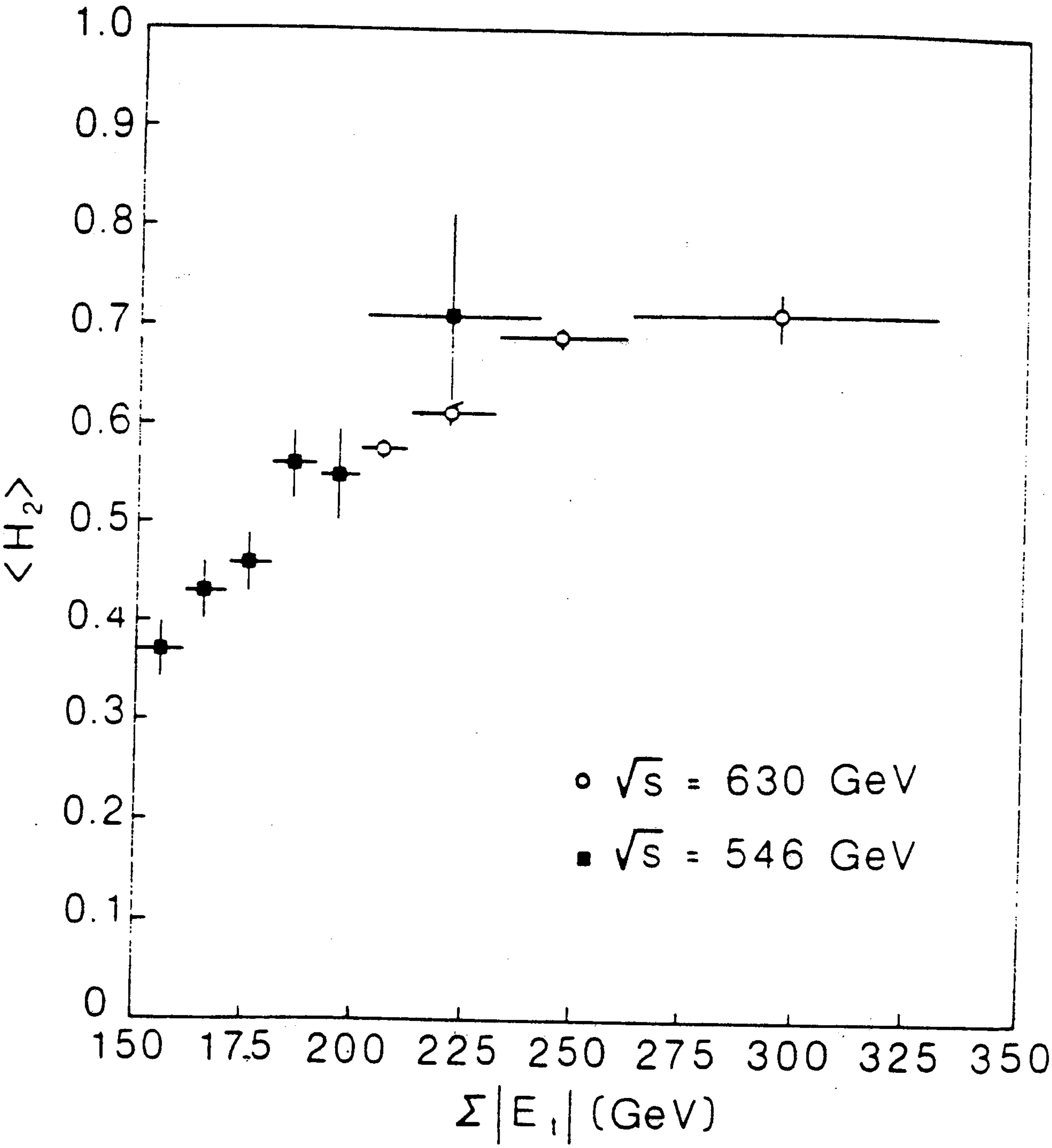
from G. Altarelli and L. Di Lella,
Proton-Antiproton Collider Physics

Figure 16



M. Banner et al (UA2)
Phys. Lett. $\underline{118B}$, 203 (1982)

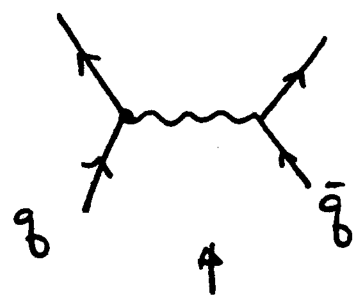
Figure 17



C. Albajar et al (UA1)

Z Phys. C 36, 33 (1987)

of mass frame and observe the angular distribution of the jets. For the UA1 data, this is shown in Figure 18. The similarity to the angular distribution for $e^+e^- \to e^+e^-$ in Fig. 1 of the previous lecture is striking. The q and $\bar{q}$ scatter by a strong-interaction analogue of Coulomb scattering

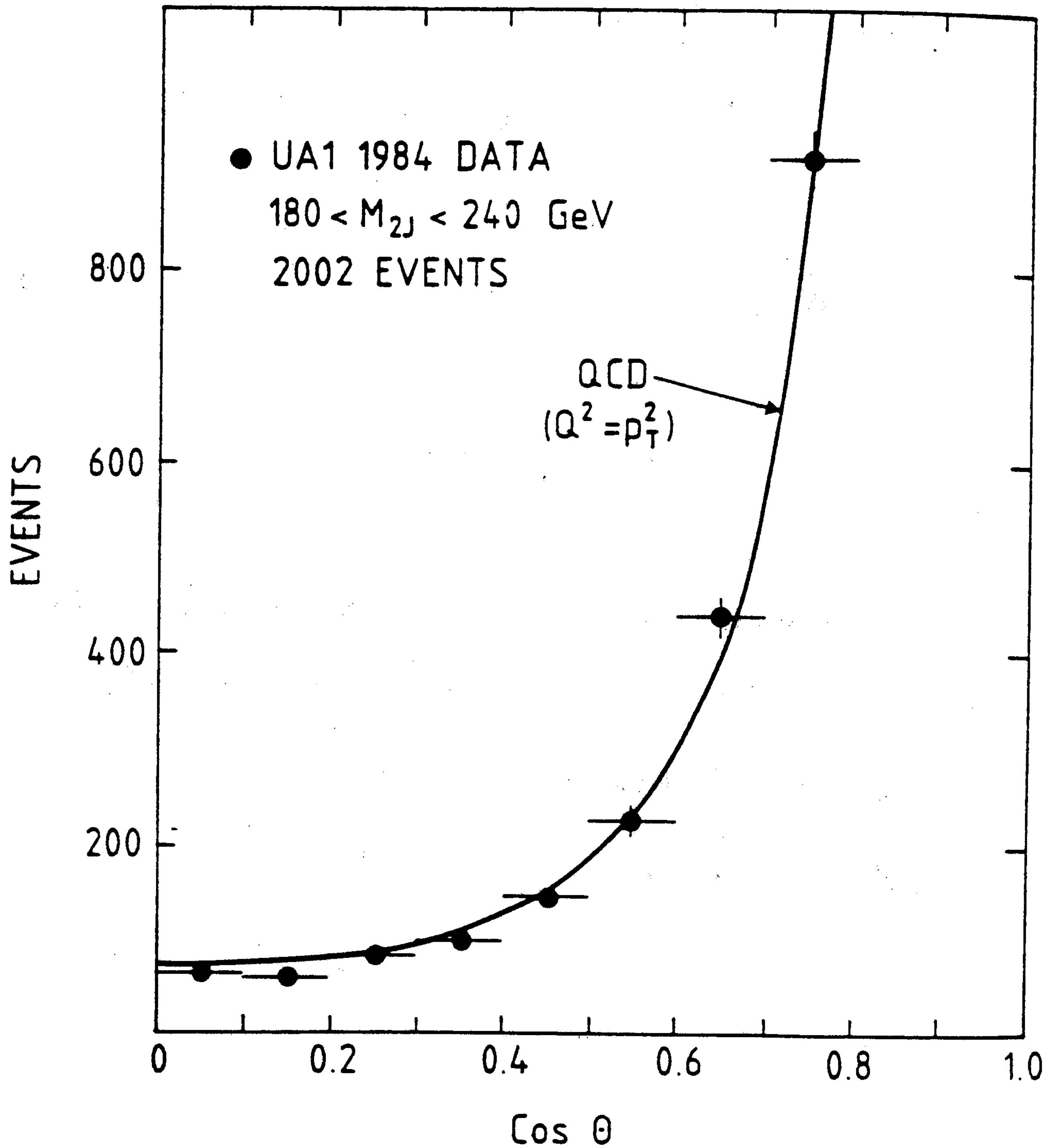


(some boson needed here)

I will discuss the physics of this diagram further in lecture 4.

Finally, I would like to explain briefly what happens in the transition regions of Fig. 11. The physics here is very rich, but I will have time to give just a brief outline. Consider first the region around 10 GeV where the process $e^+e^- \to b\bar{b}$ just becomes kinematically allowed. Fig. 19

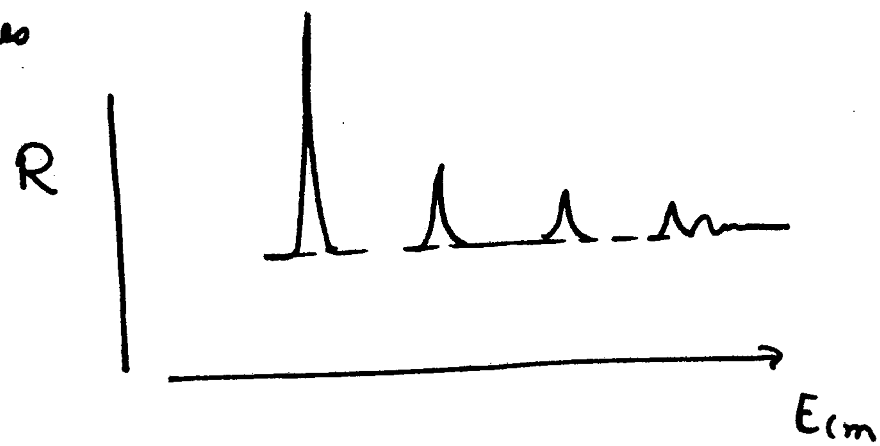
Figure 18



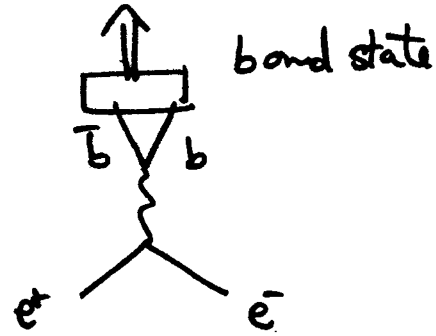
G. Arnison et al (UA1)

Phys. Lett. B177, 244 (1986)

shows the $e^+e^- \to$ hadrons cross section in a number of intervals between 9.4 and 11.3 GeV, as measured by the experimenters at the Cornell synchrotron CESR. One sees a flat value of R punctuated by a series of resonances

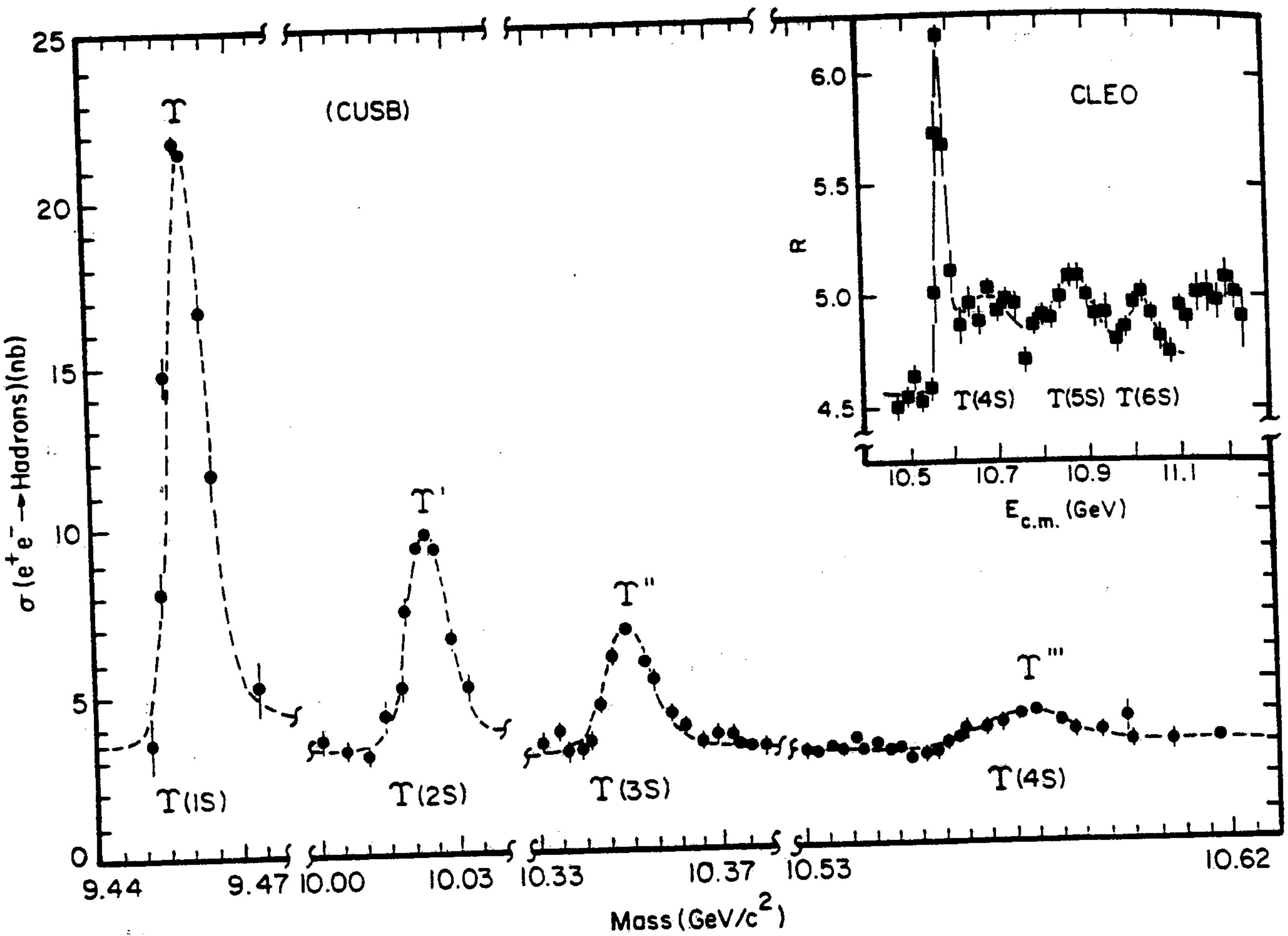


These resonances have a natural interpretation: If the b and $\bar{b}$ quarks are bound by some potential, we should see discrete bound states. The S-states should be produced in $e^+e^- \to b\bar{b}$



Thus, the peaks indicate the energies of the 1S, 2S, 3S, and 4S states of the $b\bar{b}$ atom.

Figure 19



from D. Besson and T. Skwarnicki

Ann Rev. Nucl. Part. Sci 43, 333 (1993)

Between $E_{cm} = 3$ and 4 GeV, one sees similarly resonances corresponding to the 1S, 2S, 3S, .. states of the $c\bar{c}$ atom. The 1S resonance is the celebrated J/ψ particle. By going to the energy of the 2S state and looking for photons, it is possible to observe other states of the atom, e.g.

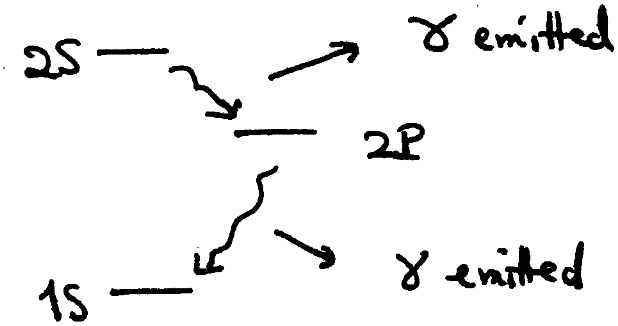
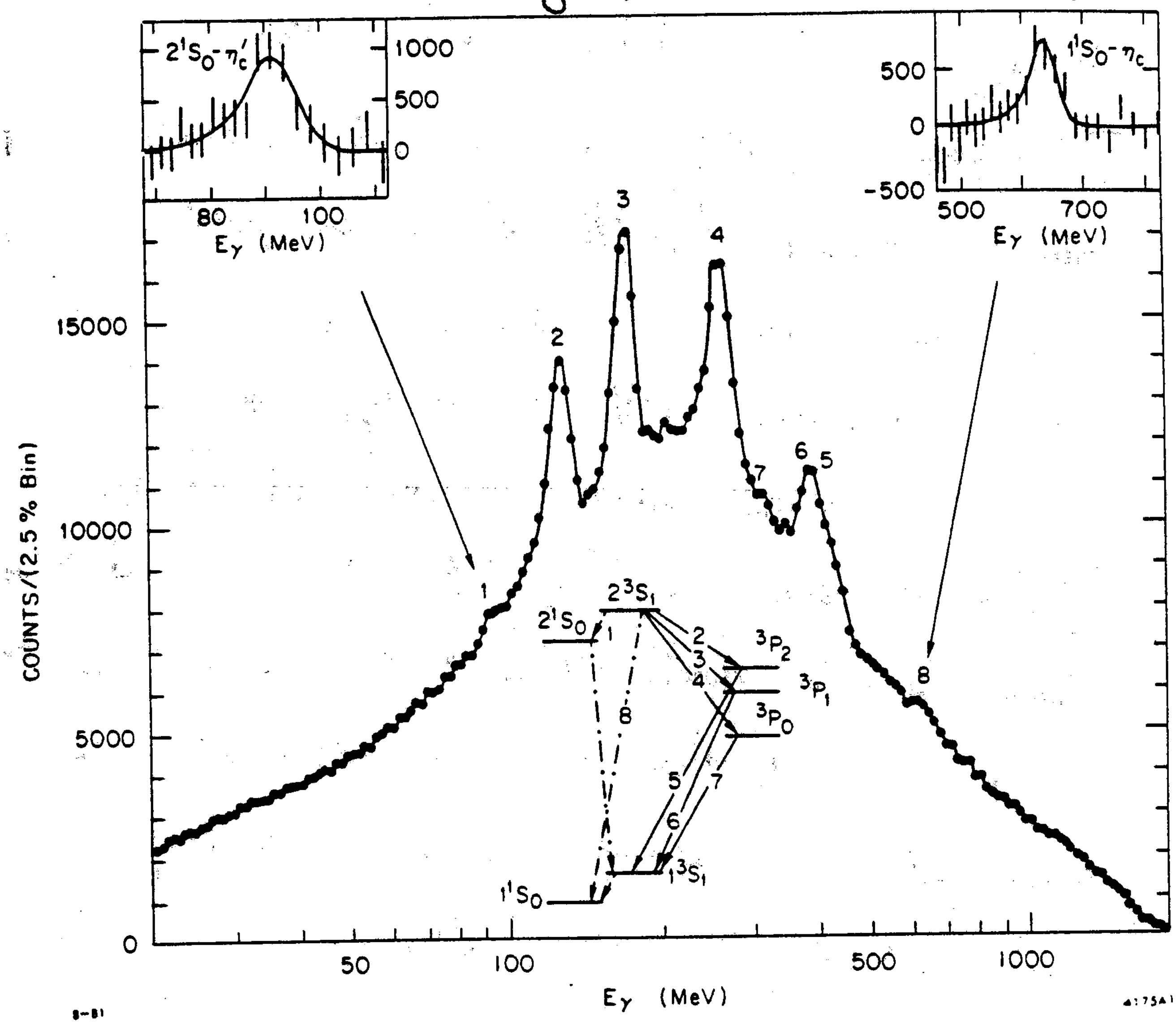


Fig. 20 shows the γ-ray spectrum measured at the 2S $c\bar{c}$ state by the Crystal Ball experiment at SPEAR. Most of the γ's come from π^0 decay ($\pi^0 \to 2\gamma$) this gives a smooth background. On top of this background you can see spectral lines corresponding to the transitions indicated in the figure.

From these and other experiments, the spectrum of $c\bar{c}$ and $b\bar{b}$ bound states is known in some detail. In Figs. 22 and 23, I show the known states

Figure 20

Crystal Ball at $E_{cm} = m(\psi')$



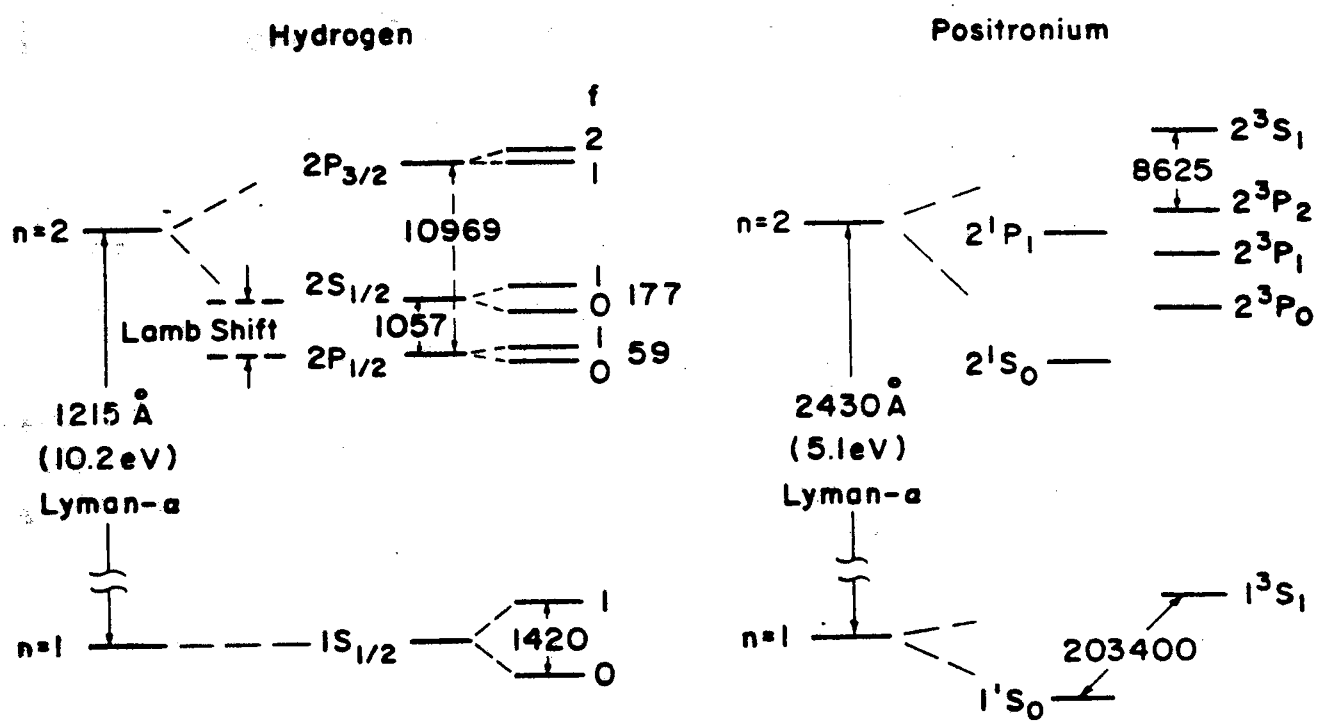
from E. Bloom and C E Peck,
Ann. Rev. Nucl. Part. Sci. 33, 143 (1983)

and the observed transitions. Fig. 21 shows the spectrum of positronium (e^+e^- bound states) for comparison. The known $c\bar{c}$ and $b\bar{b}$ levels correspond exactly in their systematics and quantum numbers. This gives very strong evidence that quarks are simple spin-½ fermions like the electron and positron.

Given the bound state energies, it is possible to work out the form of the $q\bar{q}$ binding potential. A number of potentials which successfully fit the $c\bar{c}$ and $b\bar{b}$ spectra are shown in Fig. 24. The vertical lines show the charge radii of the known $c\bar{c}$ and $b\bar{b}$ states; in the region of these radii, the potential is well constrained. The potential increases at large distances, roughly indicating the "string" behavior.

In this lecture, then, I have shown how the study of $e^+e^- \to$ hadrons gives a clean pictorial view of the quarks. These are simple spin-½ fermions which can take their place beside the leptons as fundamental constituents of matter. At the same

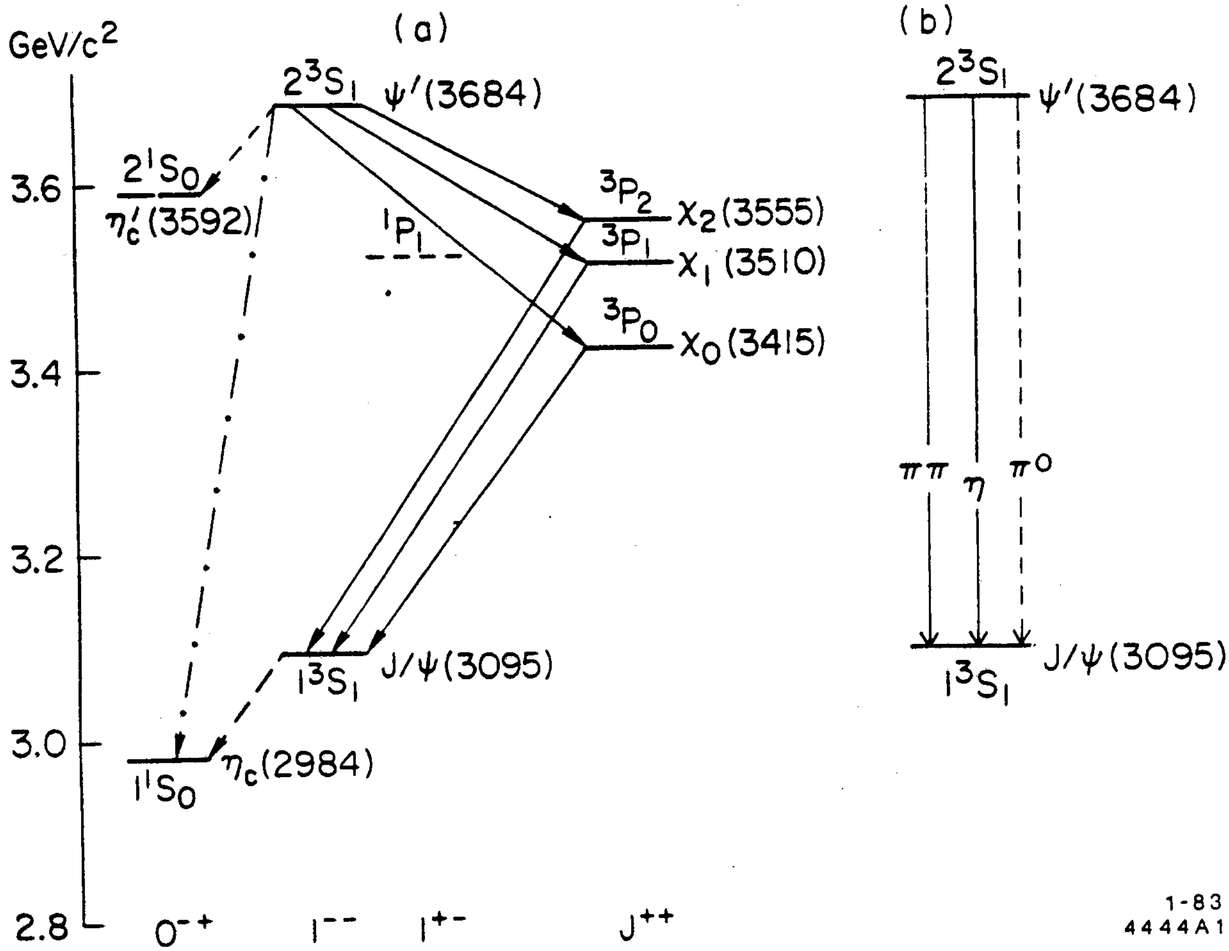
Figure 21



from S. Berko and H N Pendleton,
Ann. Rev. Nucl. Part. Sci. 30,
543 (1980)

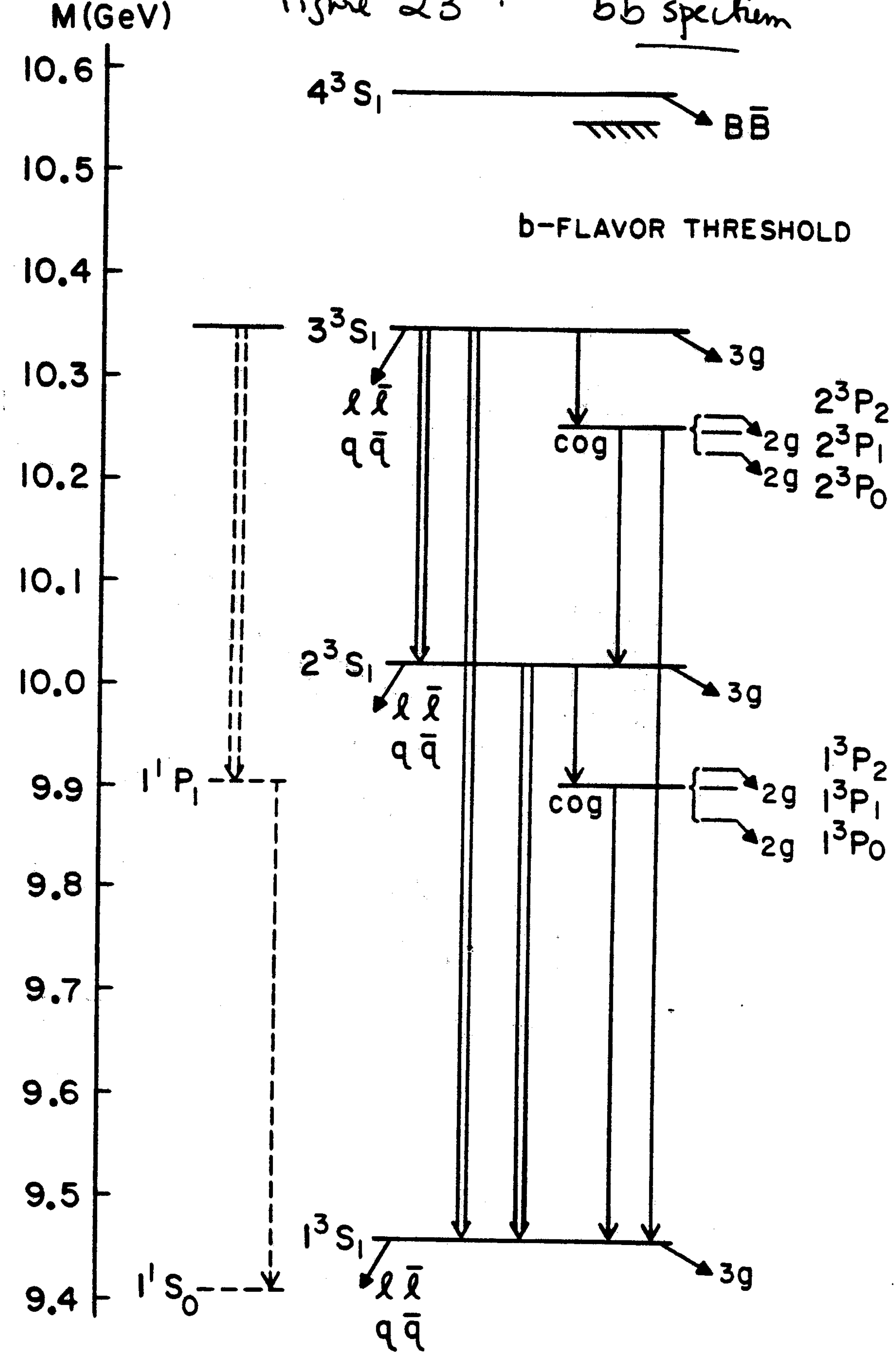
Figure 22

cc̄ spectrum



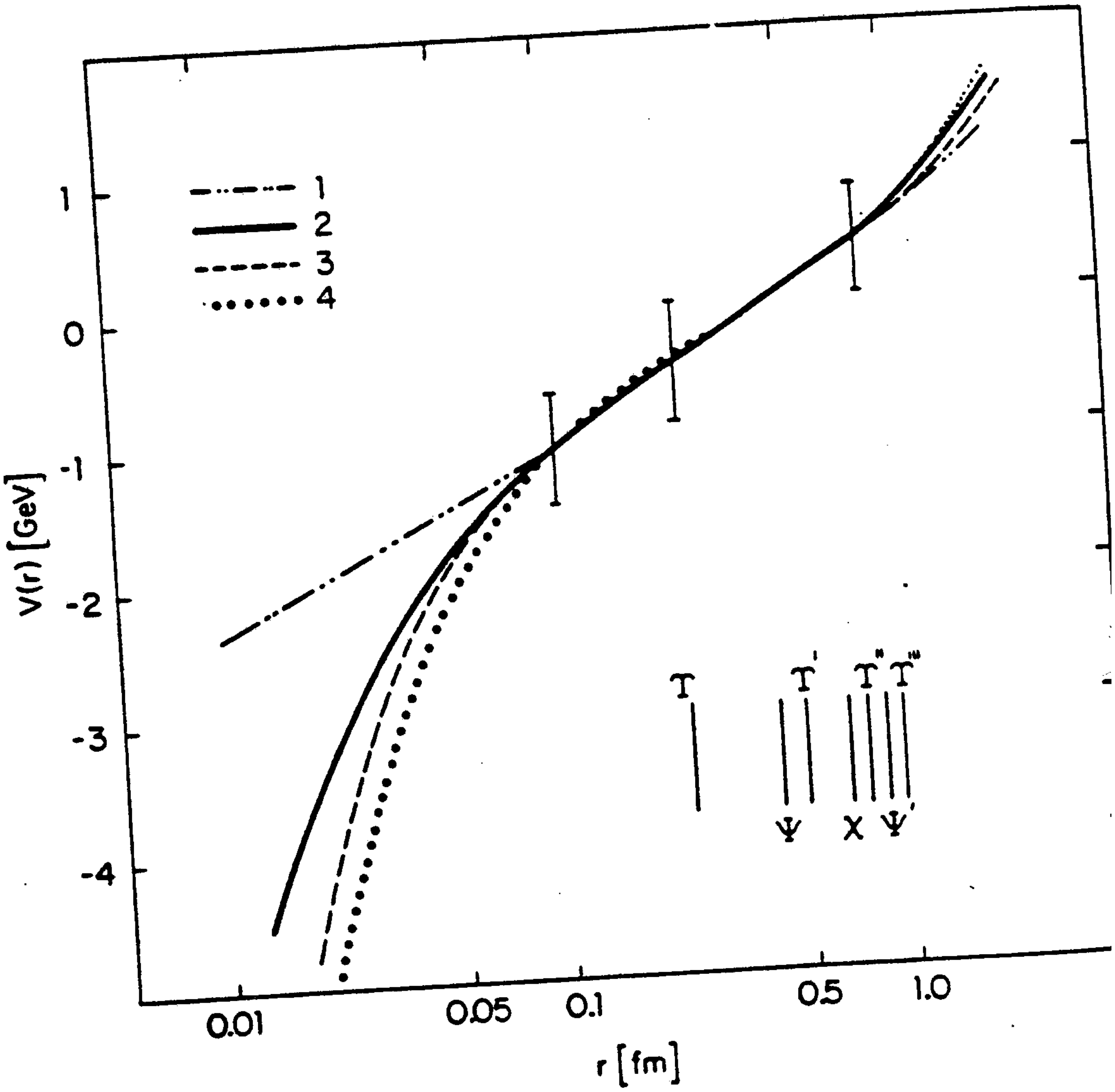
from E. Bloom and C.E. Peck
Ann. Rev. Nucl. Part. Sci. 33, 143 (1983)

Figure 23 : $b\bar{b}$ spectrum



from P. Franzini and J. Lee-Franzini,
Ann. Rev. Nucl. Part. Sci. 33, 1 (1983)

Figure 24



W. Buchmüller and S.-H. H. Tye
Phys. Rev. D 24, 132 (1981)

time, we have learned a little about the nature of the strong interactions. For the most part, these interactions are soft, binding quarks without hard scattering. However, they do have a hard component which is visible at high energy. We will see later how both of these features emerge from the Standard Model.

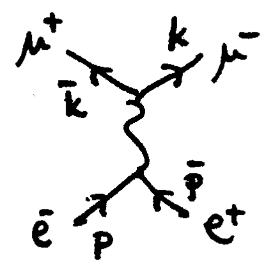
Lecture 3: July 22

In the previous lecture, I explained how we can use the electromagnetic interaction to give insight into the properties of the quarks. Now, normally quarks exist in Nature as constituents of the proton and neutron. So it is natural to ask, can we visualize experimentally how the quarks build up the structure of the proton. In this lecture, I will explain how electromagnetism allows us to do that also.

As a preliminary to this study, we need the relativistic formulae for Coulomb scattering. These can be obtained from the formulae derived in lecture 1, plus a strange trick of relativistic quantum theory. Let's recall from lecture 1 the following results for $e^+e^- \to \mu^+\mu^-$ with all particles massless:

$$\mathcal{M}(e^-_R e^+_L \to \mu^-_R \mu^+_L) = e^2(1+\cos\theta)$$

$$\mathcal{M}(e^-_R e^+_L \to \mu^-_L \mu^+_R) = e^2(1-\cos\theta)$$

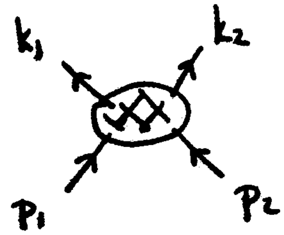


The angle θ is the CM scattering angle: $\quad E = E_{cm}/2$

$$p = (E, 0, 0, E) \qquad \bar{p} = (E, 0, 0, -E)$$

$$k = (E, E\sin\theta, 0, E\cos\theta) \qquad \bar{k} = (E, -E\sin\theta, 0, -E\cos\theta)$$

For a reaction $p_1 + p_2 \to k_1 + k_2$, it is conventional to define the Lorentz-invariants:



$$s = (p_1+p_2)^2 = (k_1+k_2)^2$$
$$t = (p_1-k_1)^2 = (p_2-k_2)^2$$
$$u = (p_1-k_2)^2 = (p_2-k_1)^2$$

"Mandelstam variables"

In the CM frame, for massless particles,

$$s = (2E)^2 = E_{cm}^2$$
$$t = -2E^2(1-\cos\theta)$$
$$u = -2E^2(1+\cos\theta)$$

so
$$s + t + u = 0$$

[More generally, $s = E_{cm}^2$ always, and $s+t+u = \underbrace{\sum_i m_i^2}_{\text{sum of external mass}}$.]

Using these variables, it is clear how to write the expressions for $\mathcal{M}$ above in a manifestly Lorentz-invariant form

$$\mathcal{M}(e^-_R e^+_L \to \mu^-_R \mu^+_L) = -2e^2 \frac{u}{s} = 4e^2 \frac{p\cdot \bar{k}}{s}$$

$$\mathcal{M}(e^-_R e^+_L \to \mu^-_L \mu^+_R) = -2e^2 \frac{t}{s} = 4e^2 \frac{p\cdot k}{s}$$

In this expression, the numerator and denominator have the following significance. An electromagnetic wave propagates according to the massless wave equation

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2 \right) A^\mu = j^\mu$$

In Fourier space

$$-\left((q^0)^2 - |\vec{q}|^2\right) A^\mu(q) = j^\mu(q) \quad \Rightarrow \quad A^\mu(q) = -\frac{1}{q^2} j^\mu(q)$$

In this process the momentum carried by the electromagnetic field is

$$q^\mu = (p+\bar{p})^\mu \quad \Rightarrow \quad q^2 = s$$

so the factor $\frac{1}{s}$ is the solution of the electromagnetic wave equation. A massive field would have the Klein-Gordon wave equation

$$\left(\frac{\partial^2}{\partial t^2} - \vec{\nabla}^2 + m^2 \right) \phi = j$$

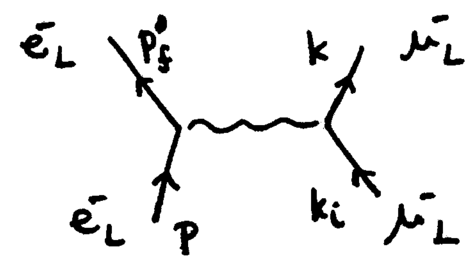
whose solution is

$$\phi(q) = -\frac{1}{(q^2-m^2)} j(q)$$

Then $\frac{1}{s}$ should be replaced with $\frac{1}{s-m^2}$. The numerator in the expression for $\mathcal{M}$ indicates how the spins are coupled and depends only on the external states.

Now I would like to write the amplitude for $e\mu$ scattering: $e^-_L \mu^-_L \to e^-_L \mu^-_L$. The

Feynman diagram for this process is



this is the same diagram, just turned on its side. Another way of saying this is that we have replaced the incoming e_R^+ by an outgoing antiparticle e_L^- and the outgoing μ_R^+ by an incoming antiparticle μ_L^-.

In relativistic quantum mechanics, the amplitude for this diagram is given by a very simple prescription. We just use the mathematical expression for $\mathcal{M}(e_L^- e_R^+ \to \mu_L^- \mu_R^+)$ with the momentum of e_R^+ $-(\bar{p})-$ replaced by $(-p_f)$ and the momentum of μ_R^+ $-(\bar{k})-$ replaced by $(-k_i)$. Again, we use the same mathematical formula, just taking account of who is incoming and who is outgoing. Now

$$q^2 = (p - p_f)^2 = t \quad \text{in the new kinematics}$$

and

$$\mathcal{M}(e_L^- \mu_L^- \to e_L^- \mu_L^-) = 4e^2 \frac{p \cdot k_i}{q^2}$$

similarly

$$\mathcal{M}(e_L^- \mu_R^- \to e_L^- \mu_R^-) = 4e^2 \frac{p \cdot k}{q^2}$$

and as before $\mathcal{M}(e_L^-\mu_L^- \to e_L^-\mu_L^-) = \mathcal{M}(e_R^-\mu_R^- \to e_R^-\mu_R^-)$

$$\mathcal{M}(e_L^-\mu_R^- \to e_L^-\mu_R^-) = \mathcal{M}(e_R^-\mu_L^- \to e_R^-\mu_L^-)$$

then, for example

$$\frac{d\sigma}{d\cos\theta_{cm}}(e_L^-\mu_L^- \to e_L^-\mu_L^-) = \frac{\pi\alpha^2}{2E_{cm}^2}\cdot\left(\frac{4p\cdot k_i}{q^2}\right)^2$$

$$4p\cdot k_i = 2(p+k_i)^2 = 2E_{cm}^2 \; ; \quad q^2 = -2E^2(1-\cos\theta) = -\tfrac{1}{2}E_{cm}^2(1-\cos\theta)$$

we find

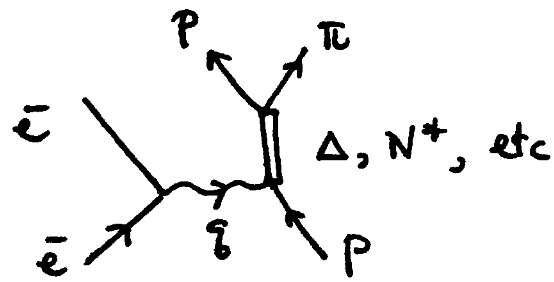
$$\frac{d\sigma}{d\cos\theta_{cm}}(e_L^-\mu_L^- \to e_L^-\mu_L^-) = \frac{8\pi\alpha^2}{E_{cm}^2}\,\frac{1}{(1-\cos\theta)^2}$$

As $\cos\theta \to 0$, the behavior is $\frac{1}{\theta^4}$, the familiar dependence of Coulomb scattering.

Now consider electron-proton scattering. If the proton were an elementary particle, we might expect to find the same formula at high energy and large momentum transfer. Instead, the result for

$$e^-p \to e^-p$$

is much smaller; it is multiplied by a "form factor" $\left|\frac{G_M}{\mu}\right|^2$, where G_M is measured to follow the dependence shown in Fig. 25. At high energy, elastic e^-p scattering is a negligible part of the total cross section. Fig. 26 shows the dependence of the e^-p scattering cross section (times $(q^2)^2$) for increasing E_{cm} and increasing $Q^2 = -q^2$. At low energy, we see clean resonances due to processes such as



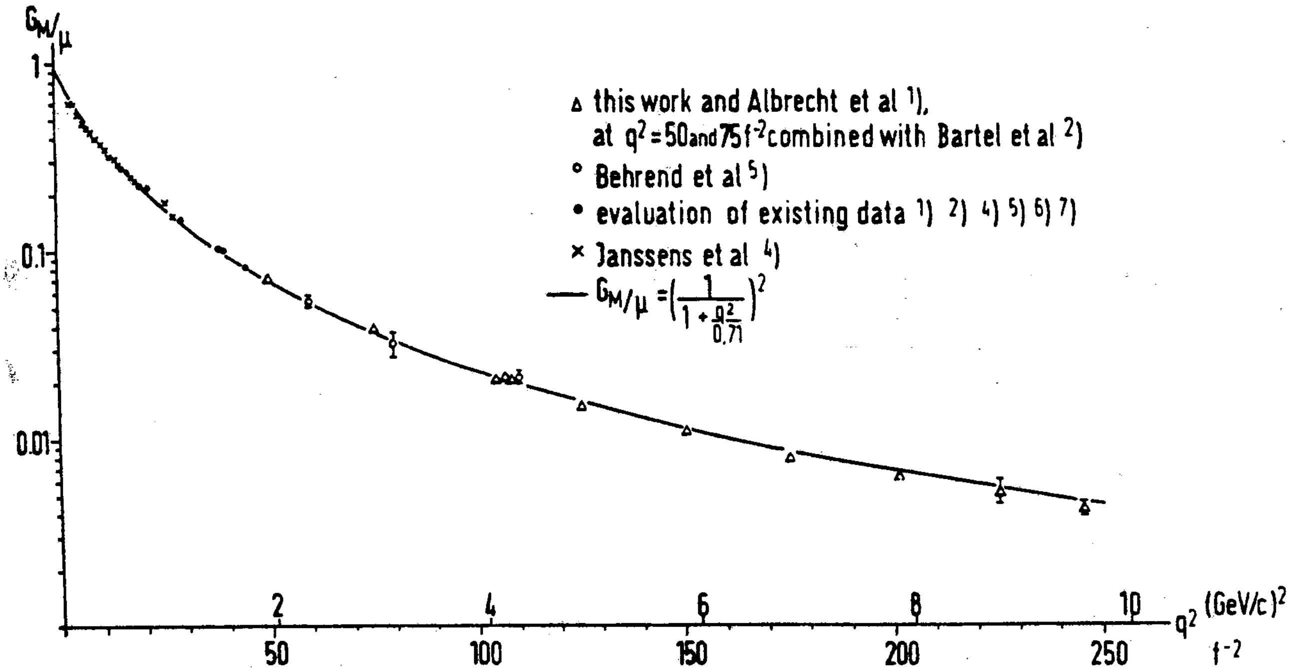
At higher energy, the individual resonances disappear, and we have only a homogeneous continuum

$$e^-p \to e^- + (\text{many hadrons})$$

This story sounds familiar. In fact, it is just the mirror of the story we saw for $e^+e^- \to$ hadrons. So, based on our intuition from that process, we might expect that, when E_{cm} and Q^2 are very large, the <u>quarks</u> appear inside the proton. This region is called the regime of "deep inelastic scattering".

We can use the ideas of the previous lecture to

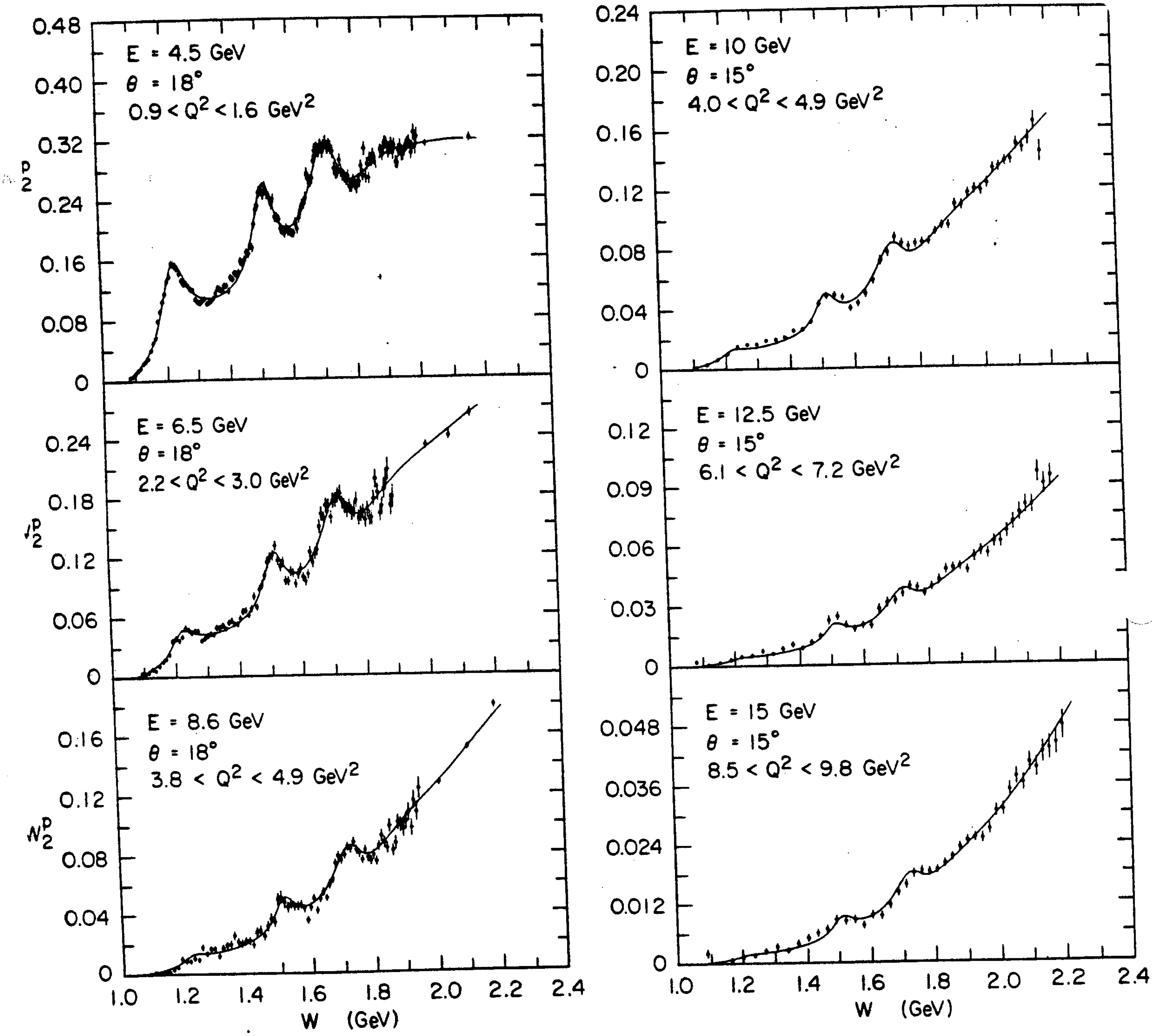
Figure 25



W. Albrecht et al.

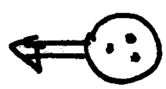
Phys. Rev. Lett. 18, 1014 (1967)

Figure 26



A. Bodek et al Phys. Rev. D 20, 1471 (1979)
(SLAC-MIT)

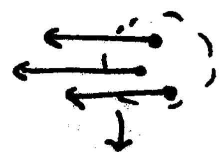
build a quantitative model of deep inelastic scattering. Look at the process from the electron-proton CM frame. Since, by our assumptions, the strong interactions are soft, the constituents of the proton carry small transverse momenta. Then, to first approximation, we can regard them as moving colinearly with the proton



momentum P $\qquad P^2 = m_p^2 \ll E^2$

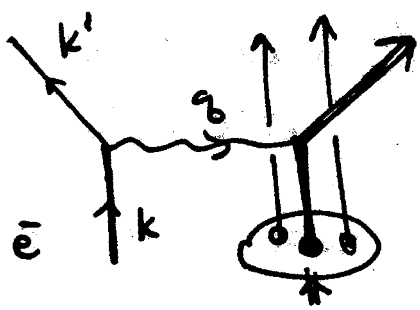
$\Uparrow$

approximate $P^2 \approx 0$



momentum ξP $\qquad 0 < \xi < 1$

Electromagnetic interactions can be hard, so the electron can scatter one quark out through a large angle:



The result of this reaction should be a jet of hadrons moving away from the proton remnant.

This picture, which is due to Feynman, is called the parton model. The essential information about the proton wavefunction in this model is encoded in parton distribution functions: e.g.

$$f_u(\xi)\, d\xi \;=\; \text{Probability that a } u \text{ quark in the proton has momentum fraction between } \xi \text{ and } \xi + d\xi.$$

These distribution functions obey the sum rules

$$\int_0^1 d\xi\, \left(f_u(\xi) - f_{\bar{u}}(\xi)\right) = 2 \qquad \text{2 u's in the proton}$$

$$\int_0^1 d\xi\, \left(f_d(\xi) - f_{\bar{d}}(\xi)\right) = 1 \qquad \text{1 d's in the proton}$$

$$\int_0^1 d\xi\, \xi \sum_i f_i(\xi) = 1 \qquad \text{Partons carry the total energy-momentum of the proton}$$

Now we can compute the cross-section for deep inelastic scattering by computing the cross section for eq scattering in the absence of strong interactions and then integrate this result over the parton distribution functions.

Let $s = (k+P)^2 = E_{cm}^2$ for electron-proton scattering. Then for the scattering of an electron from a quark of momentum fraction ξ

$$E_{cm}^2(eq) = 2k\cdot p = 2k\cdot \xi P = \xi \cdot s$$

We can take the expression for eq scattering from the formula on p. 5 for eμ scattering, only multiplying by the square of the quark charge Q_f^2:

$$\frac{d\sigma}{d\cos\theta_{cm}}(e_L^- q_L \to e_L^- q_L) = \frac{\pi\alpha^2}{2\xi s}\left(\frac{4k\cdot\xi P}{Q^2}\right)^2 \cdot Q_f^2$$

where

$$Q^2 = -q^2 = \frac{1}{2}E_{cm}^2(eq)(1-\cos\theta_{cm}) = \frac{1}{2}\xi s(1-\cos\theta_{cm})$$

Actually, $\cos\theta_{cm}$ is not a very useful variable here; trade it for the Lorentz invariant Q^2:

$$\tfrac{1}{2}\xi s \, d\cos\theta_{cm} = dQ^2 \qquad\qquad 2k\cdot P = s$$

so

$$\frac{d\sigma}{dQ^2}(e_L^- q_L \to e_L^- q_L) = \frac{4\pi\alpha^2}{s^2}\left(\frac{s}{Q^2}\right)^2 \cdot Q_f^2$$

similarly, for $e_L^- q_R \to e_L q_R$, we have a similar formula with

$$\begin{aligned} 4k\cdot p \;\to\; 4k\cdot p' &= 4k\cdot(p+q) \\ &= 4k\cdot\xi P + 4k\cdot(k-k') \\ &= 2\xi s + 2q^2 \\ &= 2(\xi s - Q^2) \end{aligned}$$

then

$$\frac{d\sigma}{dQ^2}(e_L^- q_L \to e_L^- q_L) = Q_f^2 \cdot \frac{4\pi\alpha^2}{Q^4}$$

$$\frac{d\sigma}{dQ^2}(e_L^- q_R \to e_L^- q_R) = Q_f^2 \cdot \frac{4\pi\alpha^2}{Q^4}\left(1 - \frac{Q^2}{\xi s}\right)^2$$

averaging over initial spins and summing over final spins

$$\frac{d\sigma}{dQ^2}(e^- q) = Q_f^2 \cdot \frac{2\pi\alpha^2}{Q^4}\left[1 + \left(1 - \frac{Q^2}{\xi s}\right)^2\right]$$

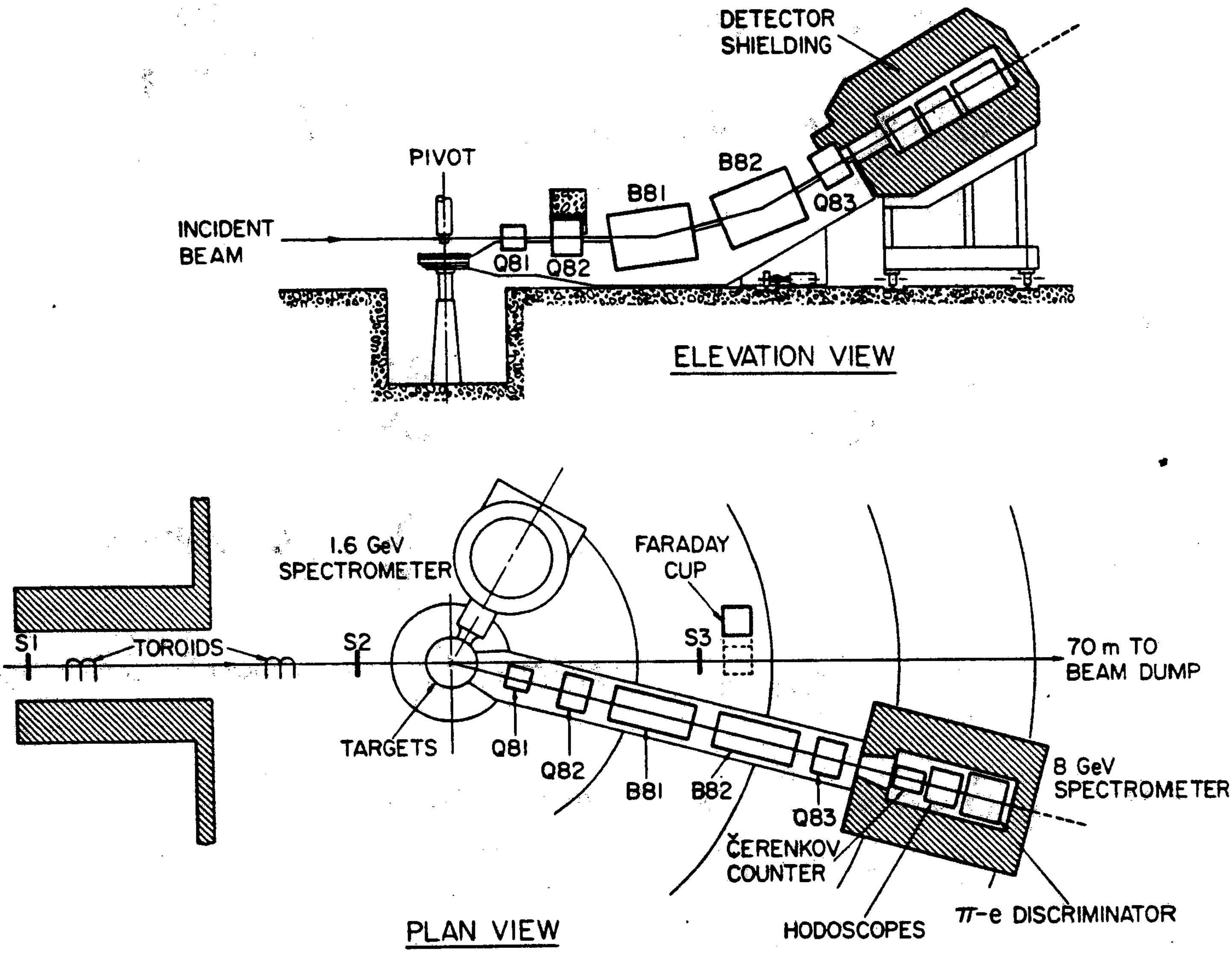
Then

$$\frac{d\sigma}{dQ^2}(e^- p \to e^- + \text{hadrons})$$

$$= \sum_f \int_0^1 d\xi \; f_f(\xi) \, Q_f^2 \cdot \frac{2\pi\alpha^2}{Q^4}\left[1 + \left(1 - \frac{Q^2}{\xi s}\right)^2\right]$$

We can get a better grasp on this formula by thinking a little more about the actual deep inelastic scattering experiment. Fig. 27 shows how the original SLAC-MIT deep inelastic scattering experiments were done. The information on the hadronic products was essentially discarded, but great pains were taken

Figure 27



A. Bodek et al (SLAC-MIT)

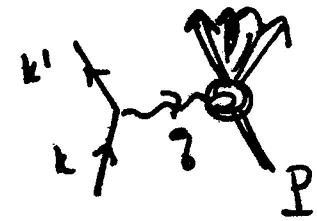
Phys. Rev. D 20 1471 (1979)

to measure the momentum of the scattered electron accurately.
Thus, for each event, we know k, k', and P. This allows us to construct several invariants

$$Q^2 = -(k-k')^2$$

$$2P\cdot q = 2P\cdot(k-k')$$

$$2P\cdot k = s$$



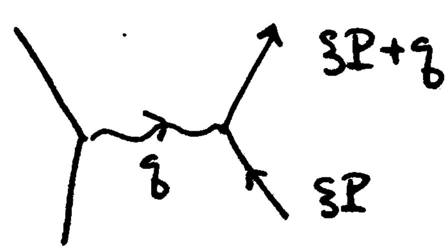
Let

$$x = \frac{Q^2}{2P\cdot q} \qquad y = \frac{2P\cdot q}{2P\cdot k}$$

$$= \frac{q^0}{k^0} \quad \text{in proton or lab fr.}$$

so y is just the fraction of energy transferred into hadrons

The significance of x is more obscure. To see it, consider the kinematics of $e^- q$ scattering



The outgoing quark is still massless, so

$$0 = (\xi P + q)^2 = \underbrace{\xi^2 m_p^2}_{\text{ignore}} + 2\xi P\cdot q + \underbrace{q^2}_{-Q^2}$$

so $$\xi = \frac{Q^2}{2P\cdot q} = x \quad !$$

For fixed x, deep inelastic scattg comes only from partons with $\xi = x$.

Now it is obvious that $0 < x < 1 \qquad 0 < y < 1$

and $$Q^2 = xys$$

The formula on p 12 becomes

$$\frac{d\sigma}{dQ^2 dx}(e^- p \to e^- + \text{hadrons})$$

$$= \left[\sum_f f_f(x)\, Q_f^2 \right] \cdot \left[\underbrace{\frac{2\pi\alpha^2}{Q^4}\left[1 + (1-y)^2\right]}_{\text{call this } \hat{\Sigma}(x,Q^2)} \right]$$

where $y = \dfrac{Q^2}{xs}$

This is a remarkable formula. It says that the deep inelastic scattg cross-section factorizes into a pure QED piece, $\hat{\Sigma}(x,Q^2)$, and a piece that relates directly to the proton structure. It comes with a remarkable prediction, that

$$\left[\frac{\frac{d\sigma}{dQ^2 dx}}{\Sigma(x, Q^2)} \right] = \sum_f Q_f^2 \, f_f(x)$$

is independent of Q^2! This is amazing, because you see from Fig. 26 that the scale of the cross section depends strongly on Q^2. Nevertheless it is true. Fig. 28 shows the whole corpus of data from the SLAC-MIT experiment with $Q^2 > 1 (\text{GeV})^2$; over many different spectrometer settings, it is a single function of x. This regularity is called "Bjorken scaling".

Electron scattering from the proton measures

$$\sum_f Q_f^2 \, f_f(x) = \frac{4}{9} \left[f_u(x) + f_{\bar{u}}(x) \right] + \frac{1}{9} \left[f_d(x) + f_{\bar{d}}(x) \right]$$

Similarly, deep inelastic electron scattering from deuterium $d = (pn)$ measures $\left(f_{u \leftarrow n}(x) = f_{d \leftarrow p}(x) \right)$

$$\frac{5}{9} \left[f_u(x) + f_d(x) + f_{\bar{u}}(x) + f_{\bar{d}}(x) \right]$$

data from J.S. Poucher et al (SLAC-MIT)
Phys. Rev. Lett $\underline{32}$, 118 (1974)

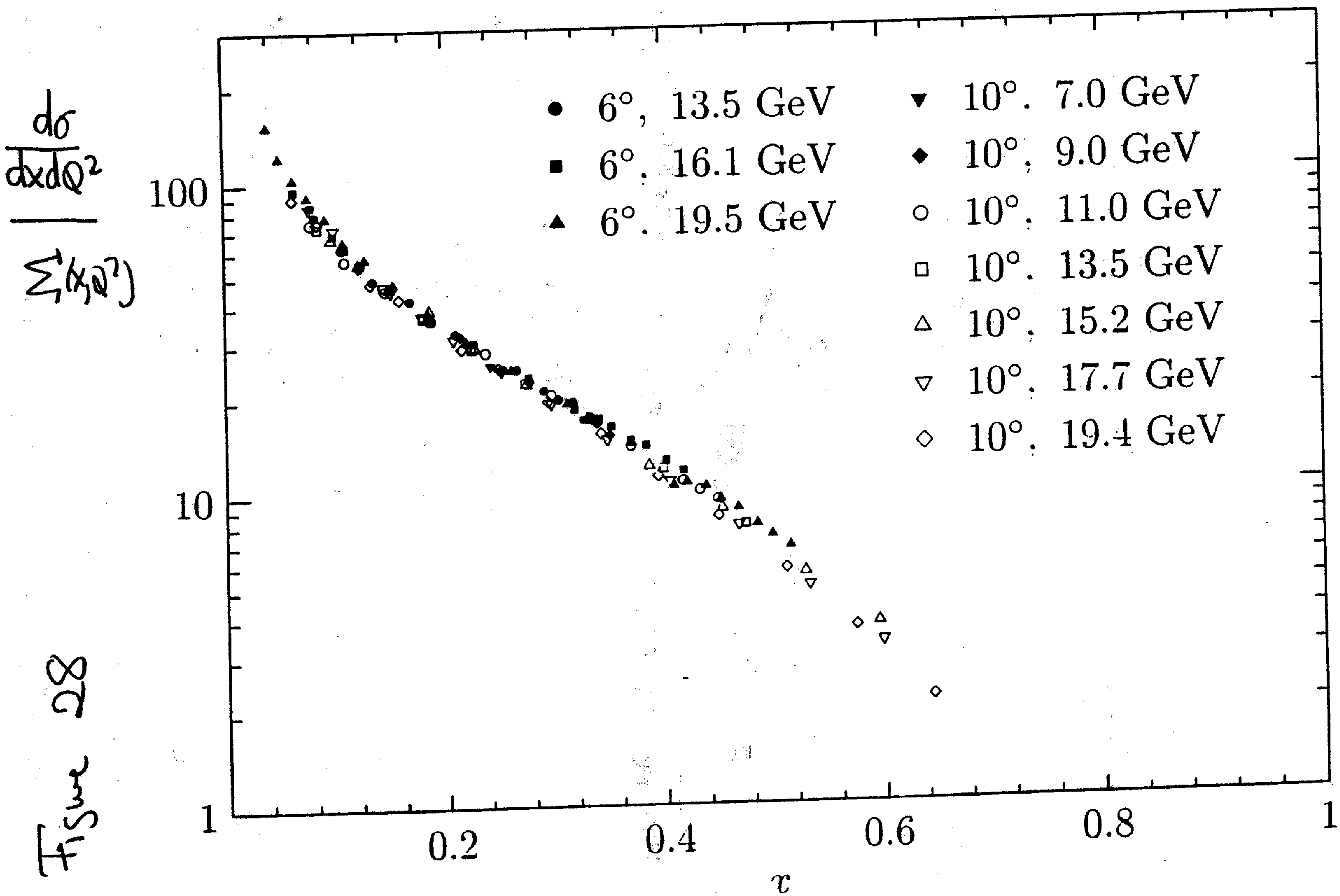
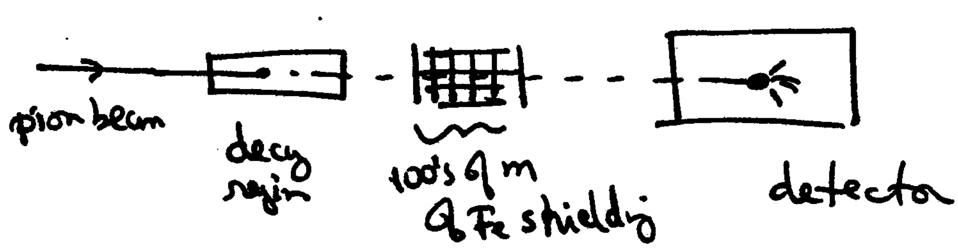
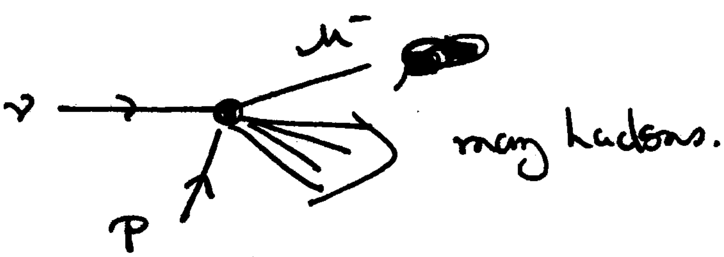


Figure 28

so from different deep inelastic scattering experiments we can learn the various components of the proton structure. To distinguish particles from antiparticles, it is useful to scatter <u>neutrinos</u>. A neutrino beam can be produced by letting a pion beam decay ($\pi^+ \to \mu^+ \nu$)



The reaction observed is the weak interaction process



I will discuss the systematics of this reaction and other weak-interaction processes in lecture 5. For now, we need to know that the process is mediated by a spin-1 particle, the W boson, so all of the spin-coupling factors given above apply directly. However, the W couples only to <u>left-handed</u> ν's and quarks and to their antiparticles.

I will show the evidence for this rule in lecture 5.

The handedness of the W couplings gives rise to an amazing effect. For νq scatt'g, the basic reaction is

$$\nu_L \, d_L \rightarrow \mu_L^- \, u_L$$

On the other hand, we could have started with a π^- beam and produced antineutrinos: $\pi^- \rightarrow \bar{\nu} \mu^-$. Then we would be studying

$$\bar{\nu}_R \, u_L \rightarrow \mu_R^+ \, d_L$$

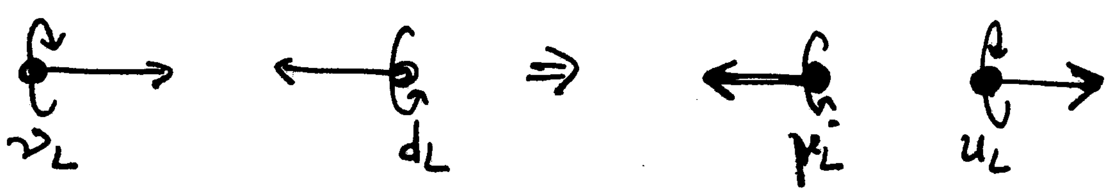
Now look at the formulae on the top of p. 12:

$$\frac{d\sigma}{dQ^2}(\nu_L d_L \rightarrow \mu_L^- u_L) = \text{indep of } y$$

$$\frac{d\sigma}{dQ^2}(\bar{\nu}_R u_L \rightarrow \mu_R^+ d_L) \propto (1-y)^2$$

The rule is easy to understand from conservation of angular momentum in backward scatt'g (which corresponds to the maximum energy transfer, $y = 1$)

For ν_L:



this is allowed; for $\bar{\nu}_R$

this violates angular momentum conservation and is forbidden.

The regularity is clearly seen in the data. Fig. 29 shows the results of the CDHS experiment at CERN. The slight deviations from the regularity are due to the <u>antiquark</u> content of the proton, for which

$$\nu \rightarrow (1-y)^2 \qquad \bar{\nu} \rightarrow 1$$

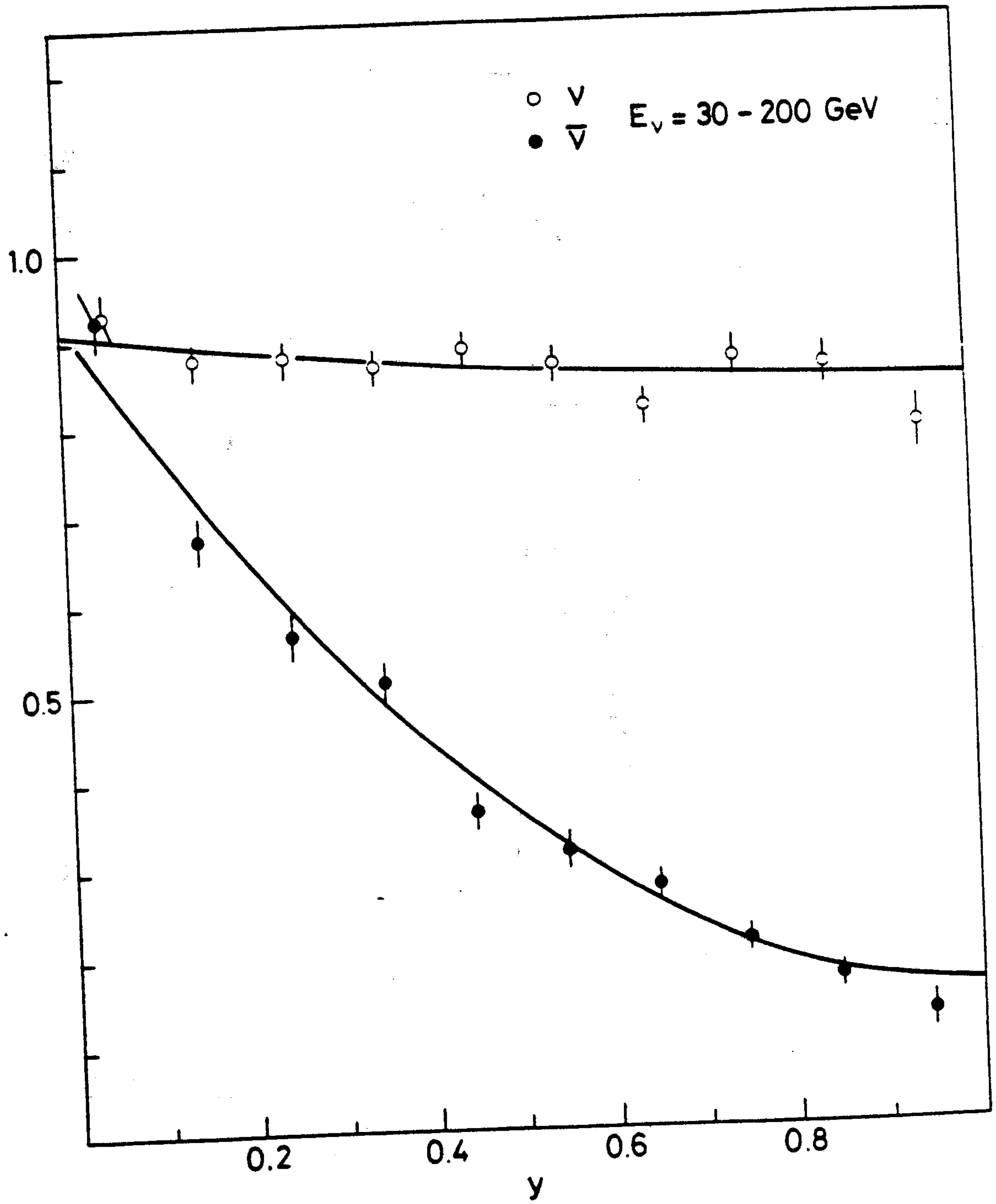
By measuring the y distributions carefully, we can distinguish quarks from antiquarks.

By measuring e^- scattering from d and ν scattering from a target with equal numbers of protons and neutrons (Fe is close enough), we can get two independent measurements of the quantity

$$x\left[f_u(x) + f_{\bar{u}}(x) + f_d(x) + f_{\bar{d}}(x)\right]$$

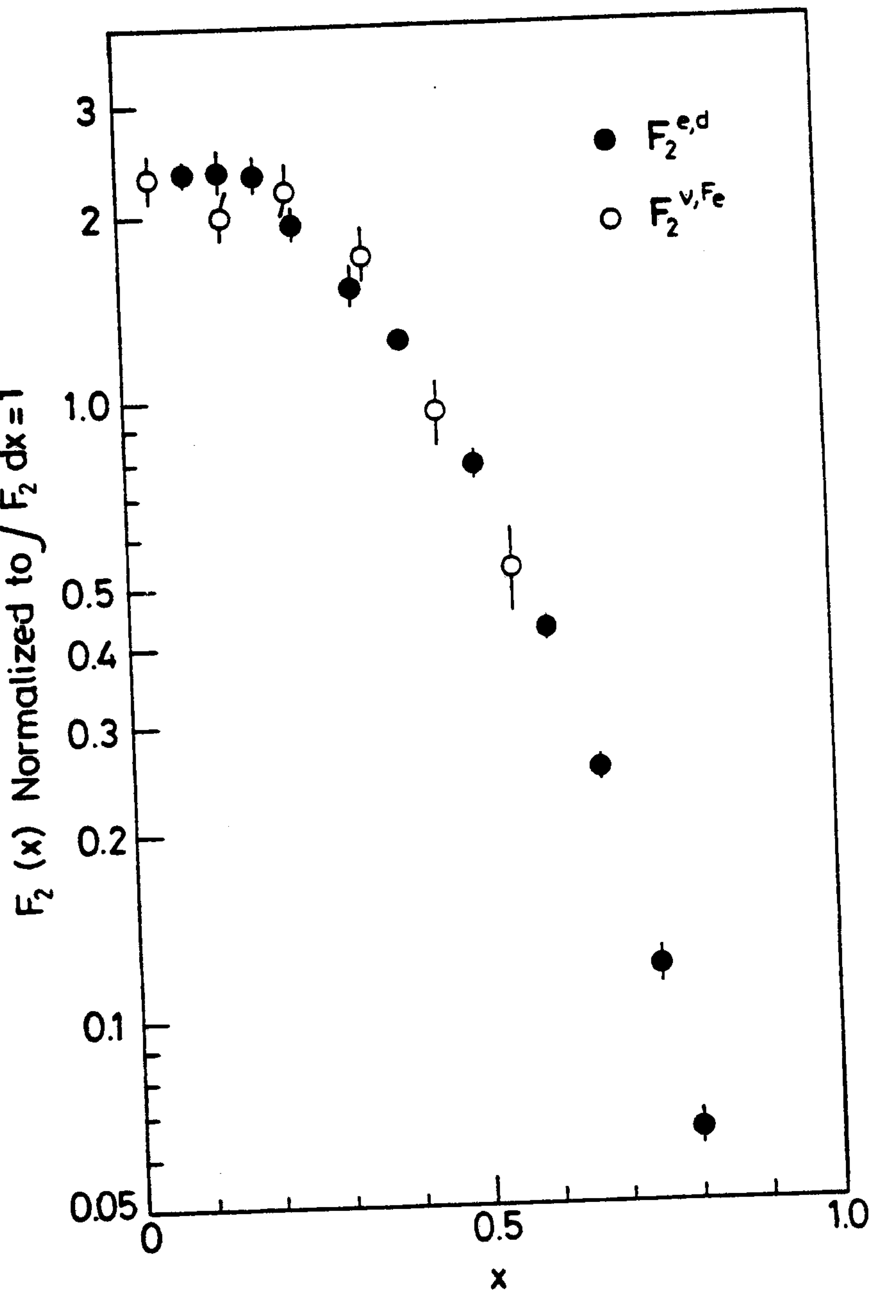
These are compared in Fig. 30. Apparently, e^- and ν deep inelastic scattering give a

Fig. 29



J G H de Groot et al (CDHS)
Z Phys C1, 143 (1979)

Fig. 30



J G H de Groot et al (CDHS)
Z Phys. C1, 143 (1979)

consistent picture of the shape of the proton wavefunction.

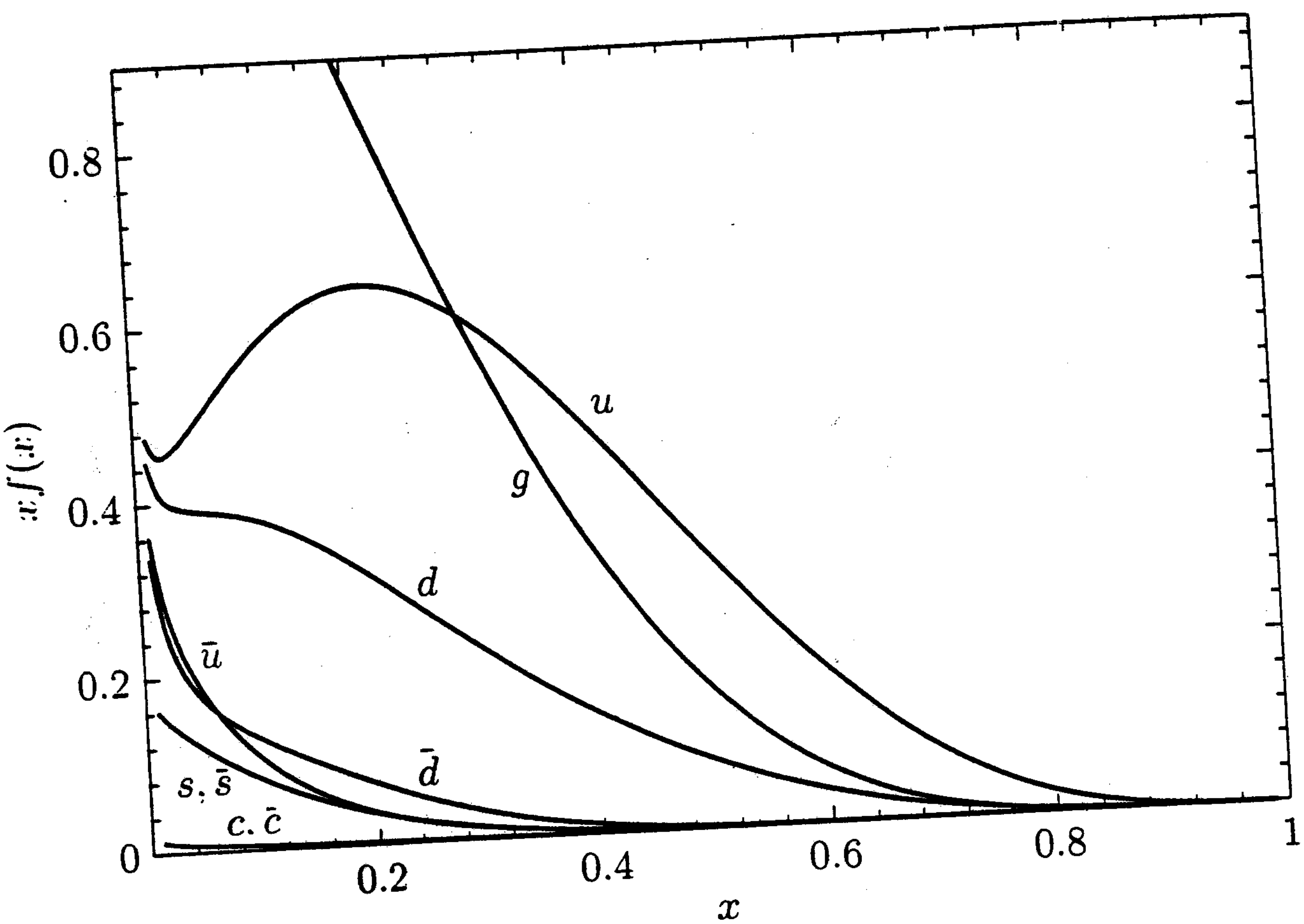
Fig. 31 shows a modern fit for the parton distribution functions of the proton. (We will see in Lecture 4 that there are small violations of Bjorken scaling; these are taken into account in the figure, which is scaled to $Q^2 = 4\ \mathrm{GeV}^2$.). Looking back at the sum rules on p. 10, the first two are satisfied. However, for the third sum rule

$$\int_0^1 dx\ x\,(f_u + f_{\bar{u}} + f_d + f_{\bar{d}}) \approx 0.5$$

Thus, there is another component of the proton which does not couple to e^- or ν. This is the "gluon", indicated in the figure as "g". I'll discuss this particle in Lecture 4.

2.

Fig. 31



J Botts et al (CTEQ)

Phys. Lett. B <u>304</u>, 159 (1993)

Lecture 4: July 28

In the previous two lectures, I explained how we can use the electromagnetic interactions to determine the properties of quarks. I showed how the process

$$e^+e^- \to \text{hadrons}$$

allows us to see that the quarks have spin $\frac{1}{2}$, and how the process

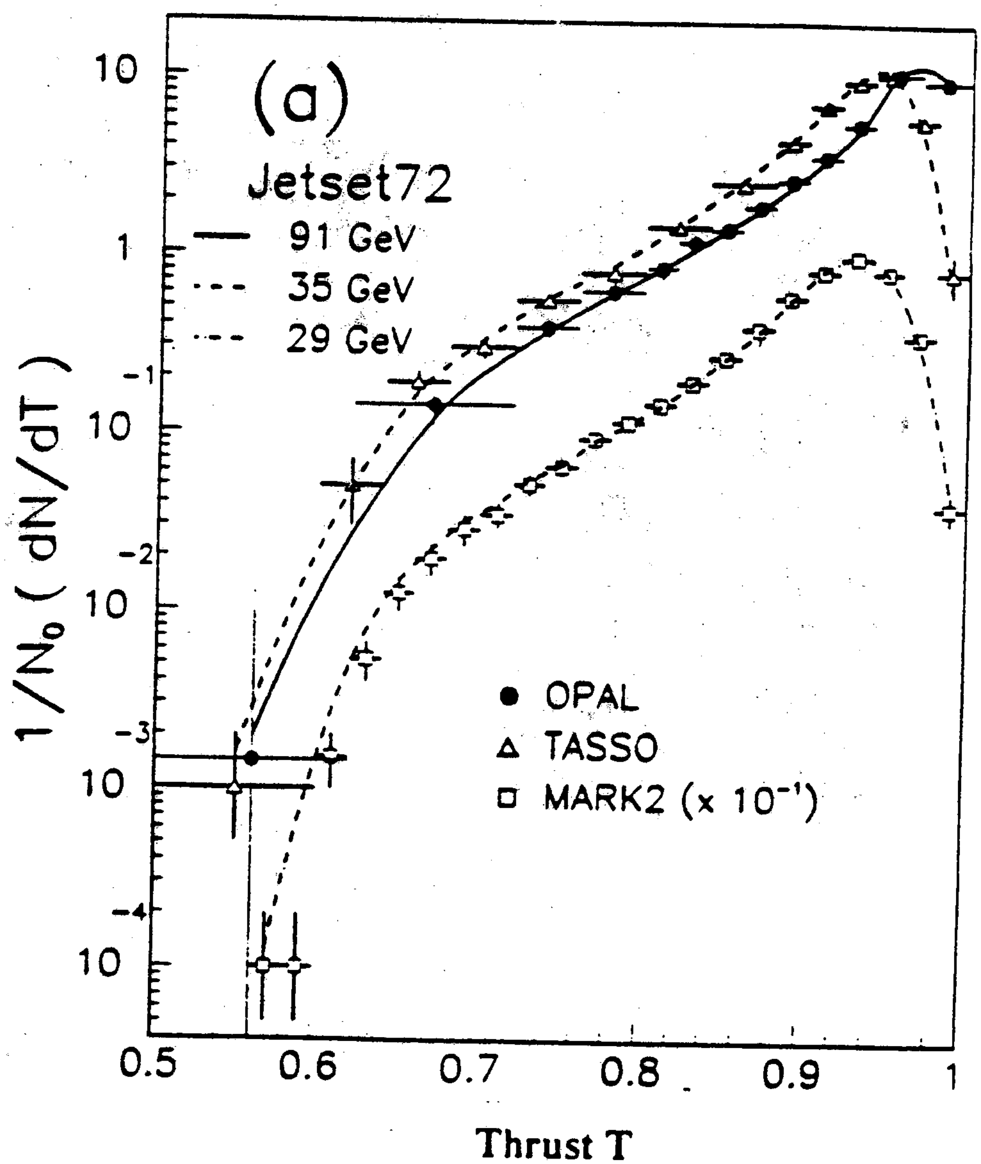
$$e^- p \to e^- + (\text{hadrons})$$

allows us to measure the momentum distribution of quarks in the proton. To analyze both of these reactions, we wrote formulae for the cross sections — which turned out to be quite accurate experimentally — that ignored the strong interactions. I justified this by arguing that the strong interactions are "soft" while the electromagnetic interactions are "hard".

On the other hand, we saw that the strong interactions do have a hard component, which causes quark-quark scattering with a distribution that looks like Coulomb scattering. In addition, we need some field or particle that causes the strong interactions, creating the quark-antiquark binding potentials and the forces that bind the proton.

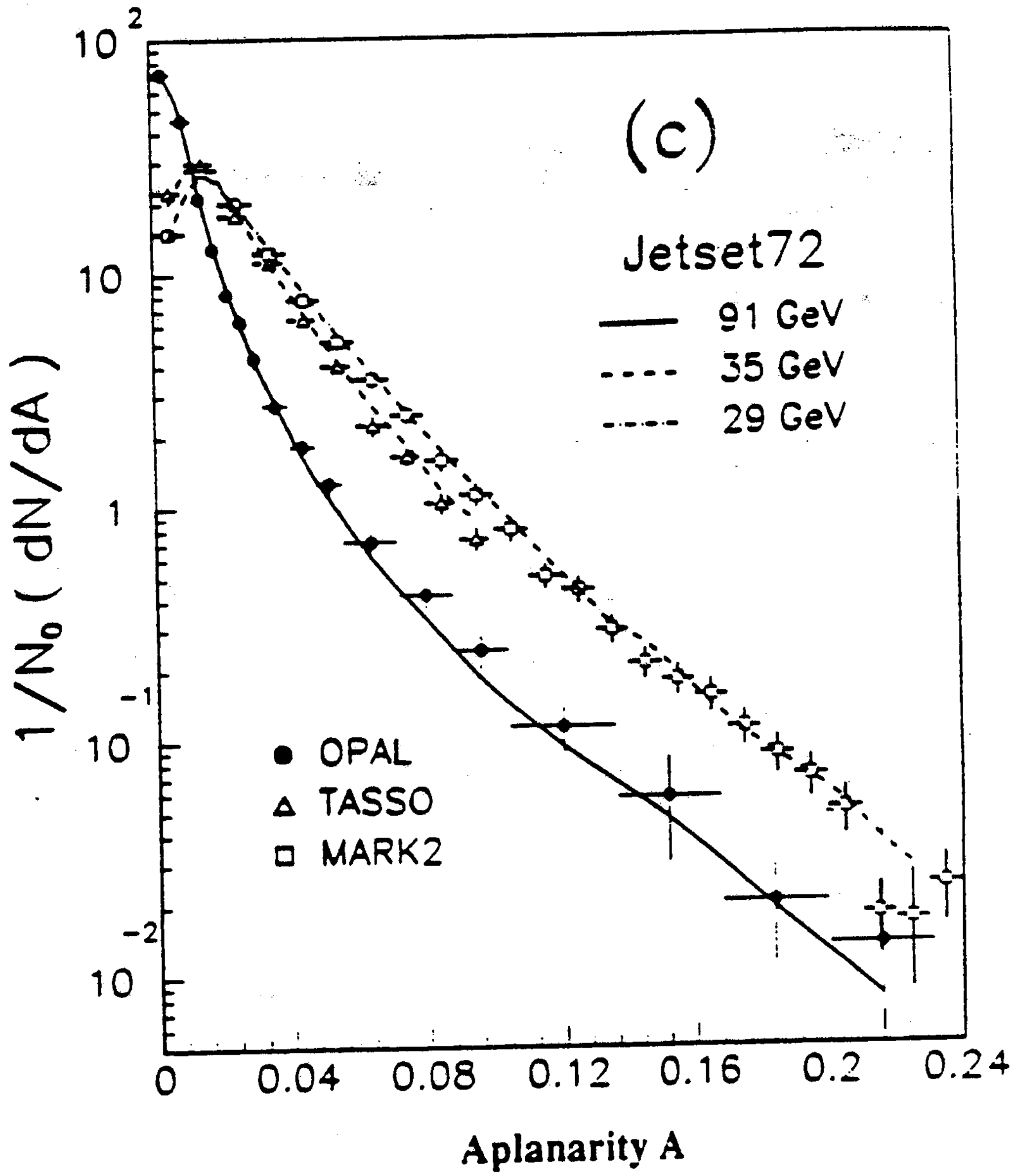
2 pages missing

Figure 33



M Z Akrawy et al (OPAL)
Z Phys. C 47, 505 (1990)

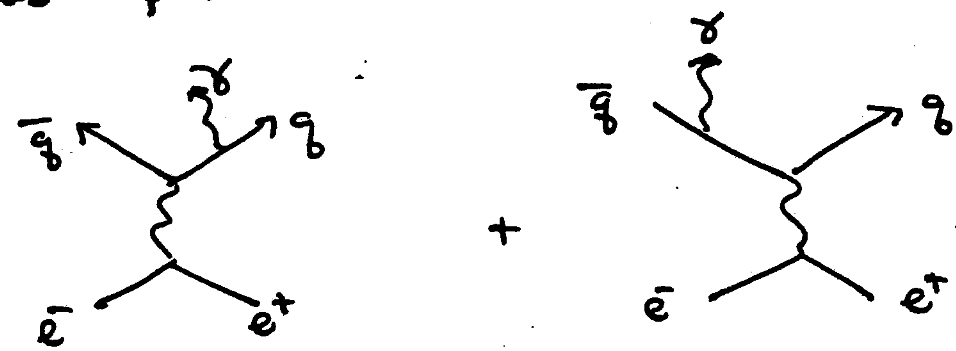
Figure 34



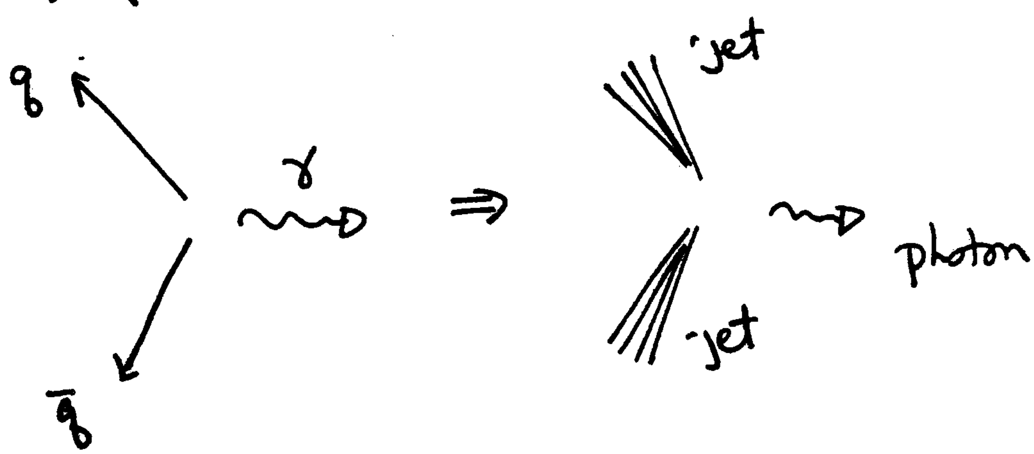
M. Z. Akrawy et al. (OPAL)
Z Phys. C47, 505 (1990)

the SLD experiment. The tracks still form clusters or jets. However, now we see three jets in the event.

To get some feeling for the nature of these events, we can compare them to an electromagnetic process which leads to three objects in the final state. If we have the reaction $e^-e^+ \to q\bar{q}$ and then the quark or the antiquark radiates a photon

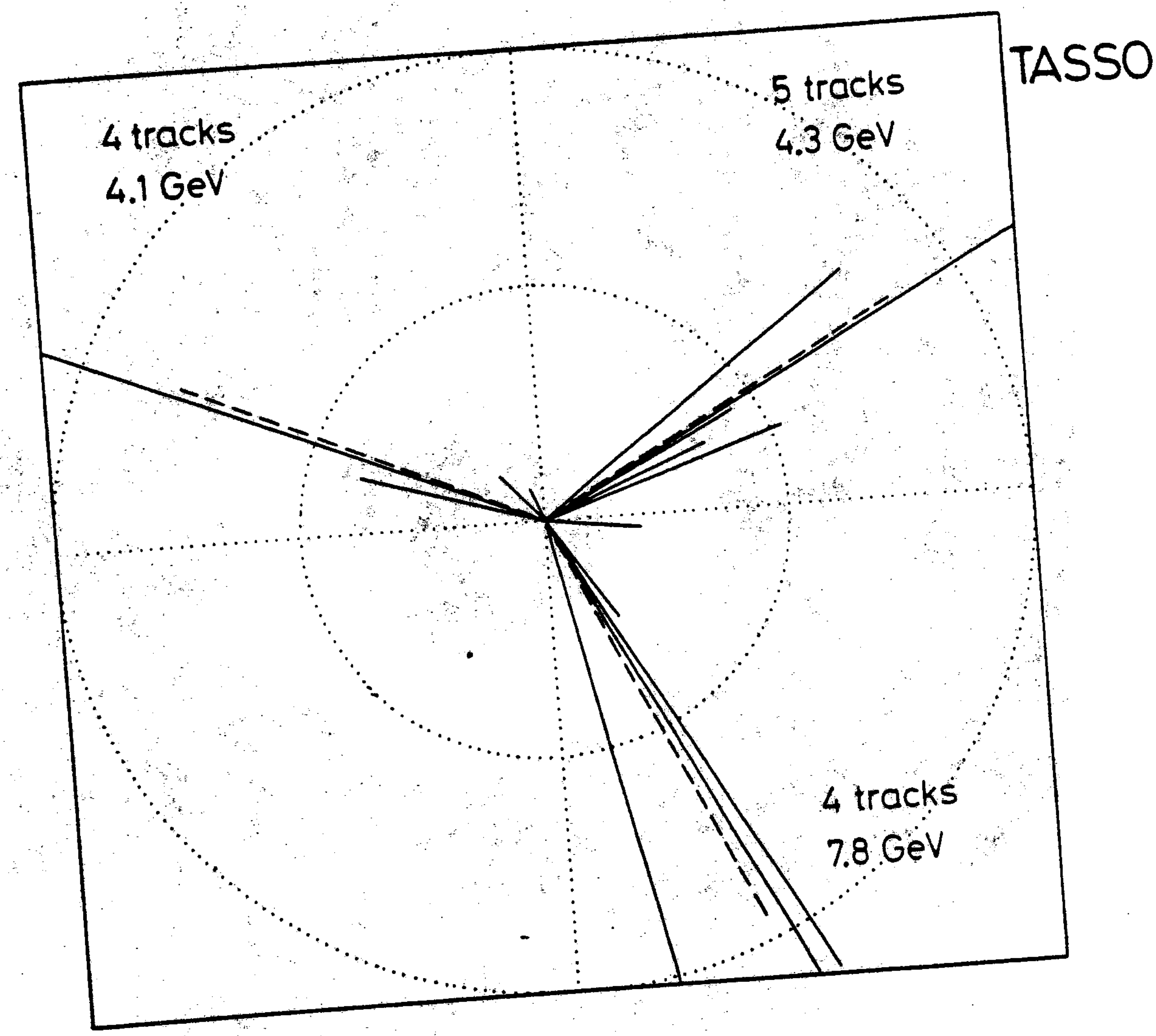


we end up with a final state that looks like



These events really do occur, at a rate about 10% that of the 3-jet events. It is interesting to compare the distribution of hadrons in the event plane for $q\bar{q}\gamma$ events in which the two jets are separated by about 150°

Figure 35

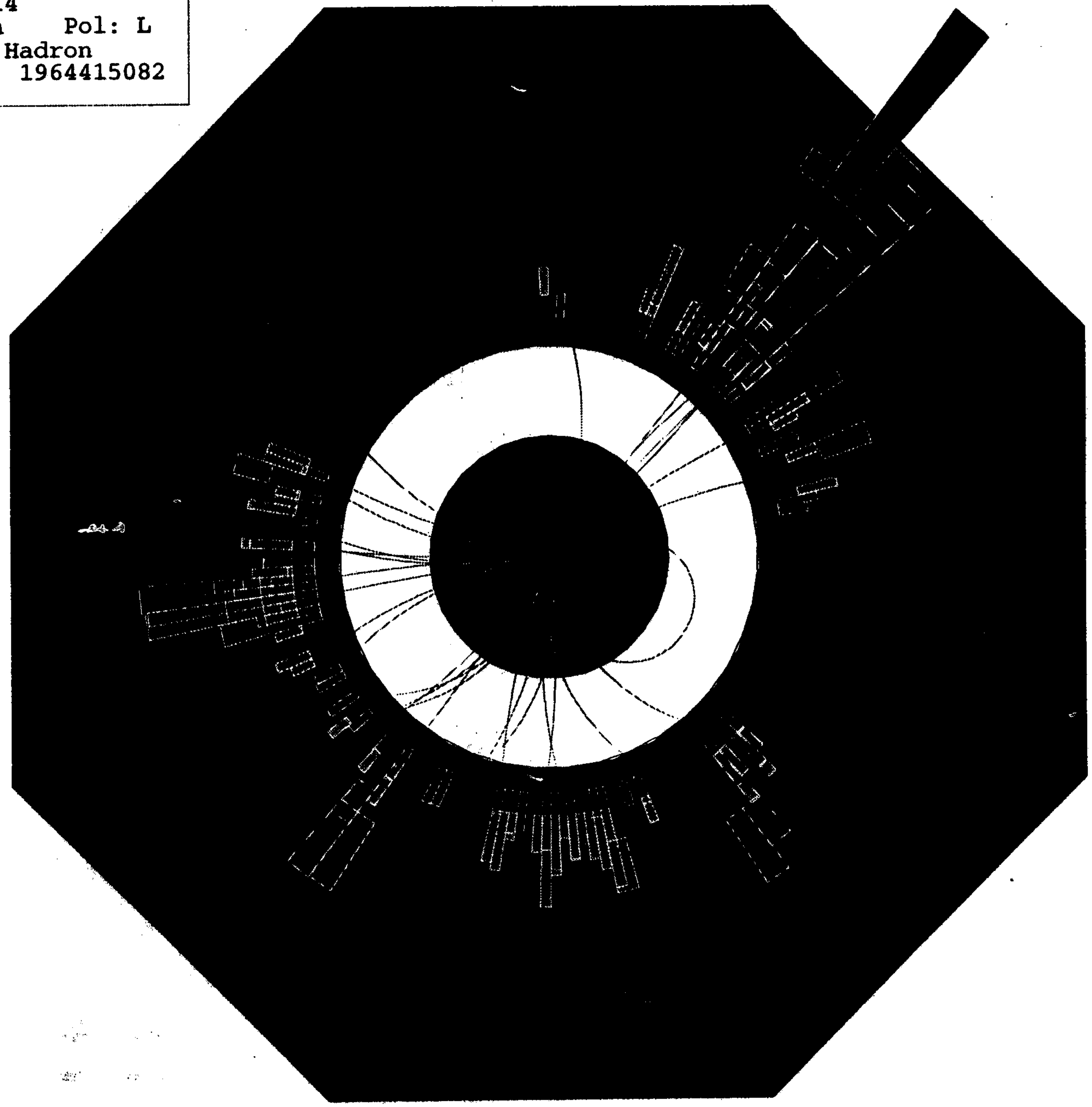


from G. Wolf, in
proceedings of the 1979 Lepton-Photon conference.

Figure 36

8

Run 12 , EVENT 6353
8-JUL-1992 10:14
Source: Run Data Pol: L
Trigger: Energy Hadron
Beam Crossing 1964415082



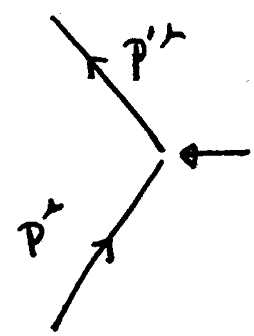
from the
SLD Web pages

$E_{cm} = 91$ GeV

SLD

ed for 3-jet events in which the two most energetic jets are separated by 150°. This comparison, from the data of the Mark II experiment at 29 GeV, is shown in Fig. 37. This figure strongly suggests that the 3-jet events correspond to a $q\bar{q}$ system which has emitted a photon-like object with strong interactions that materializes as a third jet. This is the gluon.

The analogy with photon emission is a useful one, and I would like to pursue it a little further. Let's compute the amplitude for photon emission from a relativistic charged particle. In the simplest approximation, imagine a particle that comes in with a momentum p^μ, receives a sudden, hard kick, and exits with a momentum p'^μ:



$$p^2, p'^2 \approx 0$$

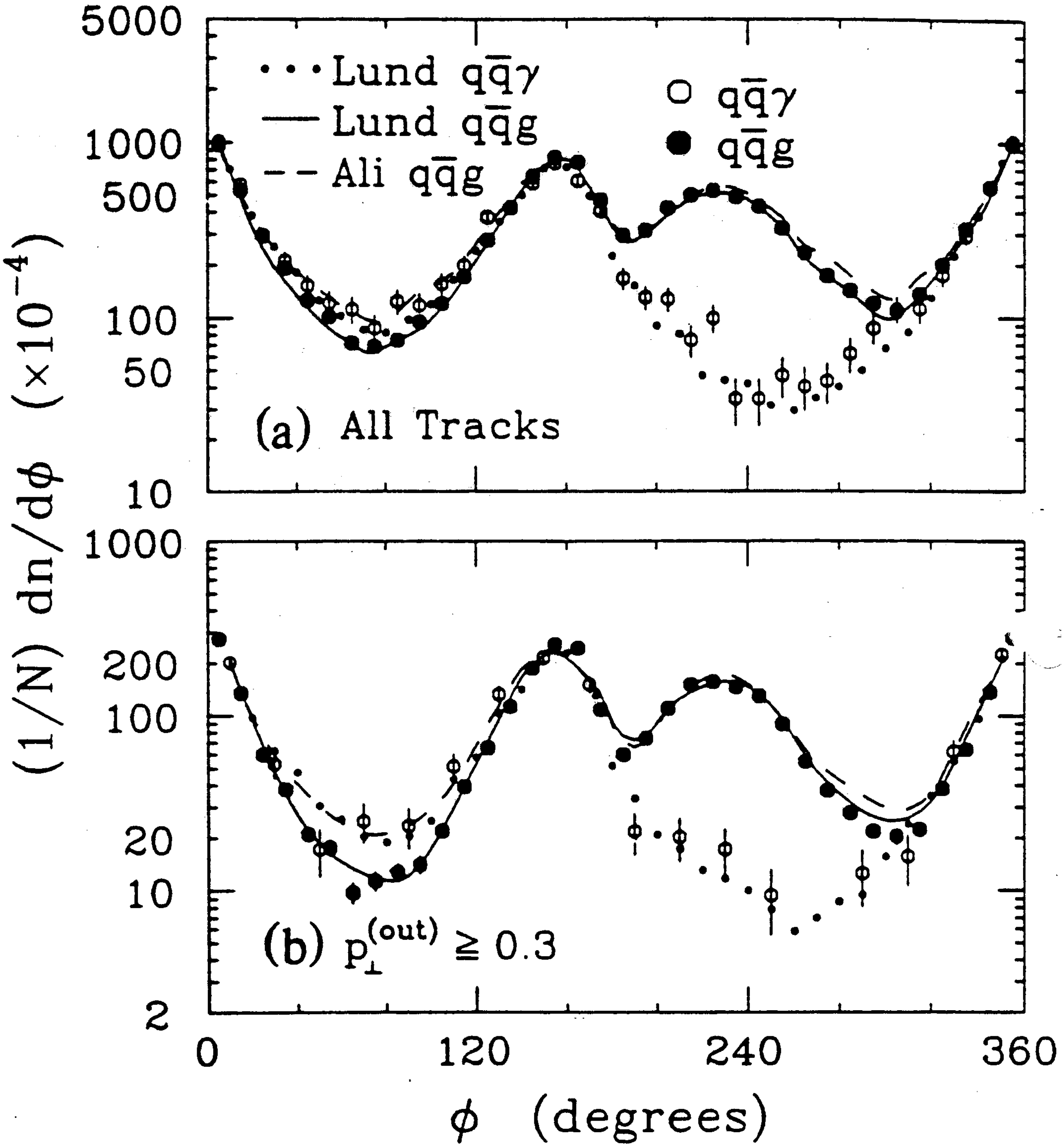
Say that the kick is at $x=0$, then the particle comes in along the trajectory

$$y^\mu(t) = V^\mu t \qquad t<0$$

$$V^\mu = (1, \vec{V}) = \frac{P^\mu}{P^0} \qquad |\vec{V}| = 1$$

and it goes out along the trajectory $y^\mu(t) = V'^\mu t$,

Figure 37



PD Sheldon et al (Mark II)
Phys. Rev. Lett. 57, 1398 (1986)

where $V^{\mu} = \frac{p^{\mu}}{p^0}$. The electric charge current is

$$j^{\mu}(x) = Q_f e \int dt \; V^{\mu}(t) \; \delta^{(4)}(x^{\mu} - y^{\mu}(t))$$

where $V^{\mu}(t) = \begin{cases} V'^{\mu} & t>0 \\ V^{\mu} & t<0 \end{cases}$ $\qquad y^{\mu}(t) = \begin{cases} V'^{\mu} t & t>0 \\ V^{\mu} t & t<0 \end{cases}$

The radiation field with momentum q^{μ} should be proportional to the Fourier transform of this current at the wavenumber q^{μ}. This is

$$j^{\mu}(q) = \int d^4x \; e^{iq\cdot x} \; j^{\mu}(x)$$

$$= Q_f e \left\{ \int_0^{\infty} dt \; e^{iq\cdot V' t} \, V'^{\mu} + \int_{-\infty}^{0} dt \; e^{iq\cdot V t} \, V^{\mu} \right\}$$

$$= i Q_f e \left(\frac{V'^{\mu}}{q\cdot V'} - \frac{V^{\mu}}{q\cdot V} \right)$$

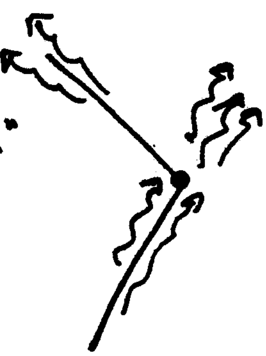
$$= i Q_f e \left(\frac{p'^{\mu}}{q\cdot p'} - \frac{p^{\mu}}{q\cdot p} \right)$$

The energy in the electromagnetic field should be proportional to $|j^{\mu}(q)|^2$. We can divide this energy into quanta with energy $(\hbar q^0)$ — photons. I hope this motivates the following expression for the probability of radiating a low-energy photon with polarization vector $\vec{\varepsilon}$:

$$\text{Probability} = \int \frac{d^3q}{(2\pi)^3} \frac{1}{2q^0} \cdot Q_f^2 e^2 \cdot \left| \frac{\vec{\varepsilon}\cdot\vec{p}\,'}{q\cdot p'} - \frac{\vec{\varepsilon}\cdot\vec{p}}{q\cdot p} \right|^2$$

As usual, an explicit analysis in QED is needed to get the overall coefficient.

If the particle which produces the radiation is relativistic ($p^2, p'^2 \cong 0$), this expression is singular when the photon momentum q becomes collinear with p or p'. Thus, we get two bursts of radiation:



Let's analyze the most singular term when q^μ is near p^μ. Write

$$p^\mu = (E, 0, 0, E)$$

$$q^\mu = (\omega, q_\perp, 0, q_\parallel) \qquad \omega^2 = q_\perp^2 + q_\parallel^2$$

$$\omega \cong q_\parallel + \frac{1}{2}\frac{q_\perp^2}{q_\parallel} + \cdots$$

then
$$q\cdot p = E(\omega - q_\parallel)$$
$$\cong E \cdot \frac{1}{2}\frac{q_\perp^2}{q_\parallel} \quad \text{if} \quad q_\parallel \gg q_\perp$$

The two transverse polarization vectors for the photon are

$$\varepsilon_1^\mu = (0, -q_\parallel, 0, q_\perp)\cdot\frac{1}{\omega} \qquad \varepsilon_2^\mu = (0,0,1,0)$$

then $\vec{\varepsilon}_2 \vec{p} = 0 \qquad \vec{\varepsilon}_1 \cdot \vec{p} = E\frac{q_\perp}{\omega} \approx \frac{q_\perp}{q_\parallel} E$

The probability of photon emission is then

$$\text{Prob} \simeq \int \frac{d^2 q_\perp dq_\parallel}{(2\pi)^3\, 2\omega} \quad Q_f^2 e^2 \cdot \left| \frac{\vec{\varepsilon}_1 \cdot \vec{p}}{q \cdot p} \right|^2$$

$$\simeq \int \frac{d^2 q_\perp}{(2\pi)^3} \frac{dq_\parallel}{2 q_\parallel} \quad Q_f^2 e^2 \left(\frac{q_\perp}{q_\parallel}\right)^2 \left(\frac{2 q_\parallel}{q_\perp^2}\right)^2$$

Let x be the fraction of the original particle's momentum carried by the photon. Then

$$q_\parallel = xE$$

$$\text{Prob} \simeq \int \frac{dx}{x} \quad Q_f^2 \; \frac{e^2}{4\pi^2} \int \pi \frac{dq_\perp^2}{q_\perp^2}$$

the integral over $q_\perp$ should be taken over the region

$$q_\perp \ll q_\parallel \sim E$$

on the other hand, if $q_\perp \sim m$, the mass of the particle emitting the photon, our extreme-relativistic limit assumptions break down. So

$$(\text{Prob of } \gamma \text{ emission}) \simeq Q_f \int dx \left(\frac{\alpha}{\pi} \frac{1}{x} \log \frac{E^2}{m^2} \right)$$

we derived this formula for soft photons: $\omega \ll E$. However, it turns out to be correct also for hard photons, with one small modification:

$$(\text{Prob of } e_L^- \to e_L^- \gamma_L) = \int dx \left(\frac{\alpha}{2\pi} \frac{1}{x} \log E^2/m^2 \right)$$

left-polarized photon

$$(\text{Prob of } e_R^- \to e_L^- \gamma_R) = \int dx \left(\frac{\alpha}{2\pi} \frac{1}{x} (1-x)^2 \log E^2/m^2 \right)$$

right polarized photon

so the total probability of photon emission from an electron is

$$\frac{d\,\text{Prob}}{dx} = \frac{\alpha}{2\pi} \frac{1}{x} \left[1 + (1-x)^2 \right] \log E^2/m_e^2$$

this formula describes the photons emitted in the direction of the incoming electron ("initial state radiation") and the photons emitted by the outgoing electron "final state radiation".

In $e^+e^- \to \mu^+\mu^-$, the outgoing μ^+ and μ^- are accompanied by collinear photons, according to the same formula with $m_e \to m_\mu$

If the gluon interacts with quarks like a photon, but with a stronger coupling, we might expect a similar formula to hold for gluon emission in $e^+e^- \to q\bar{q}$:

$$\frac{d}{dx} \text{Prob}(q \to q+g) \simeq \frac{\alpha_g}{2\pi} \frac{1}{x} \left[1 + (1-x)^2 \right] \log \left[\frac{s}{(1\,\text{GeV})^2} \right]$$

$$\alpha_g = \text{gluon coupling} \approx 10 \cdot \alpha$$

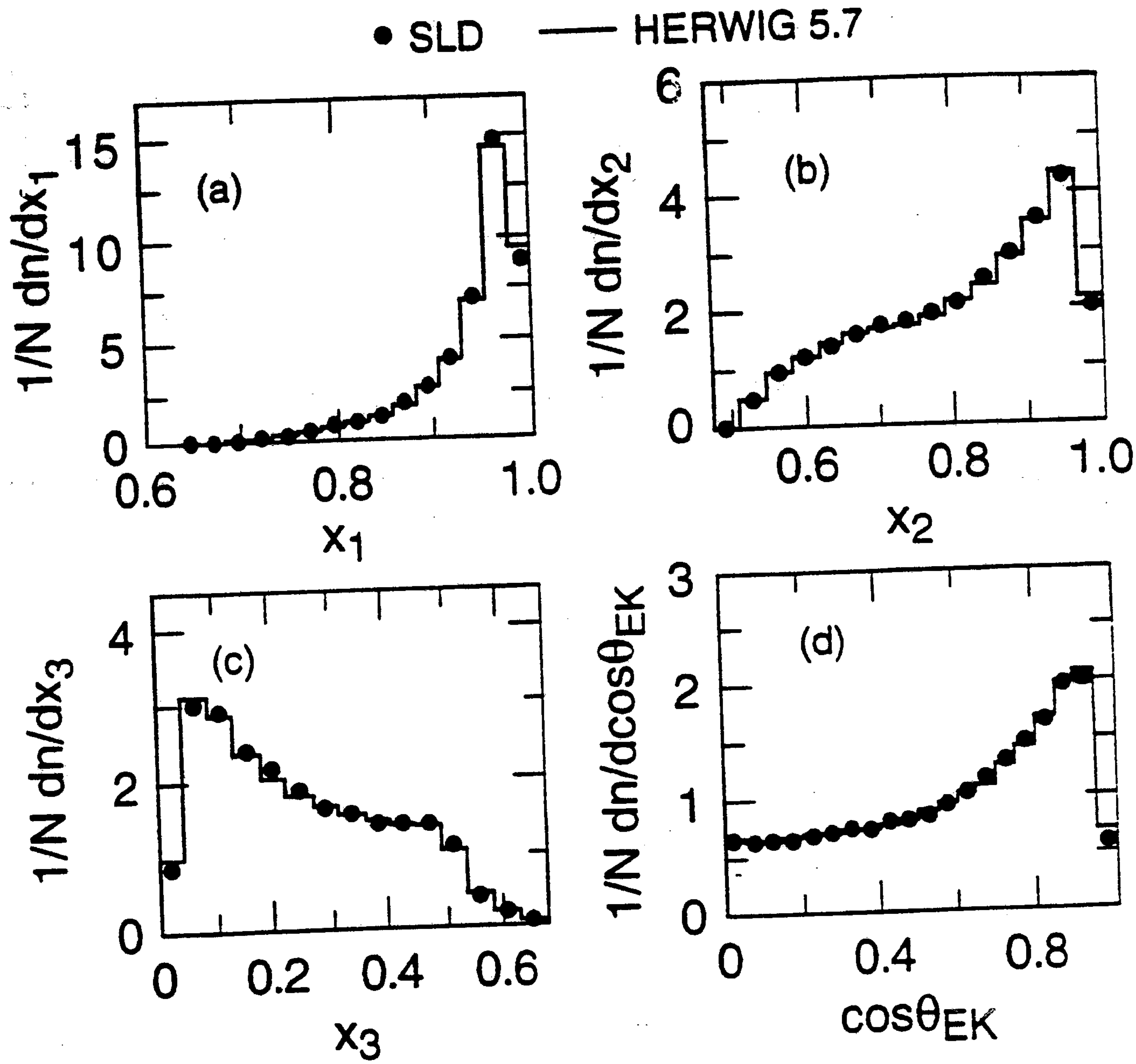
where x is the fraction of the quark's momentum taken by the gluon. Indeed, in Fig. 38, I show the energy distributions of jets in 3-jet events, as measured by the SLD experiment. The jet energies are given as

$$x_i = \left(\frac{E_i}{\frac{1}{2}E_{cm}}\right) \qquad \text{so} \qquad \begin{array}{l} x_1+x_2+x_3 = 2 \\ x_1 > x_2 > x_3 \end{array}$$

The lowest-energy jet is typically the gluon. You can see that the distribution of x_3 does have the shape of our formula for dP/dx, until x_3 becomes sufficiently large that it is not unambiguously associated with a radiated gluon.

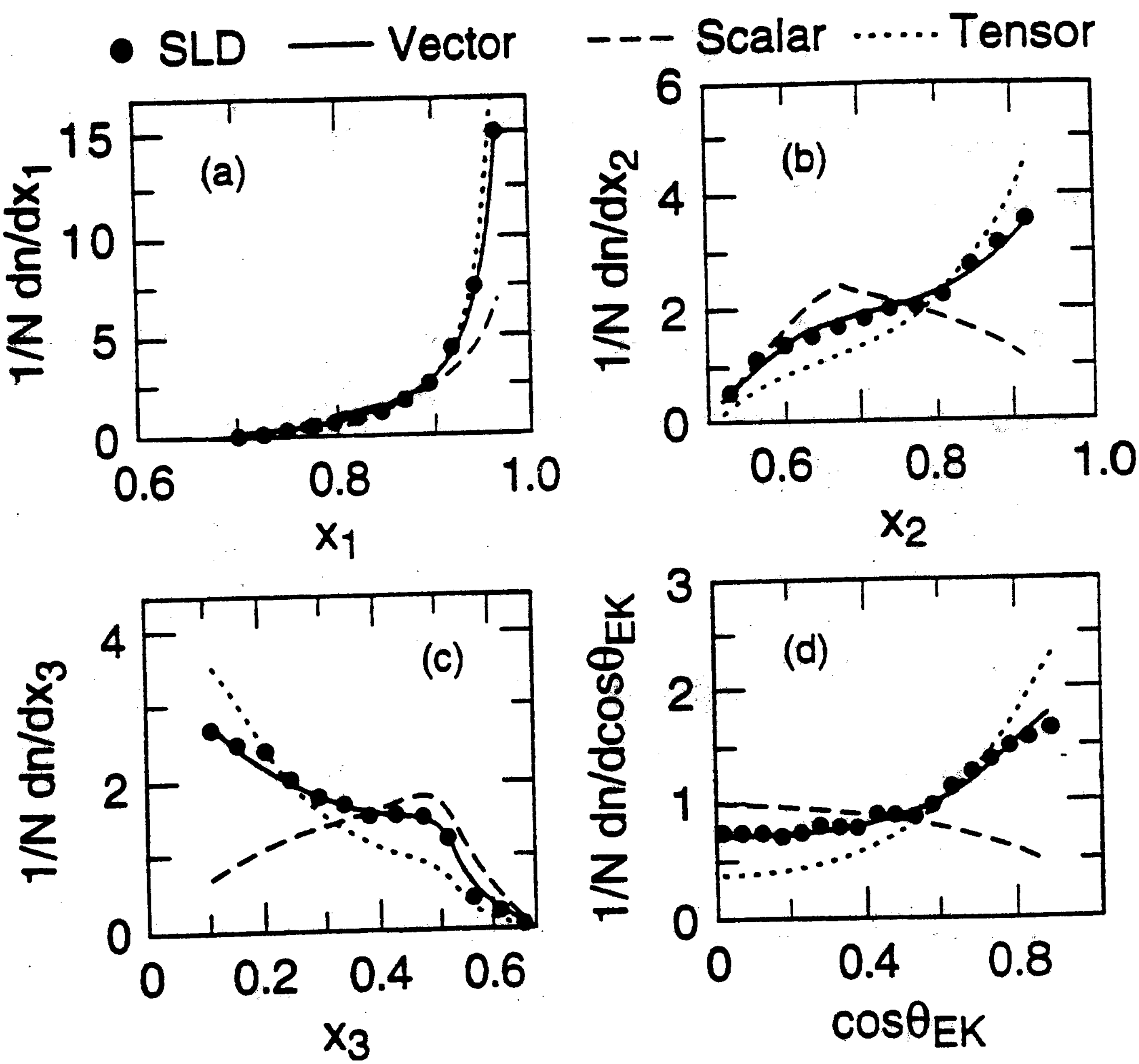
In the calculation I have just done, I have assumed that the gluon was a particle of spin 1, like the photon. One can test this hypothesis by comparing the distributions shown in Fig. 38 to models in which the emitted particle has spin = 0, 1, 2. This comparison is shown in Fig. 39. A particularly incisive quantity is θ_{EK}, the Ellis-Karliner angle, the angle between the two lowest-energy jets. You can see that the gluon is unambiguously a particle of spin 1.

Figure 38



K Abe et al, Phys. Rev. D 55, 2533 (1997)
(SLD)

Figure 39



K Abe et al. (SLD),
Phys. Rev. D 55, 2533 (1997)

In principle, quarks can radiate 2, 3, or more gluons. There is an interesting subtlety here. Given an event with particle tracks, how do we form it into jets? The simplest way is to associate tracks which are close in angle into clusters. As a cluster accumulates more and more particles, the cluster mass

$$m_c^2 = \left(\sum_i p_i\right)^2$$

increases. We can decide to stop combining tracks when the largest cluster mass would exceed

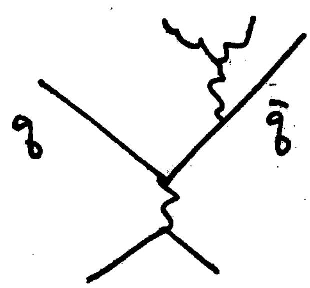
$$m_{cut}^2 = y_{cut}\, s \qquad 0 < y_{cut} < 1$$

and associate the tracks with jets. When y_{cut} is large, all of the tracks might be assembled into 2 jets. When y_{cut} is small, many jets might be visible

As we decrease y_{cut}, we resolve the structure of jets. For large y_{cut}, only gluons emitted at large $p_\perp$ relative to the quarks appear. For small y_{cut}, gluon jets with smaller $p_\perp$ become visible. A better version of the formula at the bottom of p. 14 would give the probability of emitting a gluon which is visible at the chosen value of y_{cut}:

$$\frac{d}{dx} \text{Prob}(q \to q + \text{visible gl jet}) \cong \frac{\alpha_s}{2\pi} \frac{1}{x}\left[1+(1-x)^2\right] \log\left(\frac{1}{y_{cut}}\right)$$

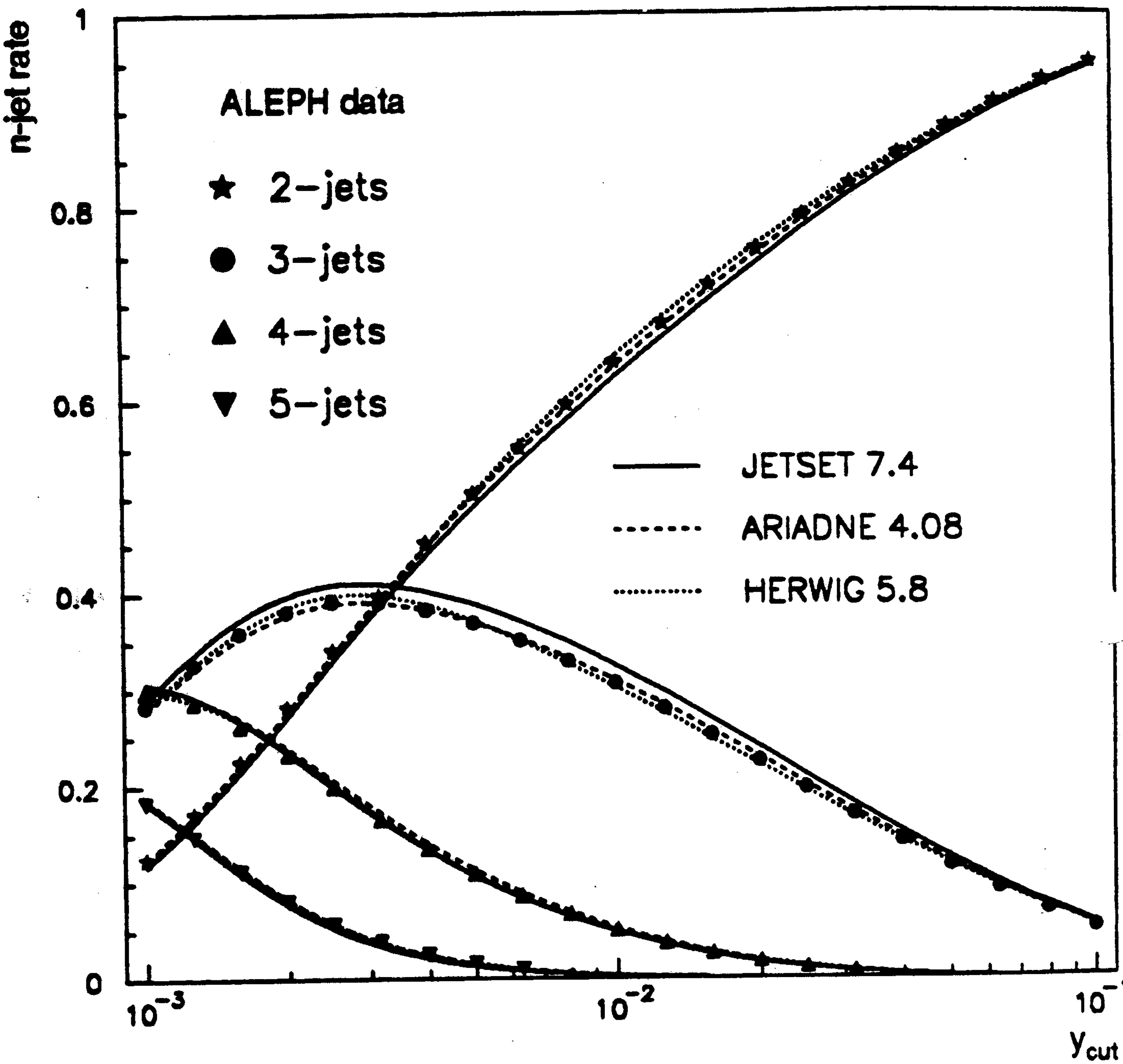
Experimentally, we do see more jets as y_{cut} is decreased. Fig. 40 shows a measurement of the number of jets in $e^+e^- \to$ hadrons at 91 GeV as a function of y_{cut}, done by the ALEPH experiment. The numbers of jets are compared to the prediction of a gluon radiation model. I should note that a process we have not discussed, the radiation of gluons by gluons,



must be added in to get quantitative agreement with experiment.

We should be able to see the effects of gluon radiation also in deep inelastic scattering. The gluon apparently does not carry electric charge and so is not scattered by an electron; otherwise we could make gg pairs in e^+e^- annihilation. Actually, we saw in lecture 3 that we need a neutral component of the proton to carry about ½ of the proton's momentum. But the radiation of gluons can also

Figure 40

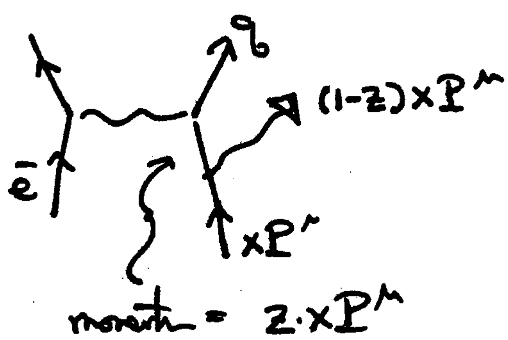


R. Barate et al (ALEPH)

Physics Reports (to appear)

affect the quark distribution functions.

Consider a quark which carries a momentum fraction x. Before this quark is scattered by an electron, it could radiate a collinear gluon:



This process brings the quark down to a lower momentum fraction $x_{new} = z \cdot x$. The probability of gluon emission increases with the hardness of the electron scattering

$$\text{Prob.} \sim \log\left(\frac{Q^2}{m_p^2}\right)$$

since the gluon may have any $p_\perp$ up to $p_\perp^2 \sim Q^2$. Thus, the quark distributions will peak at lower and lower x as Q^2 increases. This is a violation of the principle of "Bjorken scaling" introduced in the previous lecture — the principle that $f_f(x)$ is a property only of the proton wavefunction and is independent of Q^2 — but it is a slow violation, on a scale of $\log Q^2$.

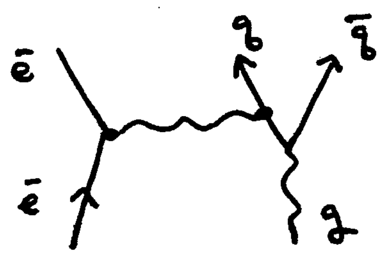
Actually, we have enough information to write a quantitative theory of the violation of Bjorken scaling:

$$\frac{d}{d\log Q^2}\, f_f(x,Q^2) = \int_x^1 \frac{dz}{z}\left[\frac{\alpha_s}{2\pi}\,\frac{1+z^2}{(1-z)} - a\,\delta(z-1)\right] f_f\!\left(\frac{x}{z},Q^2\right)$$

The first term under the integral replaces the quarks at the new x value after radiation; the δ-function removes them from the old value; a is a constant s.t.

$$\frac{d}{d\log Q^2}\left[\int dx\; f_f(x,Q^2)\right] = 0$$

There is another important process that affects the form of the parton distribution functions. Just as an initial <u>quark</u> can split into a quark + gluon, an initial gluon can split into $q\bar{q}$. This adds more quarks at low values of x:



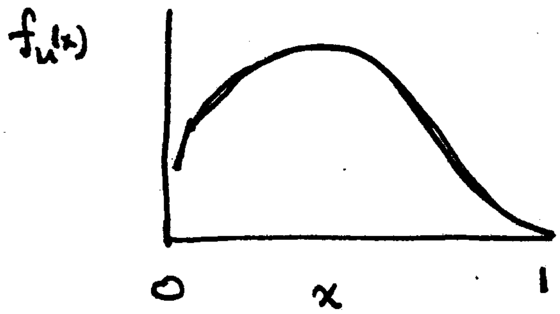
this effect is also $\propto \log Q^2$; it is described quantitatively by adding another term to the evolution equation above:

$$+\int_x^1 \frac{dz}{z}\;\; \frac{3}{8}\cdot\frac{\alpha_s}{2\pi}\cdot\left[z^2+(1-z)^2\right]\; f_g\!\left(\frac{x}{z},Q^2\right)$$

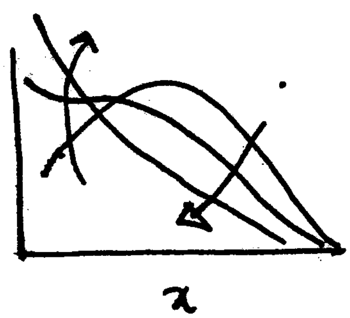
where $f_g(x,Q^2)$ is the parton distribution of <u>gluons</u> in the proton. The observation of the increase in $f_g(x,Q^2)$ at

$f_g(x, Q^2)$ at low x and large Q^2 allows us to measure the gluon distribution. The full equation on p.22, and the corresponding equation for the gluon distribution, are called the "Altarelli – Parisi equations".

The effects discussed on the previous page change the quark distributions observed in deep inelastic scattering in the following way. An initial distribution



becomes smaller at large x as quarks radiate gluons and larger at small x as gluons split into quarks. Thus.



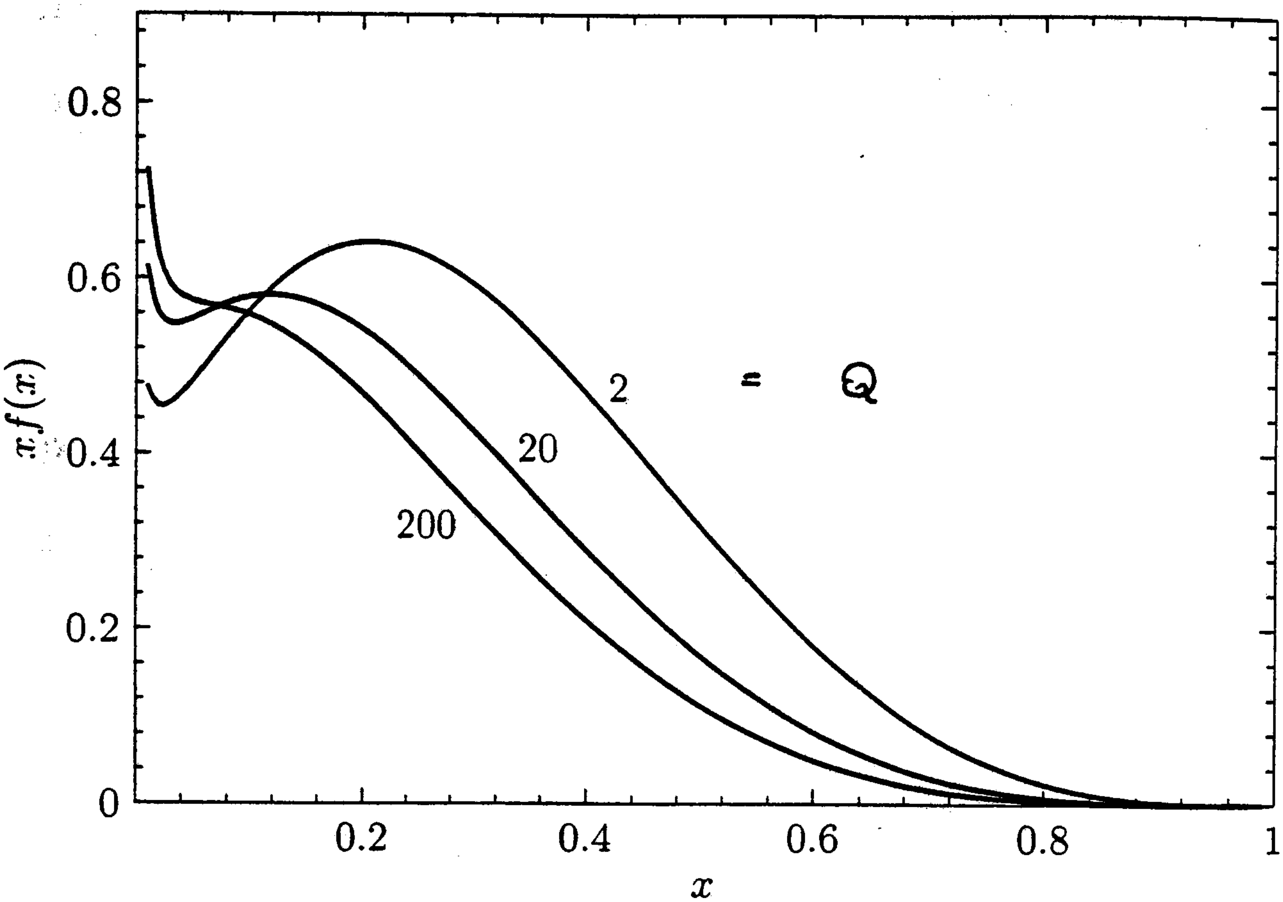
evolution w. increasing Q^2.

The distributions $f_u(x, Q^2)$ from a modern fit to the data, for $Q = 2, 20, 200$ GeV, are shown in Fig. 41.

In Fig. 42, I show the experimental measurement of the quantity $\frac{5}{9}\left[f_u(x) + f_d(x) + f_{\bar{u}}(x) + f_{\bar{d}}(x)\right]$

α /

Figure 41

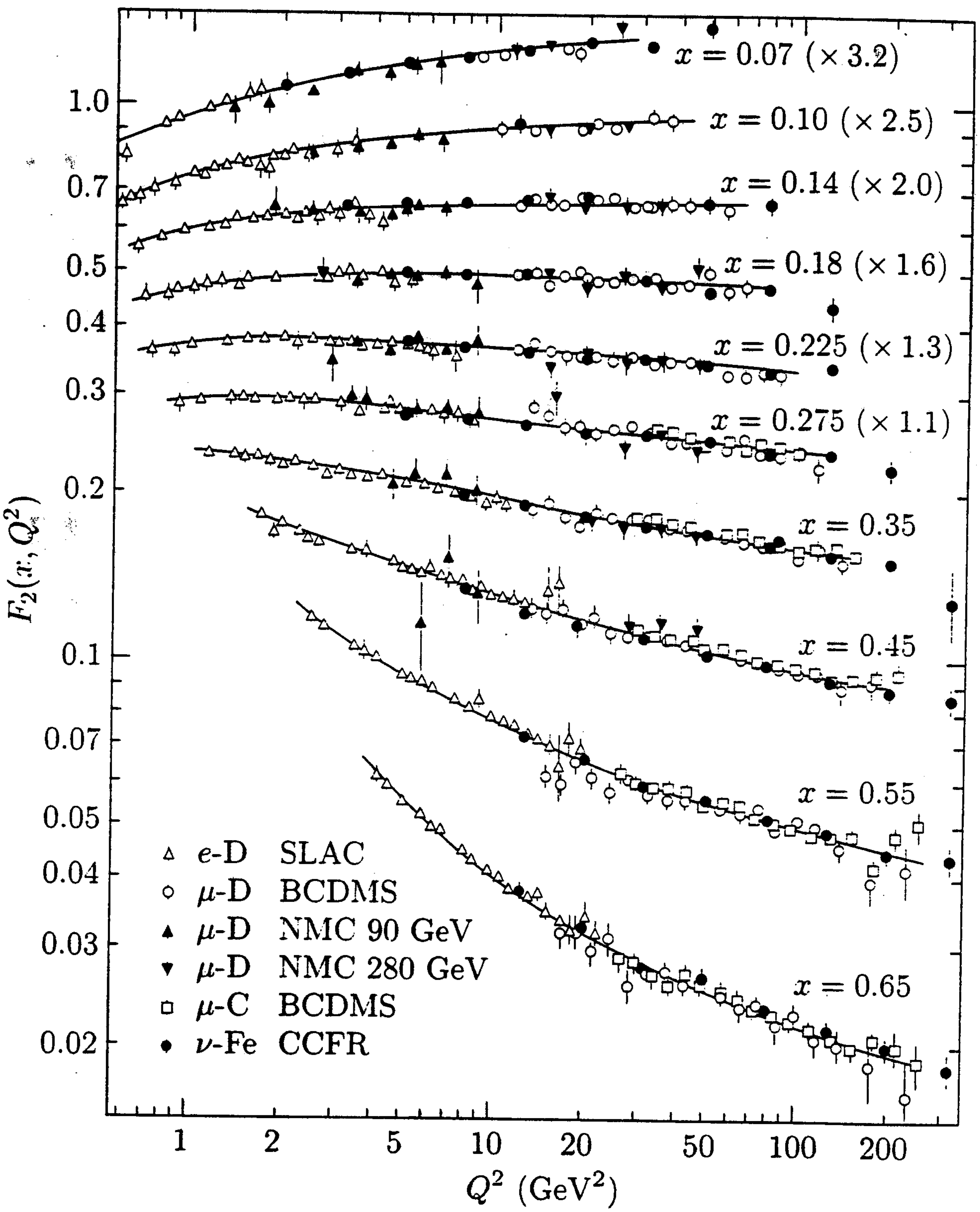


J. Botts et al (CTEQ)

Phys Lett. $\underline{B304}$, 159 (1993) as a function of Q^2, as given by e, μ, and ν deep inelastic scattering experiments. You see the effect I have just described: the distributions increase at low x and decrease at high x. Note also the log scale in Q^2. The curves are the prediction of the Altarelli-Parisi equations.

These experiments establish the gluon as an element of the Standard Model and motivate the idea that the gluon might be the basic quantum of the strong interactions. However, there is another important property needed to complete the picture. I will discuss that in lecture 8, after a detour into the theory of the weak interactions.

Figure 42



data compilation of Particle Data Group.
(Review of Particle Properties)

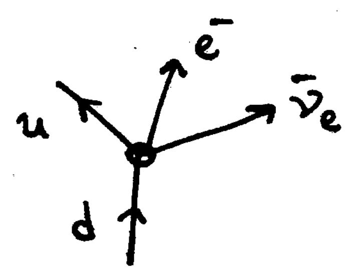
Lecture 5: July 29

Up to this point in the course, we have studied the electromagnetic and the strong interactions. We have seen that both of these interactions have as basic quanta <u>massless</u> particles of spin 1 — the <u>photon</u> and the <u>gluon</u>. We have seen how the spin of these particles has clear effects in high-energy scattering processes of quarks and leptons.

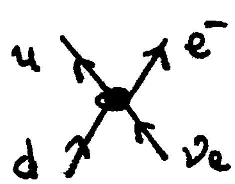
Now I would like to turn to the third of the fundamental interactions of particle physics, the weak interactions. The most familiar weak-interaction process is β-decay:

$$n \to p + e^- + \bar{\nu}_e$$

which we should now write as the process involving quarks and leptons:

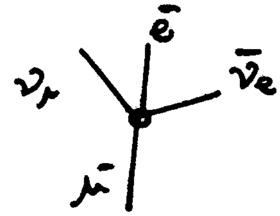


This reaction differs in two ways from those we have studied so far. First, it changes <u>flavor</u>: the d becomes a u, and in the related process (replacing the final $\bar{\nu}_e$ by an initial ν_e)

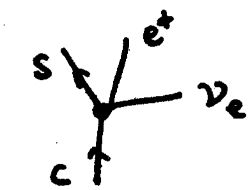


the ν_e becomes an e^-.

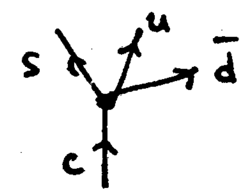
A related weak interaction process is μ decay : $\mu^- \to e^- \bar{\nu}_e \nu_\mu$



similarly, charmed quark decay $D^0 \to K^- e^+ \nu_e$ involves the process



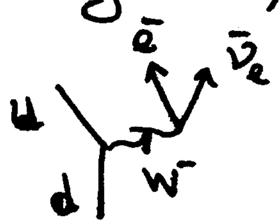
Particles containing the charmed quark can also decay to hadrons only: e.g. $D^0 \to K^- \pi^+$, which involves the process

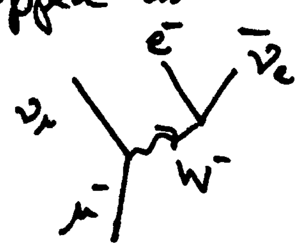


These processes are all accounted for if we postulate a heavy boson called the W^+ (and its antiparticle W^-) which couples in such a way as to make transitions between pairs of fermions

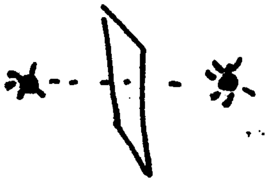
$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix} \quad \begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$$

so that β-decay and μ decay appear as





The second distinguishing characteristic of the weak interactions is ~~that they~~ violate parity, the operation of mirror reflection which converts



$x \leftrightarrow -x$

$e^-_R \longleftrightarrow e^-_L$

The strong and electromagnetic interactions conserve parity. We have seen the implication of this in these lectures through identities such as

$$\mathcal{M}(e^-_L e^+_R \to q_L \bar{q}_R) = \mathcal{M}(e^-_R e^+_L \to q_R \bar{q}_L)$$

(for electromagnetism only). It was the suggestion of Lee and Yang that the weak interactions could violate parity that opened up the experimental understanding of these phenomena. Within a few years it was realized that

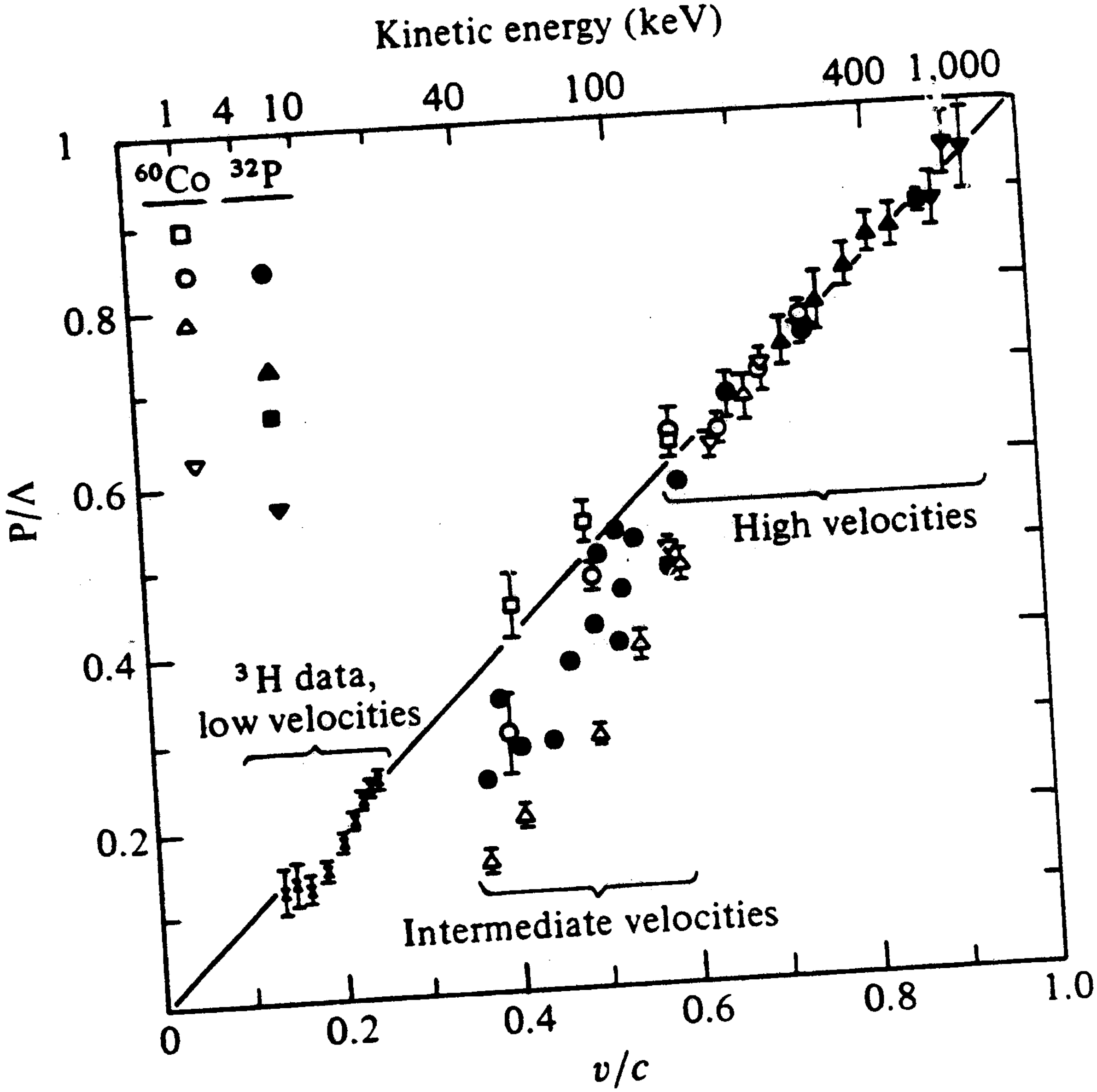
* the e^-'s emitted in β-decay are dominantly left-handed.

In Figure 43, I show a modern measurement of the helicity of the electron in β-decay. We see that $\langle h \rangle \to -\frac{1}{2}\hbar$ (1, in the figure) as the velocity of the electron goes to c.

* the e^+ emitted in μ^+ decay is right-handed.
* the ν_e emitted in $e^- p \to n \nu_e$ is left-handed.

This last measurement was done in a beautiful experiment of Goldhaber, Grodzins, and Sunyar (Phys. Rev. 109, 1015 (1958)). These authors looked at the process of "K-capture", capture of a K-shell (1S) electron by an unstable nucleus. Specifically,

Figure 43



Koks and van Klinken,
Nucl. Phys. A272 61 (1976)

they studied

$$e^- + Eu^{152}(0^-) \rightarrow \nu + Sm^{152*}(1^-) \rightarrow Sm^{152}(0^+) + \gamma$$

The γ energy is maximum when there is minimal nuclear recoil, that is, when the ν cd γ are back to back.

$$\nu \leftarrow \odot \rightsquigarrow \gamma$$

These highest-energy γ's were selected by resonant rescattering. These γ's have $\langle h \rangle = -1$, which is possible if the ν has $h = -\frac{1}{2}$, but not $h = +\frac{1}{2}$.

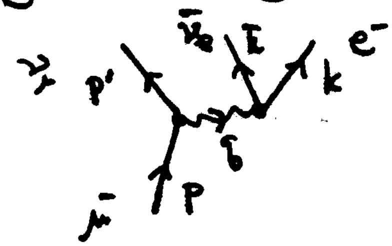
To describe these phenomena theoretically, let us make the bold statement (of Feynman cd Gell-Mann cd Sudarshan cd Marshak) that the weak interactions violate parity maximally, that is, the W boson couples only to left-handed quarks cd leptons cd their antiparticles.

There is one more distinguishing feature of weak interaction decay processes: they are weak. Weak-interaction decays proceed more slowly that strong-interaction reactions by factors of 10^{-10} to 10^{-15}. But we will see that this is not an essential feature of the weak interactions. Weak decays are slow because the W boson is very heavy. Typically

$$(\text{decay rate}) \propto \left(\frac{m}{m_W}\right)^4 = 2 \times 10^{-8} \text{ for } m \sim 1\,\text{GeV},\ m_W = 80\,\text{GeV}$$

This large suppression will disappear when we study the weak interactions at high energy.

If we assume that the W is a spin-1 particle like [redacted] the photon cd the gluon, we can make quantitative prediction for weak-interaction decay distributions. I'll begin by studying μ^- decay:



If the W were massless and the μ^- and e^- were relativistic, we could relate this amplitude to that for $\mu^-_L \nu_{eL} \to \nu_{\mu L} e^-_L$ scattering:

$$\mathcal{M}(\mu^-_L \nu_{eL} \to \nu_{\mu L} e^-_L) = (\underset{?}{e^2}) \left(\frac{4\, p\cdot \bar k}{q^2} \right)$$

For the massive W boson, we replace $\frac{1}{q^2}$, the solution of the massless wave equation, with $\left(\frac{1}{q^2 - m_W^2}\right)$, the solution of the Klein-Gordon equation. We must also replace (e^2) by the W coupling constant, which I will write as $g^2/2$. Then we have

$$|\mathcal{M}(\mu^- \to \nu_{\mu L} e^-_L \bar\nu_{eR})|^2 = \left(\frac{g^2}{2}\right)^2 \frac{|(4\, p\cdot \bar k)(4\, p'\cdot k)|}{(q^2 - m_W^2)^2}$$

In writing this equation, I have used

$(p - \bar k)^2 = (p' + k)^2$; so for massless particles $|2p\cdot\bar k| = |2p'\cdot k|$

Since $|q^2| < m_\mu^2$, $m_W^2 \gg m_\mu^2$, we can ignore q^2 in the denominator, this gives $|\mathcal{M}|^2 \propto \frac{1}{m_W^4}$ as on p. 4.

To compute the μ^- decay rate Γ, we need to know this matrix element for a μ^- at rest and ν_μ, e^- $\bar\nu_e$ relativistic. A detailed quantum field theory computation gives just the same result that I have written above, so

$$I = \frac{1}{2m_\mu}\int\frac{d^3p'}{(2\pi)^3 2p'}\int\frac{d^3k}{(2\pi)^3}\frac{1}{2k}\int\frac{d^3\bar{k}}{(2\pi)^3 2\bar{k}}(2\pi)^4\delta(p-p'-k-\bar{k})$$

$$\cdot\ \frac{g^4}{4}\frac{1}{m_W^4}\cdot 16(p\cdot\bar{k})(p'\cdot k)$$

This looks like a hard integral, but it is not so bad. Remember that

$$\int\frac{d^3p'}{(2\pi)^3 2p'}\int\frac{d^3\bar{k}}{(2\pi)^3 2\bar{k}}(2\pi)^4\delta^{(4)}(K-p'-\bar{k}) = \frac{1}{8\pi}$$

and is relativistically invariant. (We might as well integrate over the neutrino momenta, because neutrinos are massless and very penetrating, so in practice we can only observe the electron.)

Now let's try to compute

$$\mathcal{J}^{\mu\nu} = \int\frac{d^3p'}{(2\pi)^3 2p'}\frac{d^3\bar{k}}{(2\pi)^3 2\bar{k}}(2\pi)^4\delta^{(4)}(K-p'-\bar{k})\ \bar{k}^\mu p'^\nu$$

Work in the frame where $K = (K,\vec{0})$. Then

$$\bar{k}^\mu = \left(\frac{K}{2}\right)\cdot(1,\hat{n})^\mu \qquad p'^\nu = \frac{K}{2}(1,-\hat{n})^\nu$$

then

$$\mathcal{J}^{00} = \frac{1}{8\pi}\left(\frac{K}{2}\right)^2 \qquad \mathcal{J}^{ij} = \frac{1}{8\pi}\left(\frac{K}{2}\right)^2\langle-\hat{n}^i\hat{n}^j\rangle$$

$$\mathcal{J}^{0i} = 0 \qquad\qquad = -\frac{1}{3}\frac{1}{8\pi}\left(\frac{K}{2}\right)^2\delta^{ij}$$

If I write

$$\mathcal{J}^{\mu\nu} = \frac{1}{8\pi}\left(\frac{K}{4}\right)^2\left(\frac{1}{3}g^{\mu\nu} + \frac{2}{3}\frac{K^\mu K^\nu}{K^2}\right)$$

This expression agrees with the result in this particular frame but is valid in any frame. Now go to the μ^- frame.

$$p^\mu = (m_\mu, \vec{0}) \qquad \text{write} \qquad k^\mu = \underbrace{\frac{m_\mu}{2}}_{\substack{\uparrow \\ \text{max. possible value}}} \cdot x \cdot (1, \vec{v}) \qquad |\vec{v}| = 1$$

$$K^2 = (p-k)^2 = m_\mu^2 - 2p\cdot k$$
$$= m_\mu^2 (1-x)$$

max. possible value : $0 < x < 1$

then

$$\Gamma \propto \int_0^1 dx \, \frac{x^2}{x} \left[\frac{1}{3} p\cdot k \; m_\mu^2 (1-x) + \frac{2}{3} p\cdot K \; K\cdot k \right]$$

$$p\cdot k = \frac{m_\mu^2}{2} x \qquad p\cdot K = m_\mu \left(m_\mu - \frac{m_\mu}{2} x\right) = \frac{m_\mu^2}{2} (2-x)$$

$$2K\cdot k = (K+k)^2 - K^2 = m_\mu^2 - m_\mu^2 (1-x) = m_\mu^2 x$$

$$\propto \int_0^1 dx \; \frac{x^2}{x} \left[\frac{1}{3} \frac{m_\mu^2}{2} x \; m_\mu^2 (1-x) + \frac{1}{3} \frac{m_\mu^2}{2} (2-x) m_\mu^2 x \right]$$

$$= \frac{m_\mu^4}{2\cdot 3} \int_0^1 dx \; x^2 (3-2x)$$

so that

$$\frac{d\Gamma}{dx} = (\text{const}) \cdot \; x^2 (3-2x)$$

which has a maximum at $x = 1$

You can work out similarly that, if the $\bar{\nu}_e$ energy is

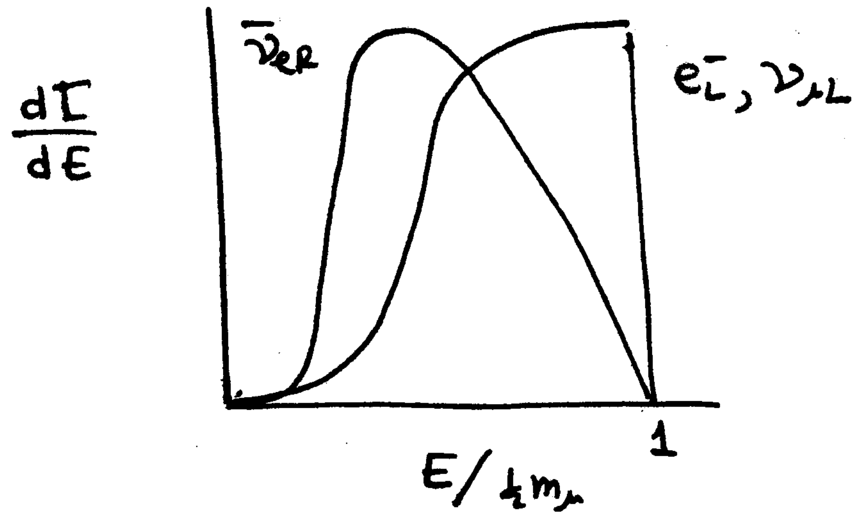
$$k^0 = \frac{m_\mu}{2} y ,$$

then

$$\frac{d\Gamma}{dy} = (\text{const.}) \; y^2 (1-y)$$

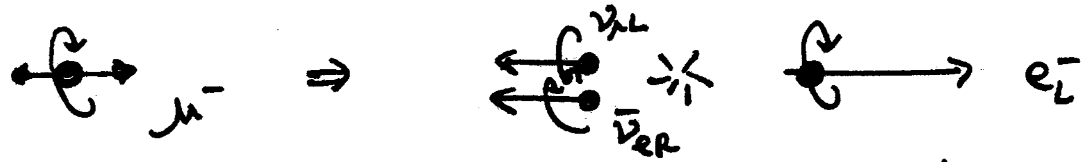
which vanishes at $y = 1$

So



This prediction for the electron energy spectrum in μ^- decay is compared to experiment in Fig. 44; it works spectacularly well!

We have computed that it is possible to have the e^- or ν_μ with an energy at the kinematic endpoint $E = m_\mu/2$, but not the $\bar{\nu}_e$. This is easy to understand from angular momentum considerations. The e^- has maximum energy when it it back-to-back with the two neutrinos. This is possible for



However, in order for the $\bar{\nu}_{eR}$ to have the maximum energy, the μ^- would have to decay to

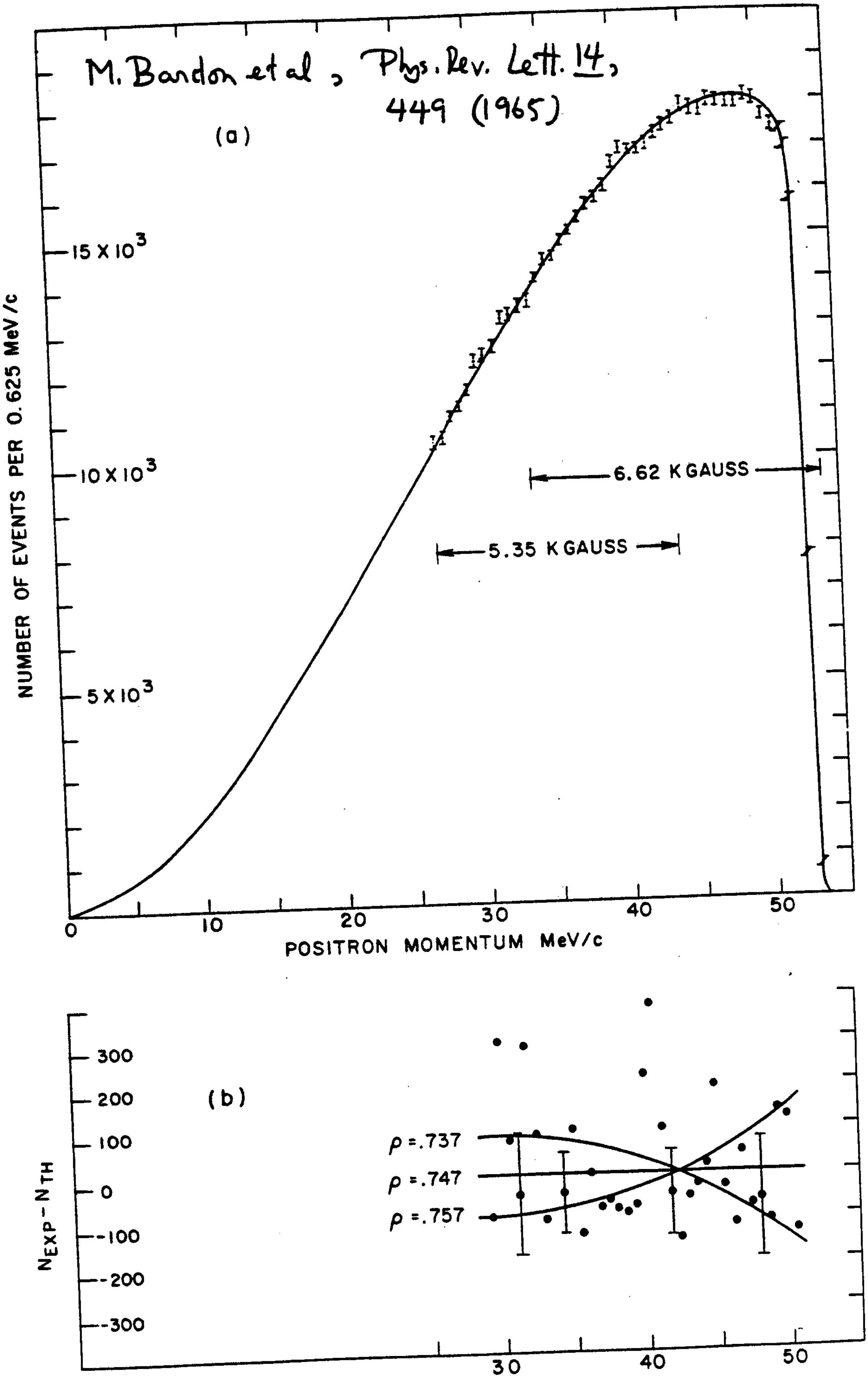
which has $J^z = +3/2 \longrightarrow$ impossible!

A corollary of this argument is that the highest energy electrons are emitted <u>backward</u> with respect to the spin

9

Figure 44

M. Bardon et al, Phys. Rev. Lett. 14, 449 (1965)



of the muon. This correlation is shown in Fig. 45. It is possible to test this idea by stopping muons and causing them to precess in a magnetic field. If we then select electrons of energy close to the endpoint, the counting rate should go to zero when the spin of the muon points to the detector. This allows a very sensitive test of the handedness of the three particles emitted in μ decay. The experiment was done very carefully at TRIUMF; the results are shown in Fig. 46. After correcting for the depolarization of muons in the stopping medium, one sees the zero to within a few percent.

You see that the theory that W bosons have spin 1 and couple only to left-handed particles is very well tested in β decay. Similar tests are available in β-decay and in high-energy neutrino scattering. I showed in lecture 3 that

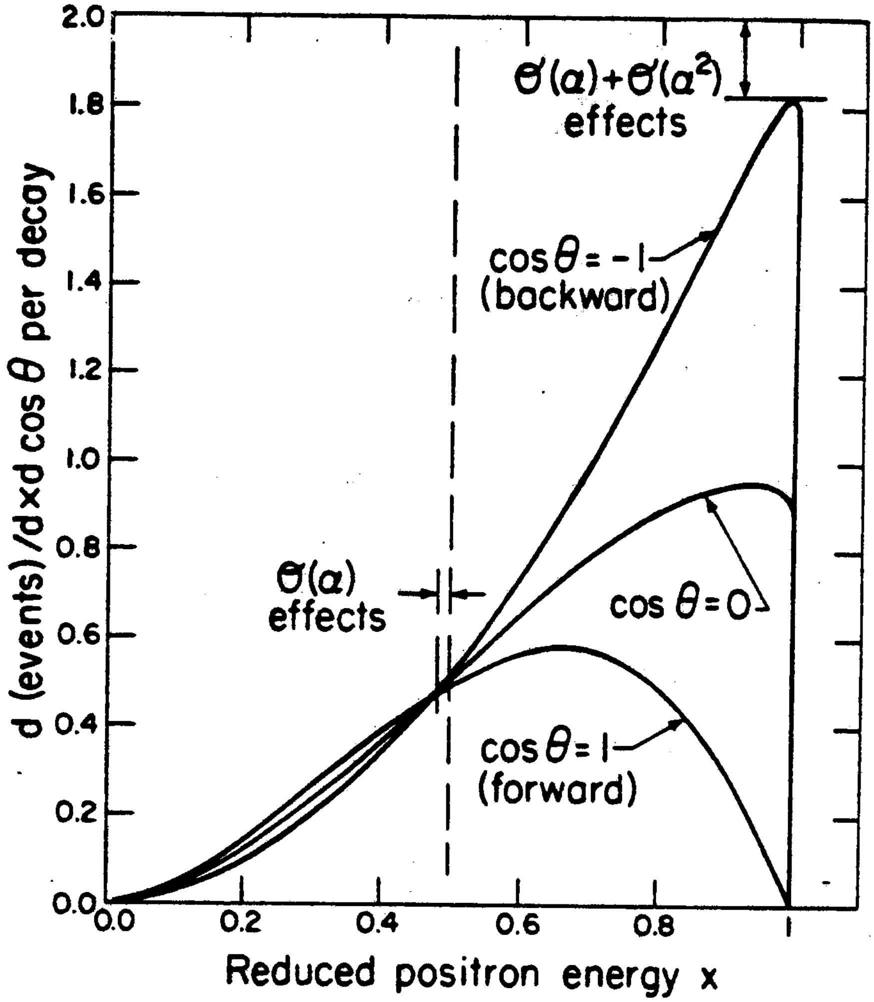
$$\text{for } \nu_\mu \qquad \frac{d\sigma}{dQ^2\,dx} \propto 1$$

$$\text{for } \bar{\nu}_\mu \qquad \frac{d\sigma}{dQ^2\,dx} \propto (1-y)^2$$

this tests the left-handed coupling both of ν_μ and of the quarks in the proton and neutron.

It turns out, though, that the W boson is not the only carrier of weak interactions. The experiments on neutrinos

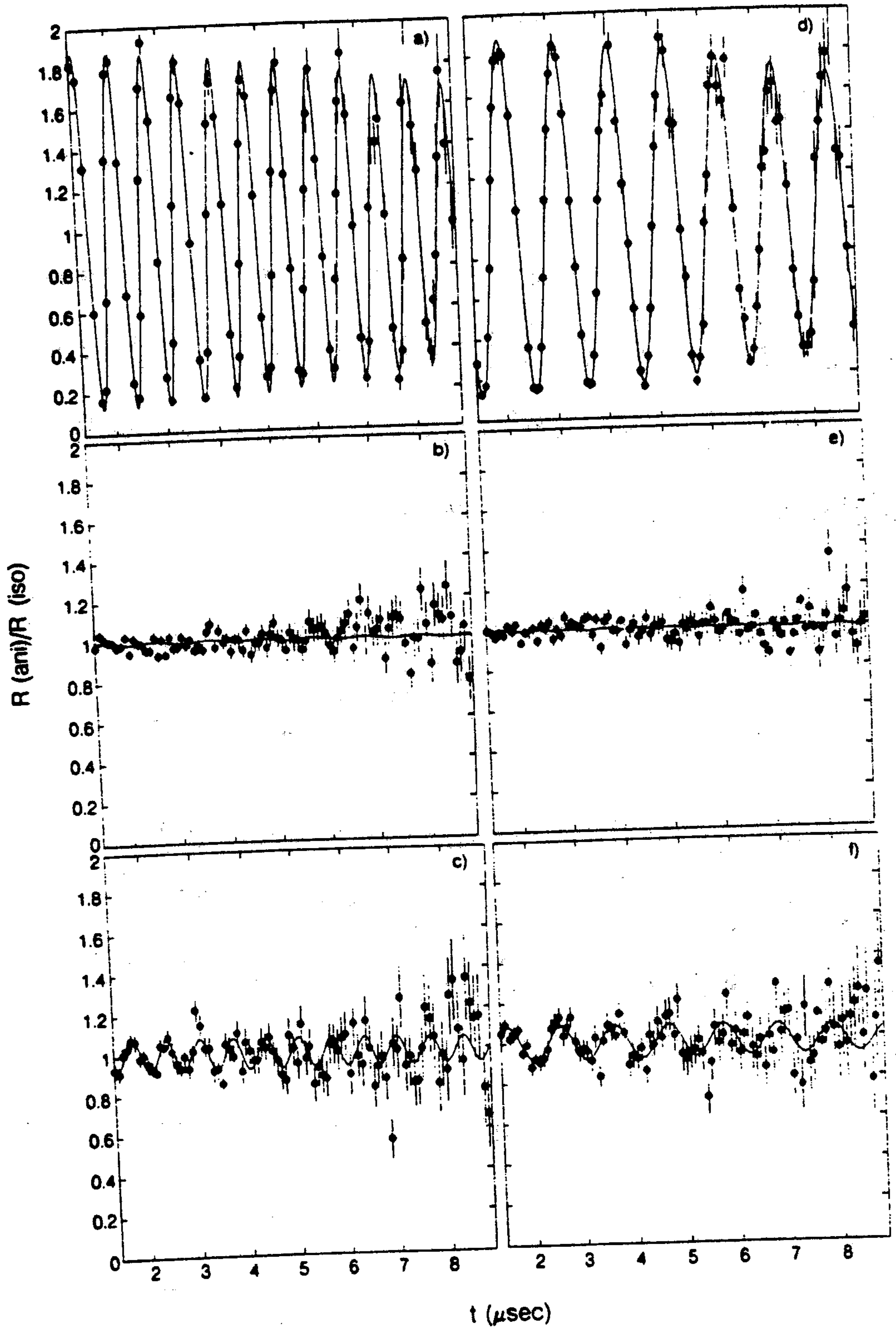
Figure 45



B. Balke et al (Berkeley-TRIUMF)

Phy. Rev. D37, 587 (1988)

Figure 46



B. Balke et al. (Berkeley-TRIUMF)
Phys. Rev. D37, 587 (1988)

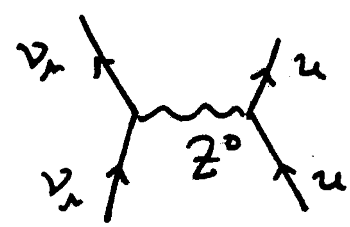
deep inelastic scattering, while they observe the reaction

$$\nu_\mu + (p,n) \rightarrow \mu^- + (\text{hadrons})$$

also observe the reaction

$$\nu_\mu + (p,n) \rightarrow \nu_\mu + (\text{hadrons})$$

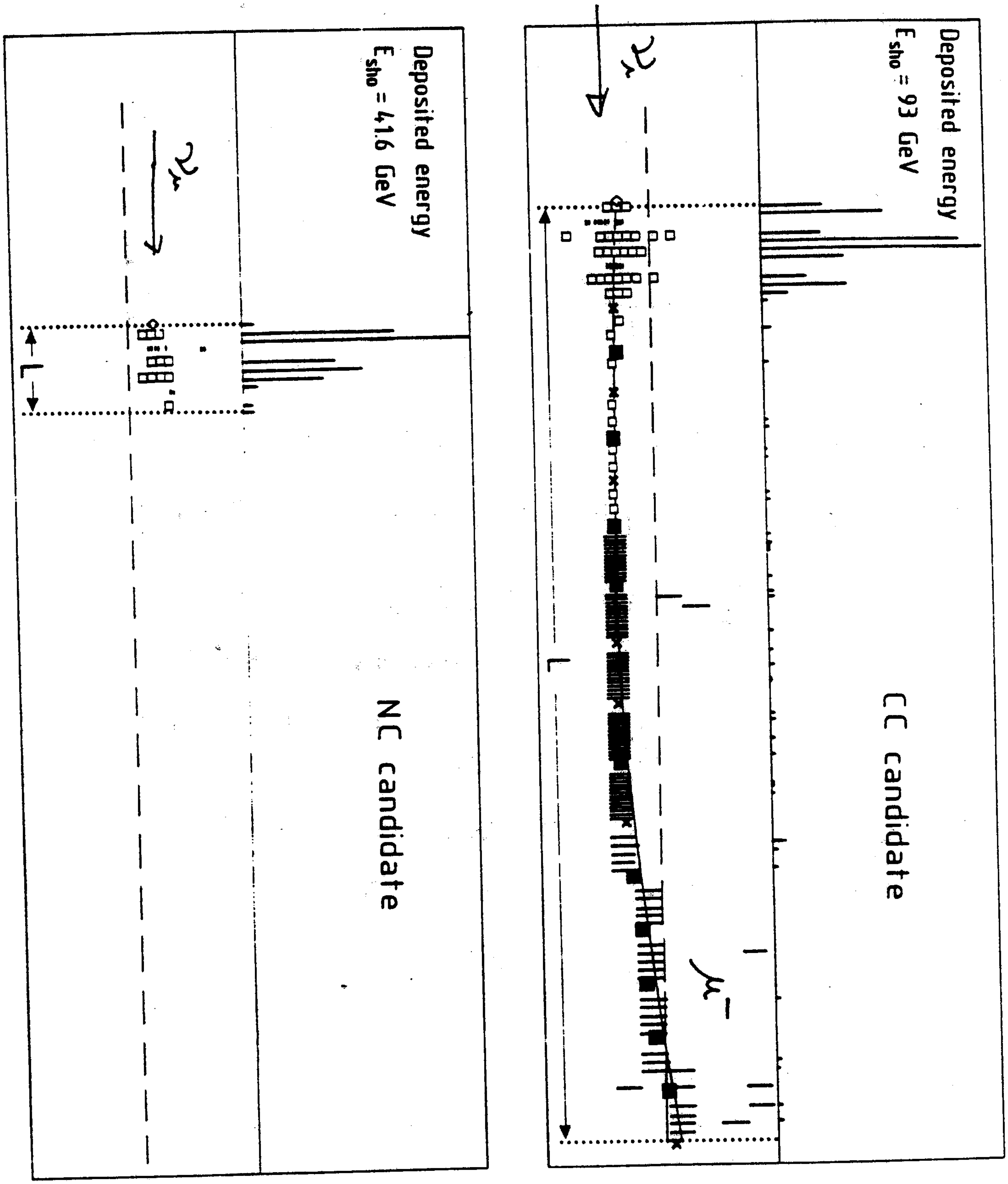
The first type of reaction is called the "charged current" process and is mediated by the W; the second type is called the "neutral current" process and is mediated by a neutral counterpart of the W called the Z°:



In neutrino scattering, the rate of neutral current reactions is about ¼ the rate of charged current reactions. Typical events observed by the CDHS experiment are shown in Fig. 47.

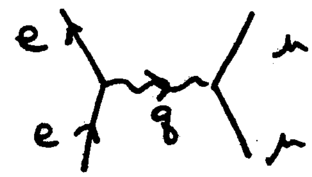
I would now like to write ~~an~~ unified expression for the matrix elements for charged current and neutral current scattering processes. In the remainder of this lecture, I will show some experimental evidence for this expression. In the next lecture, I will derive this expression from

Figure 47



A. Blondel et al (CDHS) Z.Phys. C45, 361 (1990) some deeper principles.

To begin, let's go back to the case of electromagnetic scattering processes. In $e^-\mu^-$ scattering, an electromagnetic impulse is created by one electromagnetic current and is absorbed by another:



Representing the interactions  and  schematically as matrix elements of the electromagnetic current

$$j^\mu_{EM} = (-j^\mu)_e + (-j^\mu)_\mu$$

we can write the matrix element as the matrix element of

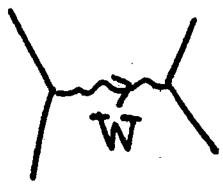
$$\mathcal{M}(e^-\mu^- \to e^-\mu^-) = \frac{e^2}{2}\, j^\mu_{EM} \,\frac{1}{q^2}\, j_{\mu\, EM}$$

The factor $\frac{1}{2}$ will be canceled because there are two terms with one $(j^\mu)_e$ and one $(j_\mu)_\mu$. If we write

$$j^\mu_{EM} = \sum_f Q_f\, j^\mu_f$$

then the same expression describes the electromagnetic interactions of all leptons and quarks.

In the same language, let $j^{\mu+}_L$ be a current operator which annihilates an e^- and produces a ν_e or, more generally, raises any of the lower states listed on p.2 to its upper partner. Let J_L^{μ} be the current that destroys the upper states ($\nu_e, \nu_\mu, u, c, \ldots$) and creates the corresponding lower state ($e^-, \mu^-, d, s, \ldots$). Then we can write the process mediated by a W boson



(L denotes left-handed) particles only.

as

$$\mathcal{M} = \frac{g^2}{2} J_L^{\mu +} \frac{1}{q^2 - m_W^2} J^-_{\mu L}$$

For the rest of this lecture, I will consider reactions with $q^2 \ll m_W^2$, and so I will ignore q^2 in this expression. One often sees the combination

$$\frac{g^2}{8m_W^2} = \frac{G_F}{\sqrt{2}} \qquad \text{the "Fermi constant"}$$

$$= \frac{1}{\sqrt{2}} \cdot (1.166 \times 10^{-5} \ \mathrm{GeV}^{-2})$$

so that

$$\mathcal{M} = - \frac{4G_F}{\sqrt{2}} J_L^{\mu +} J^-_{\mu L}$$

The doublet structure of multiplets on p.2 might remind us of spinors, and maybe even of Pauli sigma matrices. These are the 2×2 matrices that rotate spinors: $\sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\quad \sigma^y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ $\quad \sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Useful combinations are

$$\sigma^+ = \tfrac{1}{2}(\sigma^x + i\sigma^y) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$\sigma^- = \tfrac{1}{2}(\sigma^x - i\sigma^y) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

A rotationally invariant combination acting on two spinors is:

$$\vec{\sigma}\cdot\vec{\sigma} = \sigma^x\sigma^x + \sigma^y\sigma^y + \sigma^z\sigma^z$$

$$= 2\left(\sigma^+\sigma^- + \sigma^-\sigma^+ + \tfrac{1}{2}\sigma^z\sigma^z\right)$$

$$= 2\left(\sigma^+\sigma^- + \sigma^-\sigma^+ + 2\cdot\frac{\sigma^z}{2}\cdot\frac{\sigma^z}{2}\right)$$

I wrote the last term in this way because $(\sigma^z/2)$ is the z-component of spin acting on a spinor. Let's consider the multiplets on p. 2 as spinors of some fictitious rotations. Call the corresponding spin "isotopic spin" or "isospin" $\vec{I}$. Then we assign.

ν_{eL}	$I^3 = +\frac{1}{2}$	u_L	$I^3 = +\frac{1}{2}$
e^-_L	$I^3 = -\frac{1}{2}$	d_L	$I^3 = -\frac{1}{2}$

and e^-_R, u_R, d_R have $I^3 = 0$. Then we can make the expression on p. 16 invariant under isospin rotations if we write

$$\mathcal{M} = -\frac{4G_F}{\sqrt{2}}\left(j^{\mu+}_L\, j_{\mu-L} + j^{\mu 3}_L\, j^3_{\mu L}\right)$$

where $j^{\mu 3}_L$ is the current whose charges are the isotopic spin I^3. The new term in $\mathcal{M}$ is a neutral-current interaction. This result would be very simple but it is not correct experimentally. The correct result is

$$\mathcal{M} = -\frac{4G_F}{\sqrt{2}}\left[j_L^{\mu +} j_{\mu L}^{-} + (j_L^{\mu 3} - \sin^2\theta_W j_{EM}^{\mu})(j_L^{\mu 3} - \sin^2\theta_W j_{EM}^{\mu}) \right]$$

where j_{EM}^{μ} is the electromagnetic current (p.15) and ($\sin^2\theta_W$) is a constant. I will describe the origin of this expression in the next lecture.

This expression for the neutral current amplitude contains only one new parameter — $\sin^2\theta_W$ — and it makes many predictions, so there are many ways to test it. To begin, let's compute the ratio of the rates for charged-current and neutral current reactions in ν deep inelastic scattering. Consider a target with equal numbers of neutrons and protons — that is, equal numbers of u and d quarks. For simplicity, consider the situation in which there are no antiquarks in the target. Then for the charged-current reaction we have only the process $\nu_\mu u_L \to \mu^- d_L$, for which

$$\mathcal{M}(\nu_\mu u_L \to \mu^- d_L) \sim \frac{4G_F}{\sqrt{2}}$$

For the neutral-current reaction we have

$$\mathcal{M}(\nu_\mu u_L \to \nu_\mu u_L) \sim \frac{4G_F}{\sqrt{2}} \cdot 2 \cdot \underset{I^3_\nu}{\tfrac{1}{2}} \cdot \underset{I^3_u}{\left(\tfrac{1}{2} - \tfrac{2}{3}\sin^2\theta_W\right)}$$

$$\mathcal{M}(\nu_\mu d_L \to \nu_\mu d_L) \sim \frac{4G_F}{\sqrt{2}} \cdot 2 \cdot \frac{1}{2} \cdot \left(-\frac{1}{2} + \frac{1}{3}\sin^2\theta_w\right)$$

$$\mathcal{M}(\nu_\mu u_R \to \nu_\mu u_R) \sim \frac{4G_F}{\sqrt{2}} \cdot 2 \cdot \frac{1}{2} \left(-\frac{2}{3}\sin^2\theta_w\right)$$

$$\mathcal{M}(\nu_\mu d_R \to \nu_\mu d_R) \sim \frac{4G_F}{\sqrt{2}} \cdot 2 \cdot \frac{1}{2} \left(-\frac{1}{3}\sin^2\theta_w\right)$$

If we remember that the latter two processes have $(1-y)^2$ distribution, we find

$$\frac{\frac{d\sigma}{dx\,dQ^2}(\nu A \to \nu + \text{hadrons})}{\frac{d\sigma}{dx\,dQ^2}(\nu A \to \mu^- + \text{hadrons})} = \frac{\left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_w\right)^2 + \left(\frac{1}{2} - \frac{1}{3}\sin^2\theta_w\right)^2 + \frac{5}{9}\sin^4\theta_w(1-y)^2}{1}$$

$$= \frac{1}{2} - \sin^2\theta_w + \frac{5}{9}\sin^4\theta_w\left(1 + (1-y)^2\right)$$

now, $(1-y)^2$ is also the ratio of $\bar\nu$ to ν charged current deep inelastic scattering. So if

$$R^\nu = \frac{\int d\sigma/dx\,dQ^2(\nu A \to \nu + X)}{\int d\sigma/dx\,dQ^2(\nu A \to \mu^- + X)}$$

$R^{\bar\nu}$ = similar for $\bar\nu_\mu$

$$r = \frac{\int d\sigma/dx\,dQ^2(\bar\nu A \to \mu^+ + X)}{\int d\sigma/dx\,dQ^2(\nu A \to \mu^- + X)}$$

(The integrals are taken over the kinematic region observed by the detector.)

then

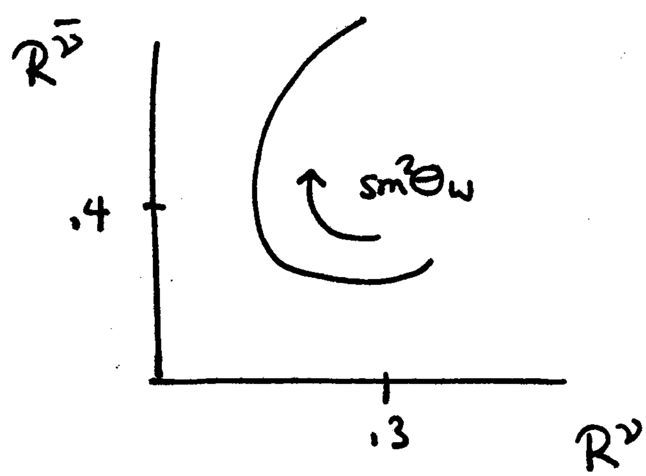
$$R^\nu = \frac{1}{2} - \sin^2\theta_w + \frac{5}{9}\sin^4\theta_w(1+r)$$

You can do the same calculations with $\bar{\nu}$ neutral current scatt'g and find

$$R^{\bar{\nu}} = \frac{1}{2} - \sin^2\theta_W + \frac{5}{9}\sin^4\theta_W \left(1+\frac{1}{r}\right)$$

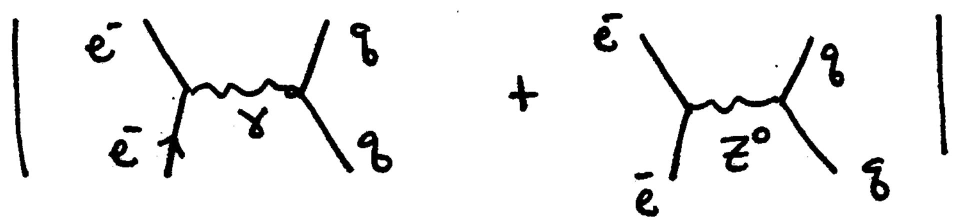
Llewellyn-Smith realized an amazing thing about these two formulae. If you now put back the antiquarks, using $f_u(x) = f_d(x)$ and $f_{\bar{u}}(x) = f_{\bar{d}}(x)$, you will see that these formulae are still true. Further, r is measured by the experiment, so we don't have to predict it. Then we get the NC/CC ratios R^{ν} and $R^{\bar{\nu}}$ in terms of one parameter, $\sin^2\theta_W$. For a reasonable measured value, $r = 0.4$, we can plot $R^{\bar{\nu}}$ against R^{ν} as a function of $\sin^2\theta_W$.

This gives a figure called "Weinberg's nose".



It is a test of the theory that the measured values fall on Weinberg's nose, and the location chosen sizes $\sin^2\theta_W$. The data is shown in Fig. 48; it agrees well with the model for $\sin^2\theta_W \sim 0.23$.

Another way to test the formula on p. 18 is to look carefully at e^- deep inelastic scattering. If we include the effect of the neutral current, we now have two diagrams:

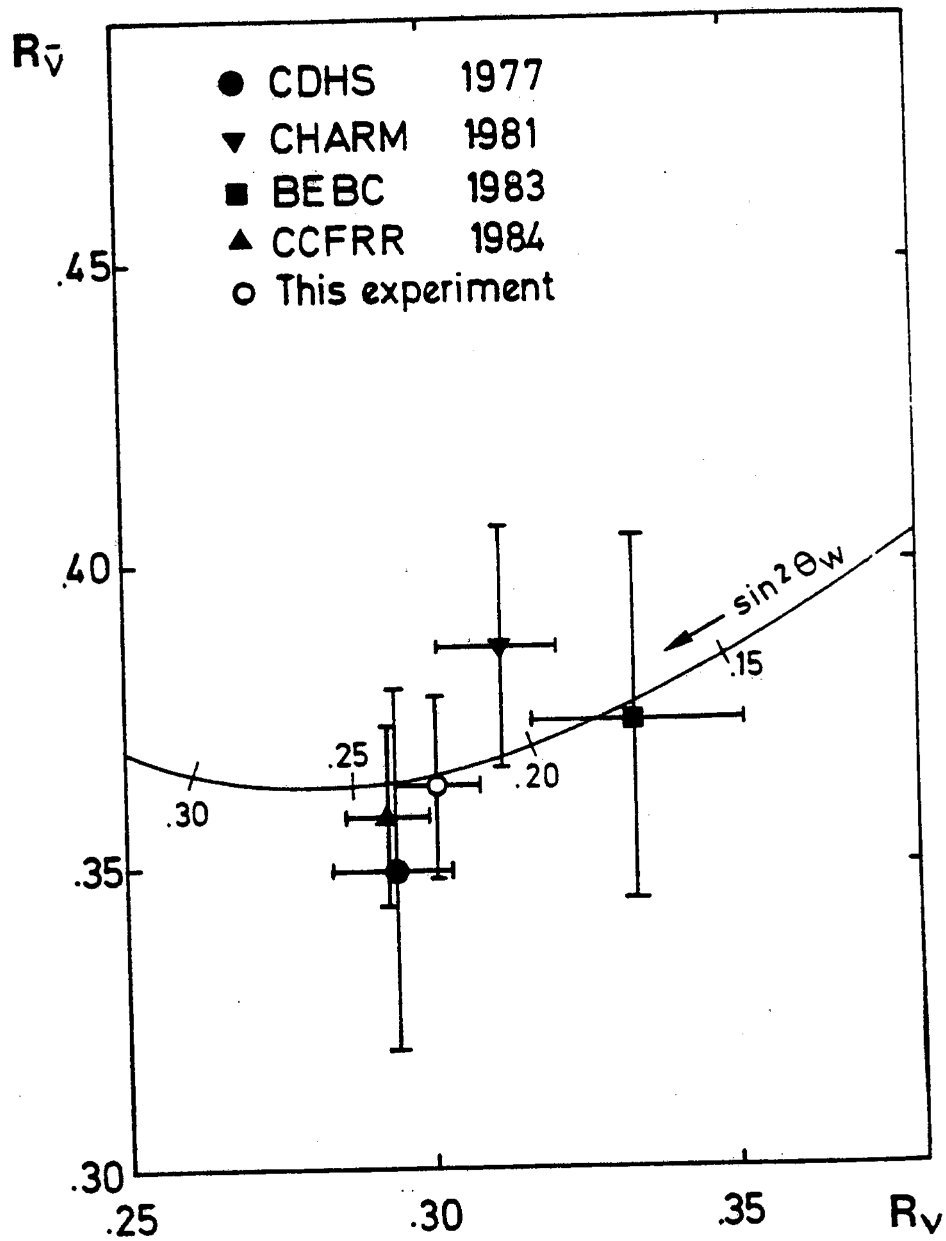
$|\mathcal{M}|^2 = \Big|$  $\Big|^2$

The first diagram is parity conserving, but the second diagram violates parity. This effect is visible as a slightly different rate for deep inelastic scattering on an unpolarized target for e^-_L as opposed to e^-_R beams. On the other hand, the second diagram is smaller by a factor

$$\frac{Q^2}{m_Z^2} \sim \left(\frac{1\,\text{GeV}}{90\,\text{GeV}}\right)^2 \sim 10^{-4}$$

so we expect an asymmetry of only this tiny magnitude. An experiment at SLAC measured this small effect. Fig. 49 shows some of the data. As the electrons are transported from the accelerator to the target,

Figure 48

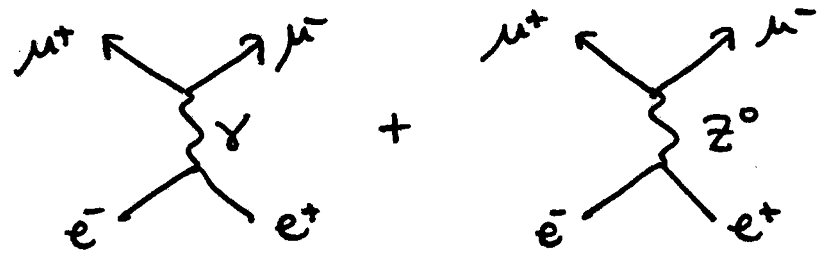


H. Abramowicz et al (CDHS)
Z Phys. C <u>28</u> 51 (1985)

they go through a small bend. In this bend, the spin precesses by an amount which depends on the beam energy. By varying the energy slightly, it is possible to reverse the spin direction, and the asymmetry also reverses; this is a nice systematic check. The final result, shown in Fig. 50, is compatible with the formula for $\sin^2\theta_W \sim 0.2$.

A similar effect can be seen in e^+e^- annihilation. Adding the diagrams:

$\mathcal{M}$ =



we have

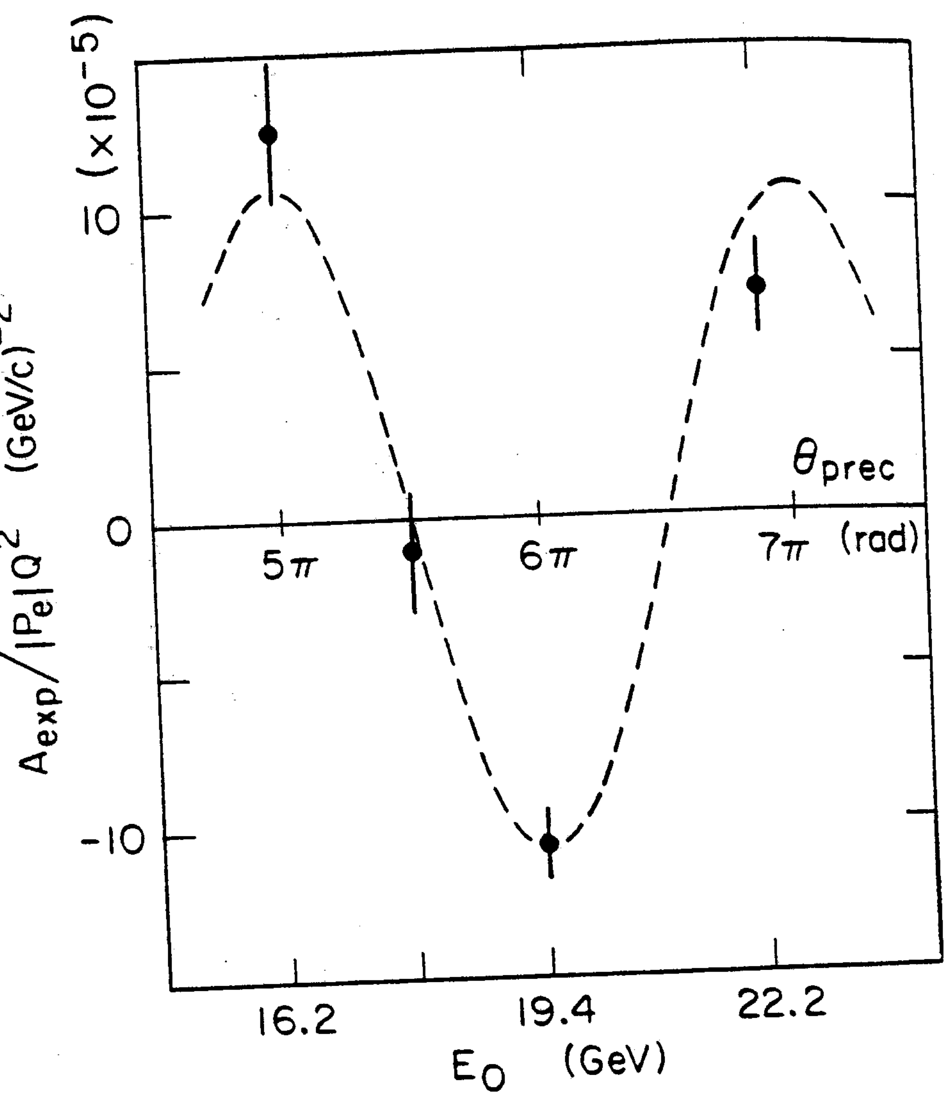
$$\mathcal{M}(e^-_L e^+_R \to \mu^-_L \mu^+_R) \sim \frac{e^2}{s} - \frac{8G_F}{\sqrt{2}}\left(-\tfrac{1}{2} + \sin^2\theta_W\right)^2$$

$$\mathcal{M}(e^-_L e^+_R \to \mu^-_R \mu^+_L) \sim \frac{e^2}{s} - \frac{8G_F}{\sqrt{2}}\left(-\tfrac{1}{2} + \sin^2\theta_W\right)\sin^2\theta_W$$

$$\mathcal{M}(e^-_R e^+_L \to \mu^-_L \mu^+_R) \sim \frac{e^2}{s} - \frac{8G_F}{\sqrt{2}}\left(\sin^2\theta_W\right)\left(-\tfrac{1}{2} + \sin^2\theta_W\right)$$

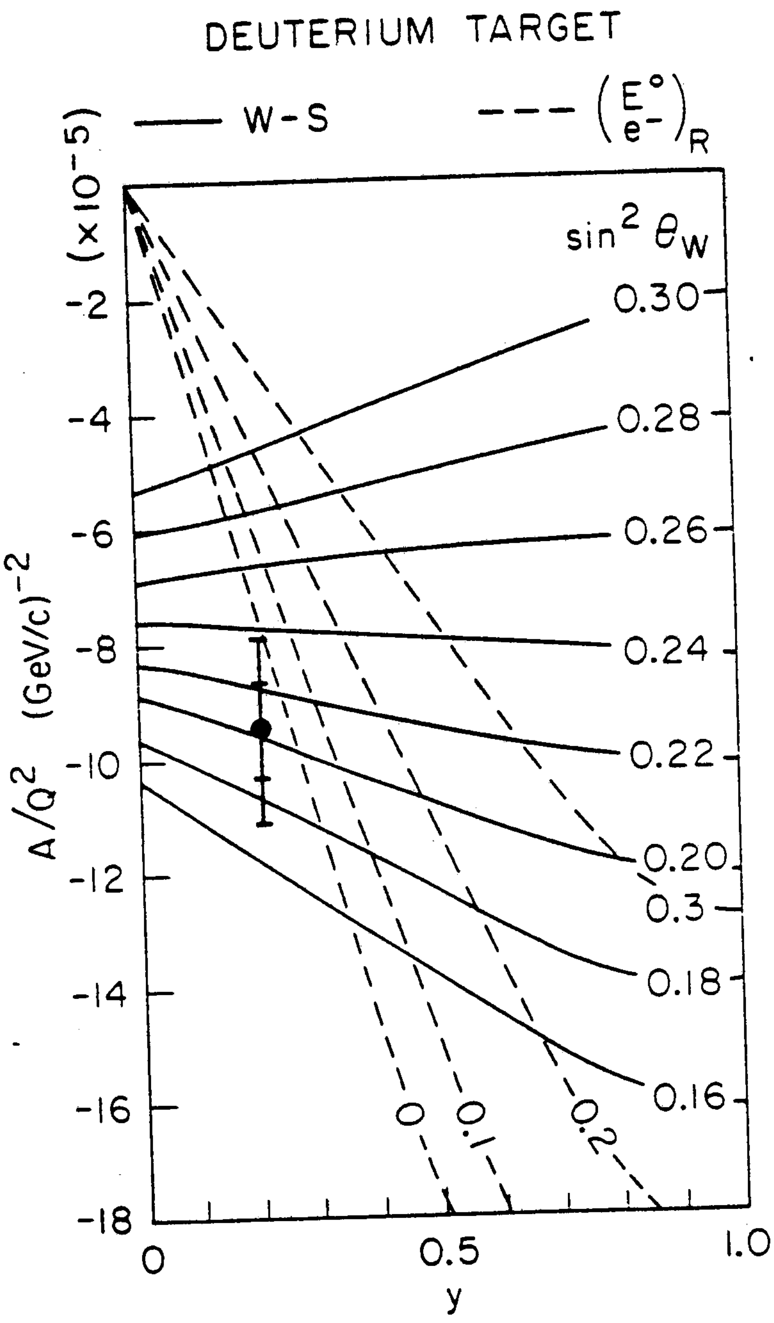
$$\mathcal{M}(e^-_R e^+_L \to \mu^-_R \mu^+_L) \sim \frac{e^2}{s} - \frac{8G_F}{\sqrt{2}}\sin^4\theta_W$$

Figure 49



C.Y. Prescott et al. (SLAC-Yale)
Phys. Lett. B 77, 347 (1978)

Figure 50



CY Prescott et al (SLAC-Yale)

Phys. Lett. B77 347 (1978)

Recall that the first and last processes have an angular distribution

$$(1+\cos\theta)^2$$

while the second and third processes have

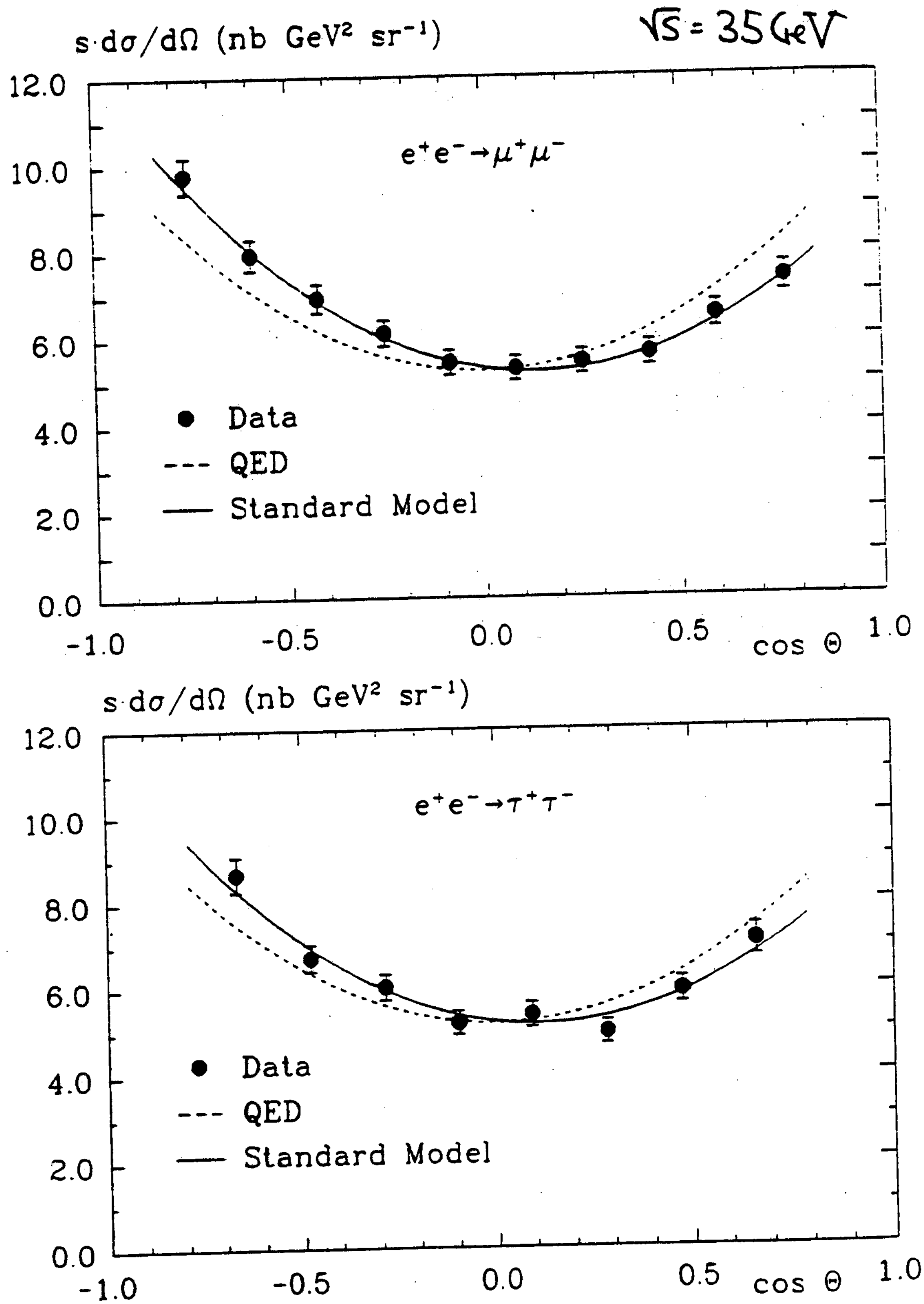
$$(1-\cos\theta)^2$$

Since the neutral-current effect makes the first and last amplitudes smaller and the second and third larger, we predict an asymmetrical angular distribution slightly skewed in the backward direction. I showed you this effect in Fig. 2; here are the graphs again in Fig. 51.

A similar effect is seen in $e^+e^- \to q\bar{q}$. The d, s, b quarks have a particularly large asymmetry, because the electromagnetic amplitude is proportional to the quark charge $Q_f = -\frac{1}{3}$. I will explain in lecture 7 how it is possible to recognize $e^+e^- \to b\bar{b}$ events using the fact that hadrons containing b quarks have small but measurable lifetimes $\tau_b \sim 1\,\mathrm{ps} = \frac{1}{3}\,\mathrm{mm}$. The b can be told from the $\bar{b}$ by a leptonic decay:

$$b \to c\,\mu^-\bar{\nu}_\mu \qquad \bar{b} \to \bar{c}\,\mu^+\nu_\mu$$
$$(e^-\bar{\nu}_e) \qquad\qquad (e^+\nu_e)$$

Figure 51

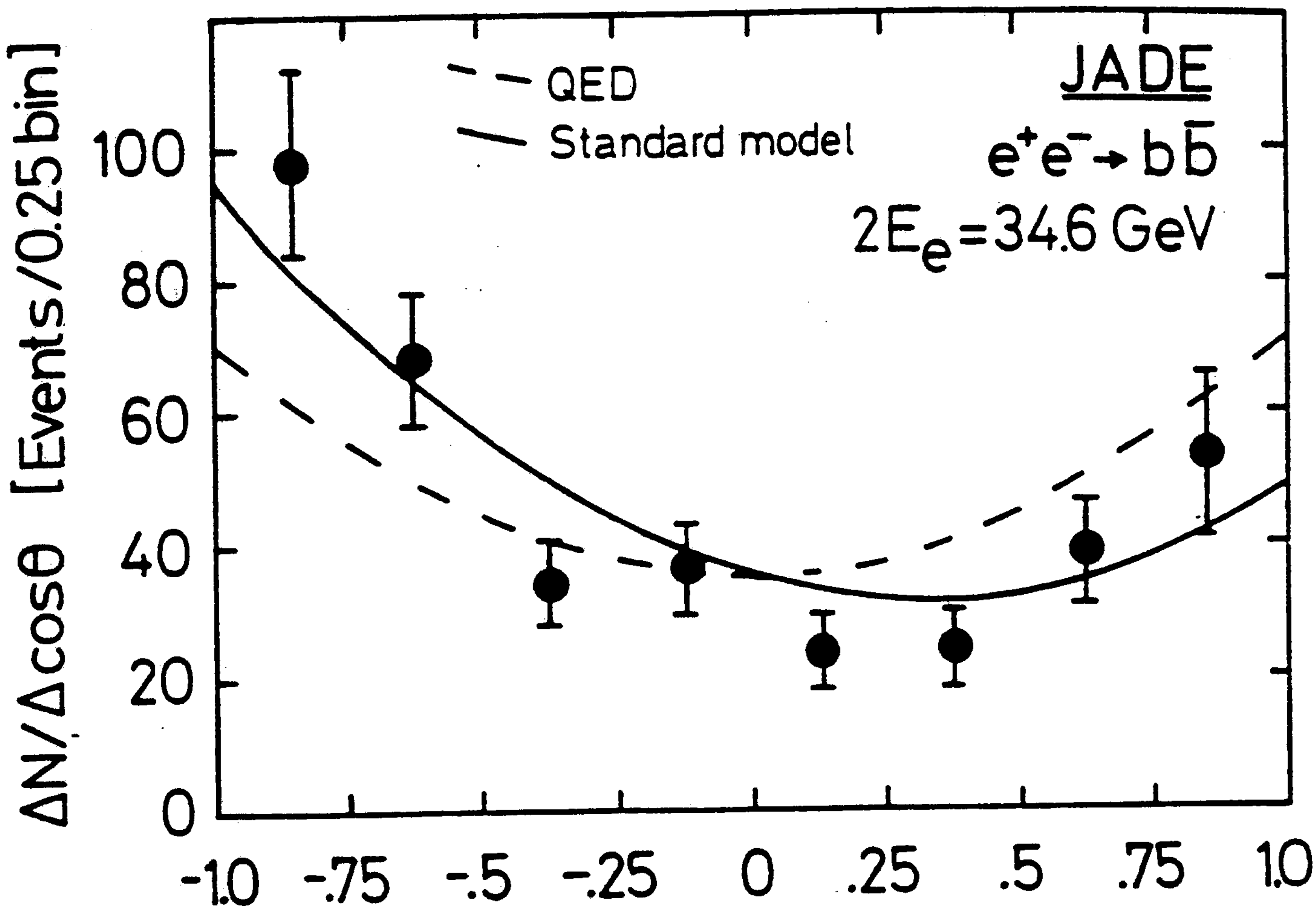


S. Hegner et al (JADE)
Z Phys. C46 547 (1990)

The JADE experiment at PETRA used these techniques to measure the forward-backward asymmetry in $e^+e^- \to b\bar{b}$; their results are shown in Fig. 52. These are again compatible with the formula on p. 18 for $\sin^2\theta_W \sim 0.2$.

The neutral-current amplitude can be checked in many other ways, including $\nu_\mu e$ scattering and the observation of parity violation in atomic transitions. So the formula is certainly right. But why is it so complicated? I'll address this question in the next lecture.

Figure 52



W. Bartel et al. (JADE)
Phys. Lett. 146B, 437 (1984)

Lecture 6: July 30

Now we have met all of the bosons which mediate the interactions of the Standard Model:

photon → electromagnetic interactions

$W^\pm, Z^0$ → weak interactions

gluon → strong interactions

All of these bosons have spin-1, a property tested by the fact that they produce fermion-fermion scattering amplitudes of the current-current ($-j_\mu j^\mu$) form.

In this lecture, I will address the question of how we write field equations describing these three forces. For electromagnetism, we have the example of Maxwell's equations. I will address the question: what is the generalization of Maxwell's equations which describes the strong and the weak interactions?

To begin, I would like to describe some principles for constructing relativistic equations. First of all, every particle should be described by a field. In the quantum theory, this field will become an operator that creates and destroys the particles, as I explained in the first lecture. Second, the field should carry indices for rotations and Lorentz transformations which reflect the spin of the particles. Spin-0 particles should be described by scalar fields $\phi(x)$. Spin-1 particles should be described by 4-vector fields $V_\mu(x)$. Spin-$\frac{1}{2}$ particles will be described by 2-component spinor fields $\psi_\alpha(x)$ $\alpha = 1,2$. Third, the field equations should be Lorentz-covariant. For example, the field equations for a vector field should naturally be a 4-vector of equations. This will guarantee that, if a particular function $V_\mu(t,\vec{x})$ solves the field equations, the Lorentz boost or rotation of $V_\mu(t,\vec{x})$ will also satisfy the field equations. Fourth, the equations should be <u>local</u>, that is, they should be simple differential equations of functions of $x^\mu = (t,\vec{x})$.

The most straightforward way to produce equations with these properties is to write a <u>Lagrangian</u> and derive the field equations from the principle that the Lagrangian should be stationary under variations. If the Lagrangian is <u>invariant</u> under a symmetry operation T and it is stationary at $V_\mu(x)$, it must also be stationary at $T[V_\mu(x)]$; thus we obtain covariant equations. As a bonus, <u>Noether's theorem</u> implies that there is an associated conservation law. If we write

$$\text{"action"} = S = \int dt\, L = \int d^4x\, \mathcal{L} \leftarrow \text{"Lagrange density"}$$

then the variational equations will be <u>local</u> and the associated conservation law will have the form

$$j^\mu = (\rho, \vec{j}) \qquad \partial_\mu j^\mu = 0 \qquad \text{or} \qquad \frac{d}{dt}\rho + \vec{\nabla}\cdot\vec{j} = 0$$

In particular, energy and momentum will always be conserved locally.

I'll give some examples with a scalar field. First, let $\phi(x)$ be real-valued. Then write

$$\partial_\mu = \frac{\partial}{\partial x^\mu} = \left(\frac{\partial}{\partial t}, \vec{\nabla}\right)$$

$$\mathcal{L} = \frac{1}{2}\left[(\partial_\mu\phi)(\partial^\mu\phi) - m^2\phi^2\right] \qquad \partial^\mu = \left(\frac{\partial}{\partial t}, -\vec{\nabla}\right)$$

The variation $\phi \to \phi + \delta\phi$ changes the action by

$$\delta S = \int d^4x \, \{ \partial_\mu(\delta\phi)\,\partial^\mu\phi - m^2\,\delta\phi\,\phi \}$$

$$= \int d^4x \;\; \delta\phi(x)\left[-\partial^2\phi - m^2\phi\right]$$

[In these lectures, I will always choose boundary conditions so that surface terms vanish.]

Then S is stationary, $\delta S = 0$, if

$$(\partial^2 + m^2)\phi = 0 \qquad \text{or} \qquad \left(\frac{d^2}{dt^2} - \nabla^2 + m^2\right)\phi = 0$$

This is the Klein-Gordon equation, the relativistic wave equation for a massive field.

Now let ϕ be complex-valued. Write

$$\mathcal{L} = \partial_\mu\phi^*\,\partial^\mu\phi - m^2\phi^*\phi$$

The variation of S is

$$\delta S = \int d^4x \left(\delta\phi^*\left[(-\partial^2 - m^2)\phi\right] + \left[(-\partial^2 - m^2)\phi^*\right]\delta\phi\right)$$

so $\delta S = 0 \Rightarrow \quad (\partial^2 + m^2)\phi = (\partial^2 + m^2)\phi^* = 0$

In addition, the Lagrangian is invariant under the symmetry transformation

$$\phi \to e^{i\alpha}\phi \qquad \phi^* \to e^{-i\alpha}\phi^*$$

where α is a constant phase. Noether's theorem implies that there must be an associated conserved current. It is

$$j^\mu = \frac{1}{2i}\left[\phi^* \partial_\mu \phi - (\partial_\mu \phi^*)\phi\right]$$

we can check that

$$\partial_\mu j^\mu = \frac{1}{2i}\left[\partial_\mu\phi^* \partial^\mu \phi + \phi^* \partial^2 \phi - \partial^2 \phi^* \phi - \partial_\mu \phi^* \partial^\mu \phi\right]$$

$$= \frac{1}{2i}\left[\phi^*(\partial^2\phi) - \partial^2\phi^*\phi\right] = \frac{1}{2i}\left[\phi^*(-m^2\phi) - (-m^2\phi^*)\phi\right]$$

$$= 0$$

The equation for a relativistic fermion is derived from

$$\mathcal{L} = \psi^\dagger i\sigma^\mu \partial_\mu \psi$$

where ψ is a 2-component field $\psi = \begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$ and $\sigma^\mu = (1, \vec{\sigma})$, where $\vec{\sigma}$ are the Pauli sigma matrices. Let $\bar{\sigma}^\mu = (1, -\vec{\sigma})$. The field equation following from this Lagrangian is

$$(i\sigma^\mu \partial_\mu)\psi = 0$$

To understand the nature of this equation, look for plane wave solution

$$\psi = \xi e^{-ip^\mu x_\mu} \qquad \xi = \text{a spinor}$$

Then $(p\cdot\sigma)\xi = 0$. First note that

$$(p\cdot\bar{\sigma})(p\cdot\sigma) = (p^0 + \vec{p}\cdot\vec{\sigma})(p^0 - \vec{p}\cdot\vec{\sigma}) = p^{0\,2} - |\vec{p}|^2 = p^2$$

So this equation implies

$$p^2 = 0 \quad \Rightarrow \text{ the fermion is } \underline{\text{massless}}$$

Now set $p^\mu = (E, 0, 0, E)$; the equation becomes

$$E\,(1 - \sigma^3)\,\xi = 0 \quad \Rightarrow \quad \xi = \begin{pmatrix}1\\0\end{pmatrix}$$

So this is the equation for a <u>massless</u>, <u>right-handed</u>, spin-½ particle. The corresponding equation for a <u>massless</u>, <u>left-handed</u> spin-½ particle is derived from

$$\mathcal{L} = \psi_L^\dagger\, i\bar{\sigma}\cdot\partial\, \psi_L$$

The Lagrangian for a <u>massive</u> spin ½ particle turns out to be

$$\mathcal{L} = \psi_R^\dagger\, i\sigma\cdot\partial\,\psi_R + \psi_L^\dagger\, i\bar{\sigma}\cdot\partial\,\psi_L - m\,(\psi_R^\dagger\psi_L + \psi_L^\dagger\psi_R)$$

The variation with respect to $\psi_R^\dagger, \psi_L^\dagger$ leads to an equation for a 4-component field

$$\Psi = \begin{pmatrix}\psi_R\\ \psi_L\end{pmatrix}$$

2×2 blocks

namely

$$\left[\, i\begin{pmatrix} & \sigma^\mu \\ \bar{\sigma}^\mu & \end{pmatrix}\partial_\mu - m\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\right]\Psi = 0$$

By multiplying by $\left[\, i\begin{pmatrix} & \bar{\sigma}^\mu \\ \sigma^\mu & \end{pmatrix} + m\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\right]$, you can show that the solutions to this equation satisfy

$$(-\partial^2 - m^2)\,\Psi = 0.$$

Now we have to fit the electromagnetic field into this structure. In quantum mechanics, the fundamental fields that couple to the Schrödinger wavefunction are not $\vec{E}$ and $\vec{B}$ but rather the potentials ϕ and $\vec{A}$. These organize themselves into a 4-vector

$$A^\mu = (\phi, \vec{A})$$

Let
$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \qquad F^{\mu\nu} = -F^{\nu\mu}$$

This object is called the "field strength tensor". Its components are

$$F^{0i} = \frac{\partial}{\partial t} A^i - (-\nabla^i) A^0 = \nabla^i \phi + \frac{\partial}{\partial t} A^i = -E^i$$

$$F^{ij} = \left(-\frac{\partial}{\partial x^i}\right) A^j - \left(-\frac{\partial}{\partial x^j}\right) A^i = -(\nabla^i A^j - \nabla^j A^i) = -\epsilon^{ijk} B^k$$

The fact that we can write $\vec{E}, \vec{B}$ in terms of $(\phi, \vec{A})$ automatically implies the homogeneous Maxwell equations

$$\vec{\nabla} \cdot \vec{B} = 0 \qquad \frac{\partial \vec{B}}{\partial t} + \vec{\nabla} \times \vec{E} = 0$$

The inhomogeneous Maxwell equations follow from the Lagrangian

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu} F^{\mu\nu}) = \frac{1}{2}[(\vec{E})^2 - (\vec{B})^2]$$

To see this, vary $\mathcal{L}$ with respect to $A^\mu \to A^\mu + \delta A^\mu$

$$\delta S = \int d^4x \; -\frac{1}{2}(\partial_\mu \delta A_\nu - \partial_\nu \delta A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu)$$

terms are equal when contracted with $F^{\mu\nu}$

so $$\delta S = \int d^4x \;\; \delta A_\nu (\partial_\mu F^{\mu\nu})$$

and we get

$$\partial_\mu F^{\mu\nu} = 0 \quad \Rightarrow \quad \begin{array}{ll} \nu = 0 & \vec{\nabla}\cdot\vec{E} = 0 \\ \nu = i & \partial_0 F^{0i} + \nabla^j F^{ji} = 0 \\ & \text{or} \;\; -\frac{d}{dt}E^i + (\nabla\times B)^i = 0 \end{array}$$

These are the inhomogeneous Maxwell equations with $j^\mu = 0$.

Now let's discuss the coupling of electromagnetic fields to matter. In the Schrödinger equation, this is achieved by the mysterious "minimal coupling" prescription, which I write relativistically as

$$\partial^2 \psi \longrightarrow (\partial_\mu - ieQA_\mu)^2 \psi \qquad [\text{Remember } \hbar = c = 1.]$$

(Q = charge of ψ particles.)

This mysterious transformation has an equally mysterious virtue: If A_μ is replaced by a new vector potential

$$A'_\mu = A_\mu + \frac{1}{e}\partial_\mu \alpha(x) \qquad (\text{a "gauge transformation"})$$

and ψ is replaced by

$$\psi'(x) = e^{iQ\alpha(x)}\psi(x)$$

then the new ψ' satisfies the Schrödinger equation with the new A'_μ. In particular,

$$(\partial_\mu - ieQA'_\mu)\psi' = e^{iQ\alpha(x)}[\partial_\mu + Q\partial_\mu\alpha(x) - ieQA_\mu - iQ\partial_\mu\alpha]\psi$$

$$= e^{iQ\alpha(x)}(\partial_\mu - ieQA_\mu)\psi$$

so the old Schrödinger equation for $\psi(x)$ just gets an overall phase $e^{iQ\alpha(x)}$.

This argument, which you find in all books on quantum mechanics, makes much more sense if you run it the other way. I will apply it to complex-valued fields $\phi(x)$. I wrote earlier a Lagrangian with a global symmetry

$$\phi(x) \to e^{i\alpha}\phi(x) \qquad \alpha = \text{constant.}$$

Let's now try to write a Lagrangian with a local symmetry

$$\phi(x) \to e^{i\alpha(x)}\phi(x)$$

Try, for example,

$$\mathcal{L} = \partial_\mu\phi^* \partial^\mu\phi - m^2\phi^*\phi$$

The mass term is invariant under the local symmetry, but the terms with derivatives change uncontrollably. To give these terms a better property, write

$$\mathcal{L} = [(\partial_\mu + ieA_\mu)\phi^*][(\partial_\mu - ieA_\mu)\phi] - m^2\phi^*\phi$$

where A_μ is a vector field that transforms as

$$A_\mu \to A_\mu + \frac{1}{e}\partial_\mu\alpha(x) \qquad \text{"gauge transformation"}$$

Then

$$(\partial_\mu - ieA_\mu)\phi \to e^{i\alpha(x)}[(\partial_\mu - ieA_\mu)\phi]$$

& $\mathcal{L}$ is completely invariant. However, we are not done, because A_μ does not have a sensible equation of motion. To produce this, we must write some terms in $\mathcal{L}$ that includes $(\partial_\mu A_\nu)^2$. But, this term must be invariant with respect to the gauge transformation of A_μ. Fortunately, we have the answer at hand:

$$F_{\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu) \quad \text{is \underline{unchanged} by } A_\mu \to A_\mu + \tfrac{1}{e}\partial_\mu \alpha$$

so

$$\mathcal{L} = -\tfrac{1}{4}(F_{\mu\nu}F^{\mu\nu}) + (\partial^\mu + ieA^\mu)\phi^*(\partial_\mu - ieA_\mu)\phi - m^2\phi^*\phi$$

is completely invariant. Varying this with respect to A_μ, we find the field equation

$$\partial_\mu F^{\mu\nu} = e j^\nu$$

where $j^\nu = -i[\phi^*(\partial^\nu - ieA^\nu)\phi - [(\partial^\nu + ieA^\nu)\phi^*]\phi]$

which is just the current of p. 4 with the "minimal coupling" modification.

The useful quantity

$$D_\mu = (\partial_\mu - ieA_\mu)$$

is called the "gauge-covariant derivative". For a field with the transformation law

$$\phi \to e^{iQ\alpha(x)}\phi$$

we write

$$D_\mu\phi = (\partial_\mu - iQeA_\mu)\phi$$

then

$$D_\mu\phi \to e^{iQ\alpha(x)} D_\mu\phi$$

Q is precisely the <u>electric</u> <u>charge</u> of the particles created or destroyed by $\phi(x)$.

To obtain a theory in which the spin-1 boson can change flavor, as the W does, we need to generalize this structure. Yang and Mills proposed a generalization in which the local symmetry is any unitary transformation group. Let me introduce a little notation for such groups, and then I'll write down the Yang-Mills Lagrangian.

Let $U(\alpha)$ be a set of continuous unitary transformations. These can be parametrized in the form

$$U(\alpha) = e^{i\alpha^a t^a}$$

where the t^a are Hermitian matrices. The $U(\alpha)$ infinitesimally close to $U=1$ are written

$$U(\alpha) = (1 + i\alpha^a t^a + \cdots)$$

Any large transformation can be built up from infinitesimal transformations:

$$U(\alpha) = \lim_{N\to\infty} \left(1 + i\frac{\alpha^a}{N} t^a\right)^N$$

The matrices t^a typically do not commute with one another. But we might have a set of t^a s.t. the commutators are linear combinations of elements of the set. I'll write this relation

$$[t^a, t^b] = i f^{abc} t^c$$

The numbers $\{f^{abc}\}$ are called the structure constants. Such a set $\{t^a\}$ is called a Lie algebra. The associated unitary transformation

$$e^{i\alpha^a t^a}$$

form a group, called a <u>Lie group</u>, whose multiplication law is determined by the f^{abc}.

As an example, consider the matrices $\tau^a = \frac{\sigma^a}{2}$, $a = x, y, z$.

Their Lie Algebra is

$$[\tau^a, \tau^b] = i\,\epsilon^{abc}\,\tau^c$$

The matrices

$$U(\alpha) = e^{i\alpha^a\tau^a} = e^{i\alpha^a\sigma^a/2}$$

form the group $SU(2)$, which is isomorphic (locally) to the group of rotations in 3 dimensions. In quantum mechanics, $U(\alpha)$ is a rotation on a spinor, $\tau^a = \frac{1}{2}\sigma^a$ represents the angular momentum J^a on a spinor, and the Lie algebra is the algebra of commutation relations of angular momentum.

For a general Lie algebra, I will normalize the generators to

$$\operatorname{tr} t^a t^b = \tfrac{1}{2}\,\delta^{ab}$$

then

$$f^{abc} = -2i\,\operatorname{tr}\left([t^a, t^b]\,t^c\right)$$

It is not hard to see that this expression is totally antisymmetric on the three indices.

Now, let $\phi(x)$ be a field with the local transformation law

$$\phi(x) \longrightarrow U(\alpha(x))\,\phi(x) = \phi'(x)$$

We can easily construct a mass term which is locally invariant: $\quad \phi^\dagger\phi \rightarrow \phi^\dagger\phi$

As in the case of electrodynamics, it is the terms with derivatives that pose a problem. To solve the problem, introduce a vector field for each matrix t^a and write the covariant derivative

$$D_\mu \phi = (\partial_\mu - ig A^a_\mu t^a)\, \phi$$

g is the new coupling constant. Now look for a transformation law

$$A^a_\mu(x) \to A^{a\prime}_\mu(x)$$

such that

$$D_\mu\phi \to (\partial_\mu - ig A'^a_\mu t^a)\phi' = U(\alpha)\,[(\partial_\mu - ig A^a_\mu t^a)\phi]$$

since

$$\partial_\mu \phi' = U(\alpha)\,\partial_\mu\phi + (\partial_\mu U(\alpha))\,\phi$$

what we need is

$$A^a_\mu t^a \to A'^a_\mu t^a = U(\alpha) A^a_\mu t^a U^{-1}(\alpha) - \frac{i}{g}\,\partial_\mu U(\alpha)\, U^{-1}(\alpha)$$

Maybe it is not so obvious that this expression retains the form $A^a_\mu t^a$. For an infinitesimal transformation $U(\alpha) = 1 + i\delta\alpha^a t^a$

$$A'^a_\mu t^a = (1 + i\delta\alpha^b t^b)\, A^a_\mu t^a\, (1 - i\delta\alpha^b t^b) - \frac{i}{g}(i\,\partial_\mu \delta\alpha^a t^a)$$

$$= A^a_\mu t^a + i\,[\delta\alpha^b t^b, A^c_\mu t^c] + \frac{1}{g}\,\partial_\mu \delta\alpha^a t^a$$

$$= A^a_\mu t^a + \frac{1}{g}(\partial_\mu \delta\alpha^a)\, t^a - f^{abc}\,\delta\alpha^b A^c_\mu\, t^a$$

so infinitesimally

$$\delta A^a_\mu = \frac{1}{g}\,\partial_\mu \delta\alpha^a - f^{abc}\,\delta\alpha^b A^c_\mu$$

and we can iterate this to build the finite transformation.

Finally, we must build the field strength tensor. Yang and Mills proposed

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

or

$$F^a_{\mu\nu} t^a = \partial_\mu A^a_\nu t^a - \partial_\nu A^a_\mu t^a - ig [A^b_\mu t^b, A^c_\nu t^c]$$

to compute the gauge transformation of F, insert

$$\delta A^a_\mu t^a = \frac{1}{g} \partial_\mu \alpha^a t^a + i [\alpha^a t^a, A^b_\mu t^b]$$

$$\begin{aligned}
\delta F^a_{\mu\nu} t^a = & \frac{1}{g} \partial_\mu \partial_\nu \alpha \cdot t + i [\partial_\mu \alpha \cdot t, A_\nu \cdot t] + i [\alpha \cdot t, \partial_\mu A_\nu \cdot t] \\
& - \frac{1}{g} \partial_\nu \partial_\mu \alpha \cdot t - i [\partial_\nu \alpha \cdot t, A_\mu \cdot t] - i [\alpha \cdot t, \partial_\nu A_\mu \cdot t] \\
& - i [\partial_\mu \alpha \cdot t, A_\nu \cdot t] - i [A_\mu \cdot t, \partial_\nu \alpha \cdot t] \\
& \quad - ig \cdot i \cdot [[\alpha \cdot t, A_\mu \cdot t], A_\nu \cdot t] + [A_\mu \cdot t, [\alpha \cdot t, A_\nu \cdot t]] \\
= & \; i [\alpha \cdot t, \partial_\mu A_\nu \cdot t - \partial_\nu A_\mu \cdot t - ig [A_\mu \cdot t, A_\nu \cdot t]] \\
= & - f^{abc} t^a \alpha^b F^c_{\mu\nu}
\end{aligned}$$

so $$\delta F^a_{\mu\nu} = - f^{abc} \alpha^b F^c_{\mu\nu}$$

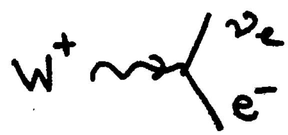
and because f^{abc} is completely antisymmetric $\delta (F^a_{\mu\nu} F^{\mu\nu a}) = 0$

Thus $$\mathcal{L} = -\frac{1}{4} (F^a_{\mu\nu} F^{\mu\nu a}) + D_\mu \phi^\dagger D^\mu \phi - m^2 \phi^\dagger \phi$$

is a locally invariant Lagrangian, and we can add in any other fields that transform under the symmetry group by replacing

∂_μ by $D_\mu = (\partial_\mu - ig A^a_\mu t^a)$.

To build a theory that includes the transition



consider a Yang-Mills theory with the symmetry group SU(2) which includes the transformation:

$$\begin{pmatrix}\nu\\ e^-\end{pmatrix}_L \to e^{-i\alpha\cdot\vec{\sigma}/2}\begin{pmatrix}\nu\\ e\end{pmatrix}_L$$

for the left-handed components of the fields only. Then if we write

$$L = \begin{pmatrix}\nu\\ e\end{pmatrix}_L$$

the kinetic terms for the electron and neutrino will be

$$\mathcal{L} = L^\dagger i\bar{\sigma}^\mu D_\mu L + e_R^\dagger i\sigma^\mu \partial_\mu e_R$$

Notice that this is perfectly consistent and invariant to gauge transformations. There is a problem in writing a mass term for the electron if e_L and e_R are assigned different quantum numbers. I will discuss this problem in Lecture 9.

To see the structure of the $W\nu e$ coupling, write out the covariant derivative more explicitly. Let

$$\sigma^+ = \tfrac{1}{2}(\sigma^x + i\sigma^y) \qquad \sigma^- = \tfrac{1}{2}(\sigma^x - i\sigma^y)$$

$$W^+_\mu = \tfrac{1}{\sqrt{2}}(A^x_\mu - iA^y_\mu) \qquad W^-_\mu = \tfrac{1}{\sqrt{2}}(A^x_\mu + iA^y_\mu)$$

(These fields <u>destroy</u> the W^+ and W^-, respectively) then $D_\mu = \partial_\mu - ig[A_\mu^x \frac{\sigma^x}{2} + A_\mu^y \frac{\sigma^y}{2} + A_\mu^z \frac{\sigma^z}{2}]$

$$= \partial_\mu - i\frac{g}{\sqrt{2}}(W_\mu^+ \sigma^+ + W_\mu^- \sigma^-) - ig\, A_\mu^3 \frac{\sigma^3}{2}$$

Inserting this into the Lagrangian, we have

$$\mathcal{L} = \ldots + \frac{g}{\sqrt{2}} W_\mu^+ (\nu_L^\dagger \bar{\sigma}^\mu e_L) + \frac{g}{\sqrt{2}} W_\mu^- (e_L^\dagger \bar{\sigma}^\mu \nu_L)$$
$$+ g A_\mu^3 \left(\tfrac{1}{2} \nu_L^\dagger \bar{\sigma}^\mu \nu_L - \tfrac{1}{2} e_L^\dagger \bar{\sigma}^\mu e_L\right)$$

In the notation of lecture 5,

$j_{\mu L}^+ = \nu_L^\dagger \bar{\sigma}^\mu e_L + \cdots$ is a current which changes flavor and raises the charge by 1 unit

$j_{\mu L}^- = e_L^\dagger \bar{\sigma}^\mu \nu_L + \cdots$ is a current which changes flavor and lowers the charge by 1 unit

$j_{\mu L}^3 = \frac{1}{2} \nu_L^\dagger \bar{\sigma}^\mu \nu_L - \frac{1}{2} e_L^\dagger \bar{\sigma}^\mu e_L + \cdots$

is the neutral current corresponding to I^3.

By assigning all of the left-handed species to SU(2) doublets

$$\begin{pmatrix}\nu_e\\ e^-\end{pmatrix}_L \quad \begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix}_L \quad \begin{pmatrix}\nu_\tau\\ \tau^-\end{pmatrix}_L \quad \begin{pmatrix}u\\ d\end{pmatrix}_L \quad \begin{pmatrix}c\\ s\end{pmatrix}_L \quad \begin{pmatrix}t\\ b\end{pmatrix}_L$$

we add the other left-handed quarks and leptons to these currents in a straightforward way. Then, for example, the full structure of $j_{\mu L}^+$ is

$$j_{\mu L}^+ = \nu_{Le}^\dagger \bar{\sigma}^\mu e_L + \nu_{L\mu}^\dagger \bar{\sigma}^\mu \mu_L + \nu_{L\tau}^\dagger \bar{\sigma}^\mu \tau_L$$
$$+ u_L^\dagger \bar{\sigma}^\mu d_L + c_L^\dagger \bar{\sigma}^\mu s_L + t_L^\dagger \bar{\sigma}^\mu b_L$$

It is clear that we are very close to deriving the formula for the weak interaction amplitudes that I wrote in the previous lecture. In fact, if I assign a common mass m_W to $W^+ W^-$ and A^3 and approximate

$$\text{[illegible]} = \frac{1}{q^2 - m_W^2} \cong -\frac{1}{m_W^2}$$

I will derive

$$\mathcal{M} = -\frac{g^2}{2}\frac{1}{m_W^2}\left(j_L^{+\mu} j^-_{\mu L} + j^3_{\mu L} j_L^{\mu 3}\right)$$

This is the symmetrical but experimentally incorrect expression that appears on p. 17 of the previous lecture.

To get the correct expression, Glashow, Weinberg, Salam and Ward proposed the following structure. Add a phase rotation symmetry acting on all fields:

$$L \to e^{iY_L\beta} L \qquad e_R \to e^{iY_e\beta} e_R \quad \text{etc.}$$

Make $\beta(x)$ a local symmetry. The group of phase rotations is called $U(1)$ so now we have an $SU(2)\times U(1)$ model. The charges Y_i are called the "hypercharges".

If $\alpha^a(x)$ and $\beta(x)$ are local symmetries, we need 4 vector fields

$$A^a_\mu \quad a = x, y, z \qquad B_\mu$$

The A^a_μ couple as above; the B_μ couples like a photon, with charges Y_i. Call its coupling constant g'.

Now write the following mass matrix for these four fields:

$$\underset{\sim}{m}^2 \begin{pmatrix} A^x \\ A^y \\ A^z \\ B \end{pmatrix} = \frac{1}{4} v^2 \left(\begin{array}{ccc|c} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & g^2 & -gg' \\ \hline 0 & 0 & -gg' & (g')^2 \end{array}\right) \begin{pmatrix} A^x \\ A^y \\ A^z \\ \hline B \end{pmatrix}$$

where v^2 is a parameter with the dimensions of (mass)^2. This matrix assigns A^x and A^y — or the linear combinations W^+ and W^-, the mass

$$m_W^2 = \frac{1}{4} g^2 v^2$$

On the other hand, A^z is no longer a mass eigenstate. To find the eigenstates we must diagonalize

$$\left(\begin{array}{c|c} g^2 & -gg' \\ \hline -gg' & (g')^2 \end{array}\right) \begin{pmatrix} A^z \\ B \end{pmatrix}$$

The eigenstates are:

$$A_\mu = \frac{1}{\sqrt{g^2+(g')^2}} \left(g' A^z_\mu + g B_\mu \right) \qquad m_A^2 = 0$$

$$Z_\mu = \frac{1}{\sqrt{g^2+(g')^2}} \left(g A^z_\mu - g' B_\mu \right) \qquad m_Z^2 = \frac{v^2}{4}[g^2+(g')^2]$$

Since A_μ has zero mass, we can identify it with the <u>electromagnetic</u> field. Z_μ is the field of the neutral currents. The $SU(2)\times U(1)$ model thus has turned out to be a model of both <u>weak</u> and electromagnetic interactions.

To finish this discussion, let's derive the couplings of

A_μ and Z_μ. The covariant derivative is

$$D_\mu = \left(\partial_\mu - i g A^3_\mu \tau^3 - i g' B_\mu Y \right)$$

with $\tau^3 = \sigma^3/2 = I^3$ in the terminology of the previous lecture.

Substitute

$$A^3_\mu = \frac{1}{\sqrt{g^2+g'^2}} \left(g' A_\mu + g Z_\mu \right)$$

$$B_\mu = \frac{1}{\sqrt{g^2+g'^2}} \left(g A_\mu - g' Z_\mu \right)$$

this gives

$$D_\mu = \Big(\partial_\mu - i A_\mu \frac{g g'}{\sqrt{g^2+g'^2}} \left(I^3 + Y \right)$$

$$- i Z_\mu \left[\frac{g^2}{\sqrt{g^2+g'^2}} I^3 - \frac{(g')^2}{\sqrt{g^2+g'^2}} Y \right] \Big)$$

The coupling constant of A_μ must be

$$e = \frac{g g'}{\sqrt{g^2+g'^2}}$$

and the coefficient must be the electric charge.

$$Q_f = \left(I^3_f + Y_f \right)$$

Writing in the second term

$$Y_f = Q_f - I^3_f$$

The Z_μ coupling becomes: $Z_\mu \sqrt{g^2+g'^2} \left[I^3 - \left(\frac{(g')^2}{g^2+g'^2} \right) Q \right]$

now define $\quad \cos\theta_w = \frac{g}{\sqrt{g^2+g'^2}} \qquad \sin\theta_w = \frac{g'}{\sqrt{g^2+g'^2}}$

now the complete covariant derivative is

$$\left(\partial_\mu - ieQ A_\mu - i \underbrace{\frac{e}{\cos\theta_w \sin\theta_w}}_{= \frac{g}{\cos\theta_w}} (I^3 - \sin^2\theta_w Q) Z_\mu \right)$$

If we use this coupling to Z_μ instead of the coupling of A^3 on p.15 to derive the neutral current amplitude, we obtain

$$\mathcal{M} = - \frac{g^2}{2} \frac{1}{m_W^2} j_L^{+\mu} j^-_{\mu L} - \frac{g^2}{2\cos^2\theta_w} \frac{1}{m_Z^2} \left(j^3_\mu - \sin^2\theta_w j_{\mu EM} \right)^2$$

Finally, note from the mass relations on p. 17 that

$$m_W^2 = m_Z^2 \cos^2\theta_w$$

Thus we do obtain

$$\mathcal{M} = - \frac{g^2}{2m_W^2} \left[j_L^{+\mu} j^-_{\mu L} + \left(j^3_\mu - \sin^2\theta_w j_{\mu EM} \right)^2 \right]$$

from the $SU(2) \times U(1)$ model.

To derive this formula, we needed a big assumption in addition to the gauge-invariant structure: We needed the special form of the mass matrix given on p. 17. On the other hand, the mass term is in any event a mystery, because a term like

$$m^2 A_\mu A^\mu$$

cannot be gauge invariant. Experiment guides us to this particular form of the mass term. The origin of this term, though, is still a mystery. I will have more to say about this issue in lecture 9.

Lecture 7: July 31

In the previous lecture, I described the mathematical form of the field equations for the photon and the $W^\pm$ and Z bosons. These equations arose in a unified way from a locally-gauge-invariant Lagrangian based on the group $SU(2) \times U(1)$. In case these equations went by very quickly, I would like to recapitulate the main results:

The $SU(2) \times U(1)$ theory contained two coupling constants, g and g'. We defined the "weak mixing angle" θ_W by the formulae:

$$\cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}} \qquad \sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}}$$

We derived that the coupling constant of the photon is given by

$$e = \frac{gg'}{\sqrt{g^2+g'^2}} = \frac{g}{\cos\theta_W} = \frac{g'}{\sin\theta_W}$$

The photon comes out massless, and the $W^\pm$ and Z^0 bosons have masses

$$m_W = \frac{g}{2}v \qquad m_Z = \sqrt{g^2+g'^2}\,\frac{v}{2}$$

where v is a parameter with the dimensions of mass.

Each handed fermion in the Standard Model is assigned $SU(2)\times U(1)$ quantum numbers. The left-handed particles have $SU(2)$ quantum number corresponding to spin-$\frac{1}{2}$;

in particular, $I^3 = \pm\frac{1}{2}$. Right-handed particle have $I^3 = 0$. Each particle is also assigned a hypercharge Y. We derived that the electric charge Q is given by

$$Q = I^3 + Y.$$

Thus, we have the hypercharge assignments:

	ν_{eL}	e^-_L	e^-_R	u_L	d_L	u_R	d_R
I^3	$\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	0
Y	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$

and similarly for the heavier quarks and leptons. Finally, the coupling of the Z_μ to fermions has the form (after we eliminate Y in favor of Q):

$$Z_\mu \cdot \frac{e}{\cos\theta_W \sin\theta_W}\left(I^3 - \sin^2\theta_W Q\right)$$

The $SU(2) \times U(1)$ model received strong support in 1982 when the W and Z bosons were discovered at the CERN $p\bar{p}$ collider, in the reactions such as

$$u\bar{d} \to W^+ \to e^+\nu \text{ or } \mu^+\nu$$

$$u\bar{u} \to Z^0 \to e^+e^- \text{ or } \mu^+\mu^-$$

The neutrino is not observed, but its presence is inferred from the presence of unbalanced transverse momentum in visible particles. The best value of the W mass now comes from the $p\bar{p}$ collider experiments at Fermilab, and the best Z^0 mass comes from e^+e^- annihilation experiments at LEP. These are

$$m_W = 80.3 \text{ GeV} \qquad\qquad m_Z = 91.187 \text{ GeV}$$

The mass formulae above predicted

$$\frac{m_W}{m_Z} = \cos\theta_W \quad \text{or} \quad \sin^2\theta_W = 1 - \frac{m_W^2}{m_Z^2}$$

from these values, we find

$$\sin^2\theta_W = 0.224$$

in reasonable agreement with the value of $\sin^2\theta_W$ from the neutrino experiments discussed in lecture 5.

From the W and Z masses, we can also infer the intrinsic strength of the weak interactions. From β-decay rates and neutrino scattering, we can find the coefficient of the low-energy current-current amplitude

$$G_F = 1.166 \times 10^{-5} (\text{GeV})^{-2}$$

From the relation

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 m_W^2}$$

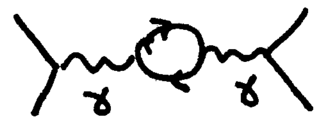
we find

$$g^2 = 0.43 \Rightarrow \quad \alpha_w = \frac{g^2}{4\pi} = \frac{1}{29.5}$$

This is another prediction of the $SU(2)\times U(1)$ model: the weak-interaction coupling should be larger than the electromagnetic coupling α. In fact, we should have

$$\frac{e^2}{g^2} = \frac{\alpha}{\alpha_w} = \sin^2\theta_w$$

Evaluating with $\alpha = 1/137$, we find $\sin^2\theta_w = 0.215$. However, I will argue in lecture 8 that electromagnetism gets stronger at short distances, due to processes such as



in which the photon converts, as a quantum fluctuation, to e^+e^-, $\mu^+\mu^-$, $\tau^+\tau^-$, $q\bar{q}$ pairs. I will calculate this effect in the next lecture and defend the estimate

$$\alpha \cong \frac{1}{128}. \qquad \text{at distances} \sim \frac{1}{m_Z}$$

Then we get $\quad \sin^2\theta_w = \dfrac{\alpha}{\alpha_w} = 0.230$

and all of our estimates of this quantity are in close agreement.

To make more precise studies of the electroweak interactions, experiments at LEP and at the SLC collider at SLAC have observed the Z^0 in e^+e^- annihilation. The Z^0 is a <u>resonance</u> in e^+e^- annihilation; at $E_{cm} \simeq m_Z$, the cross section for e^+e^- annihilation increases by a factor of several hundred. If we tune the collider to this energy, we have a huge sample of events to study. I have already shown you data from this sample which bears on the properties of the strong interactions. But the most important feature of these events is that they correspond (to an excellent approximation) to the process

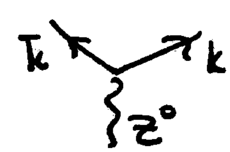
$$e^+e^- \rightarrow Z^0 \text{ (at rest)} \rightarrow \ell^+\ell^-,\ \nu\bar{\nu},\ q\bar{q}$$

Thus, these experiments allow us to hold the Z^0 in our hands and visualize its properties.

Before discussing these experiments, I would like to calculate the basic properties of the Z^0 from the $SU(2)\times U(1)$ theory. This is not very difficult. The Z^0 is a spin-1 boson which couples to a current, so its amplitudes are very similar to those of the photon which we discussed in the first lecture. Let $\vec{\varepsilon}_Z$ be the polarization of the Z^0; for Z^0's created in e^+e^- annihilation

$$e^-_L e^+_R \Rightarrow \quad \vec{\varepsilon}_Z = \tfrac{1}{\sqrt{2}}(1,-i,0)$$

$$e^-_R e^+_L \rightarrow \quad \vec{\varepsilon}_Z = \tfrac{1}{\sqrt{2}}(1,+i,0)$$



Then the amplitude for Z^0 decay to left-handed species is

$$\mathcal{M}(Z^0 \rightarrow f_L\, \bar{f}_R) = \sqrt{2}\ \frac{e\, m_Z}{\cos\theta_w \sin\theta_w}\left(I^3_f - \sin^2\theta_w Q_f\right)\vec{\varepsilon}_Z\cdot\vec{\varepsilon}^{\,*}_-(k)$$

where $\vec{\varepsilon}_-(k) = \frac{1}{\sqrt{2}}(\cos\theta, -i, -\sin\theta)$ is the vector which gives helicity $=(-1)$ along the $f\bar{f}$ production axis.

(The factor $\sqrt{2}$ is the square root of the coefficient from p.19 of lecture 1.) Similarly, for the decay to right-handed species:

$$\mathcal{M}(Z^0 \to f_R \bar{f}_L) = \sqrt{2}\, \frac{e\, m_Z}{\cos\theta_w \sin\theta_w} \left(-\sin^2\theta_w Q \right) \vec{\varepsilon}_Z \cdot \vec{\varepsilon}_+^{\,*}(k)$$

The relative strength of the couplings are controlled by the quantities

$$\mathcal{Q}_{L,R}(f) = I^3_f - \sin^2\theta_w\, Q_f$$

(with $I^3 = 0$ for f_R). The squared matrix elements are then

$$|\mathcal{M}(Z^0 \to f\bar{f})|^2 = \frac{1}{2} \frac{e^2 m_Z^2}{\cos^2\theta_w \sin^2\theta_w} \mathcal{Q}^2_{L,R}(f) \, (1 \pm \cos\theta)^2$$

with $(1+\cos\theta)^2$ for $e^-_L e^+_R \to f_L \bar{f}_R$, $e^-_R e^+_L \to f_R \bar{f}_L$ and $(1-\cos\theta)^2$ for the opposite cases.

The decay rate of the Z^0 into a handed species is given by

$$\Gamma(Z^0 \to f\bar{f}) = \frac{1}{2m_Z} \int \frac{d^3k}{(2\pi)^3} \frac{1}{2k^0} \int \frac{d^3\bar{k}}{(2\pi)^3} \frac{1}{2\bar{k}^0} (2\pi)^4 \delta(p_Z - (k+\bar{k})) \cdot |\mathcal{M}(Z^0 \to f\bar{f})|^2$$

$$= \frac{1}{2m_Z} \frac{1}{16\pi} \int_{-1}^{1} d\cos\theta \; \frac{1}{2} \frac{g^2 m_Z^2}{\cos^2\theta_w} \mathcal{Q}^2(f) \, (1 \pm \cos\theta)^2$$

$$= \frac{\alpha_w}{16 \cos^2\theta_w} m_Z \cdot \frac{8}{3} \mathcal{Q}^2(f)$$

so

$$\Gamma(Z^0 \to f\bar{f}) = \underbrace{\frac{1}{6} \frac{\alpha_w}{\cos^2\theta_w} m_Z}_{0.67\ \text{GeV}} \cdot \mathcal{Q}^2(f)$$

so we can tabulate the contribution to the Z^0 width:

	Q_L	Q_R	$Q_L^2+Q_R^2$	$\Gamma(Z\to f\bar{f})$
ν_e	$\frac{1}{2}$	x	$\frac{1}{4}$	167 MeV
e^-	$-\frac{1}{2}+\sin^2\theta_w$	$\sin^2\theta_w$	0.126	84 MeV
u	$\frac{1}{2}-\frac{2}{3}\sin^2\theta_w$	$-\frac{2}{3}\sin^2\theta_w$	0.144	298 MeV
d	$-\frac{1}{2}+\frac{1}{3}\sin^2\theta_w$	$\frac{1}{3}\sin^2\theta_w$	0.185	384 MeV

For the quarks, I should include in Γ the color factor 3. In fact, I multiplied by 3.1, incorporating a small correction due to gluons that I will discuss in Lecture 8.

Considering Z decays to (t is too heavy)

$\nu_e \quad e^- \quad \nu_\mu \quad \mu^- \quad \nu_\tau \quad \tau^- \quad u \quad d \quad s \quad c \quad b$

we find the prediction:

$$\Gamma_{Z,tot} = 2.501 \text{ GeV}$$

We also predict the ratios of the various contributions, e.g.

$$R = \frac{\Gamma(Z\to \text{hadrons})}{\Gamma(Z\to\mu^+\mu^-)} \cong 20.8$$

Because Q_L^2 is larger than Q_R^2, the Z^0 decays more often to $f_L\bar{f}_R$ than to $f_R\bar{f}_L$. We can codify this as a helicity asymmetry:

$$A_f = \frac{\Gamma(Z\to f_L\bar{f}_R) - \Gamma(Z\to f_R\bar{f}_L)}{\Gamma(Z\to f_L\bar{f}_R) + \Gamma(Z\to f_R\bar{f}_L)} = \frac{g_L^2 - g_R^2}{g_L^2 + g_R^2}$$

From the table above

	$\underline{e,\mu,\tau}$	$\underline{u,c}$	$\underline{d,s,b}$
$A_f =$	0.159	0.67	0.94

For the leptons, a good approximate formula is

$$A_\ell \cong 8\cdot\left(\tfrac{1}{4} - \sin^2\theta_W\right)$$

For d, s, b, the asymmetry is almost maximal.

I would like to set one more formula, that for the forward-backward asymmetry in $e^+e^-\to Z^0\to f\bar{f}$. This quantity is defined as:

$$A_{FB}(f) = \frac{\sigma(\cos\theta>0) - \sigma(\cos\theta<0)}{\sigma(\cos\theta>0) + \sigma(\cos\theta<0)}$$

A distribution $(1+\cos\theta)^2$ leads to

$$\int_0^1 d\cos\theta\,(1+\cos\theta)^2 = \frac{7}{3} \qquad \int_{-1}^0 d\cos\theta\,(1+\cos\theta)^2 = \frac{1}{3}$$

so $\quad A_{FB} = \dfrac{7-1}{7+1} = \dfrac{3}{4}$ for this distribution

Then for unpolarized $e^+e^-\to f\bar{f}$, A_{FB} is given by

$$A_{FB} = \frac{3}{4}\,\frac{\sigma(e^-_L e^+_R \to f_L \bar f_R) + \sigma(e^-_R e^+_L \to f_R \bar f_L) - \sigma(e^-_L e^+_R \to f_R \bar f_L) - \sigma(e^-_R e^+_L \to f_L \bar f_R)}{\sigma(\quad) + \sigma(\quad) + \sigma(\quad) + \sigma(\quad)}$$

$$= \frac{3}{4}\,\frac{\left(\sigma(e^-_L e^+_R \to Z^0) - \sigma(e^-_R e^+_L \to Z^0)\right)\left(\sigma(Z^0 \to f_L \bar f_R) - \sigma(Z^0 \to f_R \bar f_L)\right)}{\left(\sigma(\quad) + \sigma(\quad)\right)\left(\sigma(\quad) + \sigma(\quad)\right)}$$

so that

$$A_{FB}(f) = \frac{3}{4} A_e A_f$$

Then we predict

$$A_{FB}(\mu \text{ or } \tau) = .02 \qquad A_{FB}(b) = 0.11$$

Now we have a very detailed theoretical description of the Z^0. But how many of these quantities can we measure?

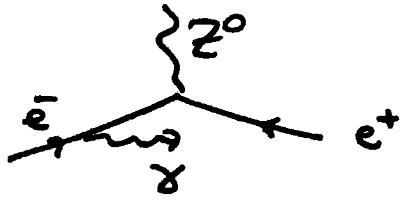
Let's first discuss the Z^0 width. In quantum mechanics, a resonance has the Breit-Wigner form

$$\sigma(E) \propto \left| \frac{1}{E - E_0 + i\Gamma/2} \right|^2$$

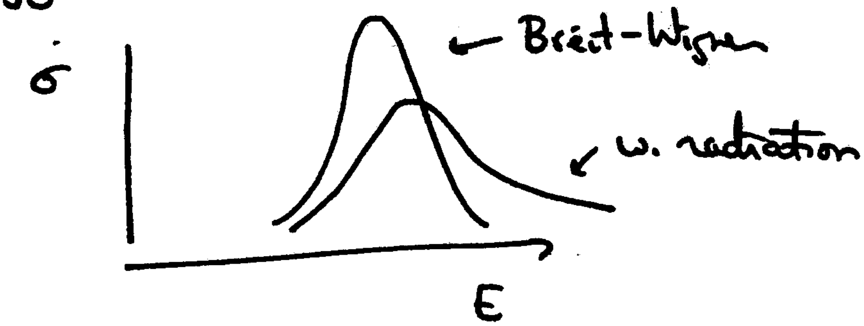
The relativistic version of this formula is given by multiplying the denominator by $2E$ and write $s = E_{cm}^2$:

$$\sigma(E) \propto \left| \frac{1}{s - m_Z^2 + i m_Z \Gamma_Z} \right|^2$$

This would produce a symmetrical resonance peak. However, the shape of the Z^0 is distorted by initial-state electromagnetic radiation



which requires the radiating electron or positron to have a higher energy. This distorts



The effect is quite significant; Fig. 53 shows the shape of the Z^0 resonance undistorted, including radiation of 1 photon (B-M), and including radiation of many collinear photons (F-[redacted]R). The final curve gives a very good fit to the cross-section measurements at LEP (Fig. 54). The best current value of Γ_Z is

$$\Gamma_Z = 2.4946 \pm 0.0027$$

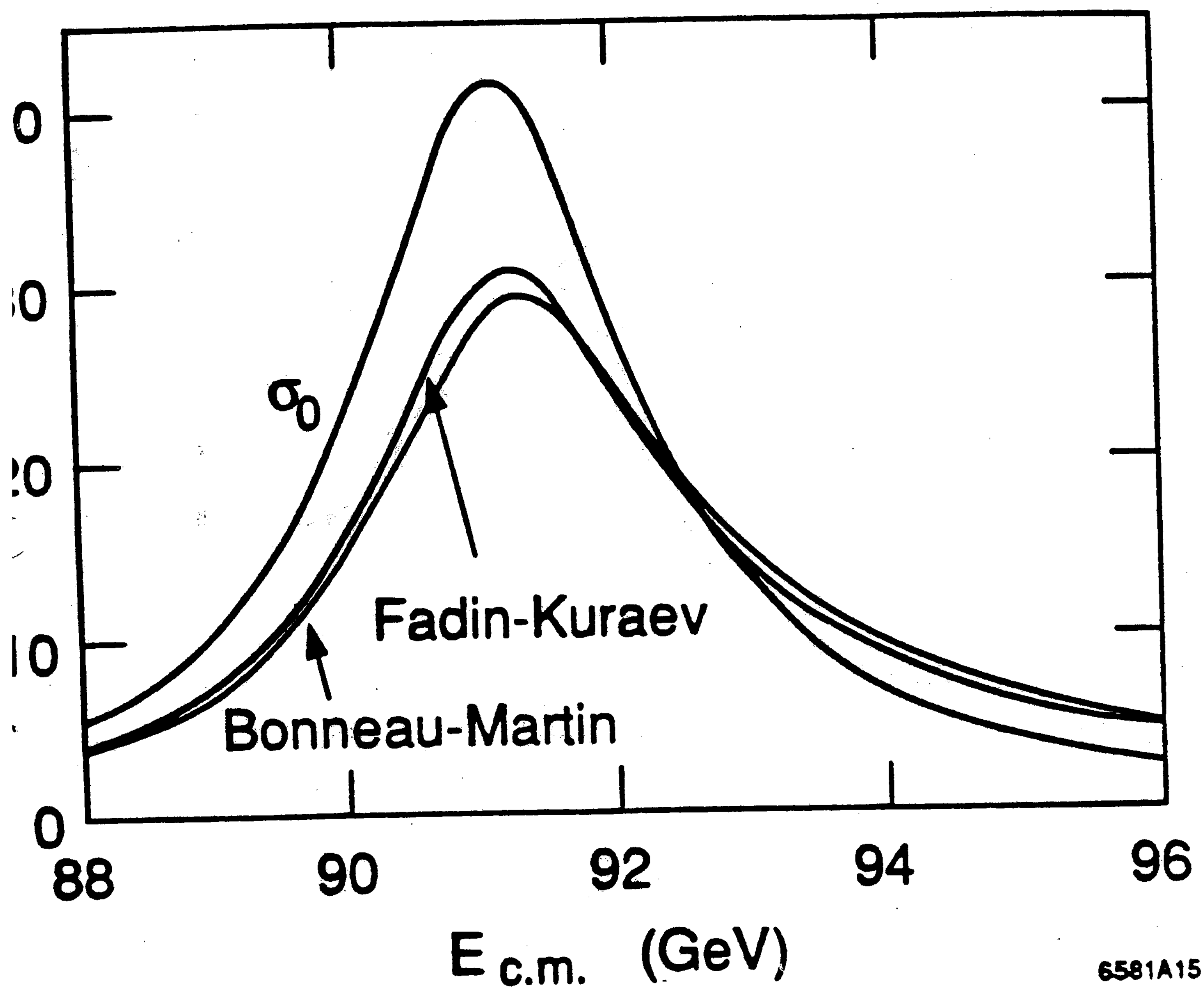
in good agreement with the prediction on p. [redacted]7.

It is interesting to note that the processes

$$Z^0 \to \nu_i \bar{\nu}_i \qquad i = e, \mu, \tau$$

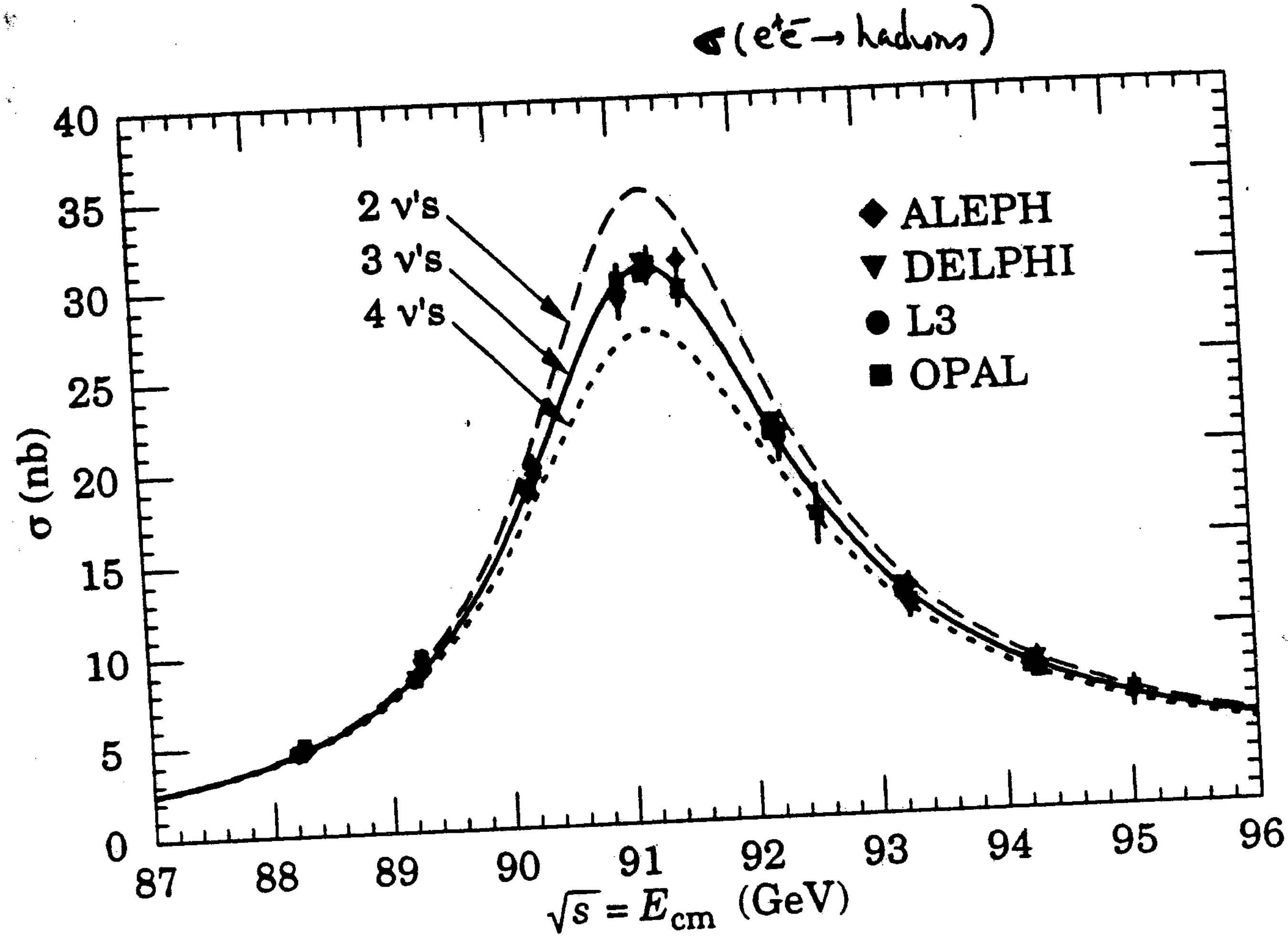
contribute $\Gamma(Z \to \nu\bar{\nu}) = 0.50$ GeV to this width. Thus,

Figure 53



0
0
20
10
0
σ_0
Fadin-Kuraev
Bonneau-Martin
88
90
92
94
96
E c.m. (GeV)
6581A15

Figure 54



data compilation of the Particle Data Group.

though the Z^0 decays to $\nu\bar{\nu}$ are invisible, we know these decays are there. If there were an extra neutrino species (perhaps associated with an undiscovered heavy lepton), the Z^0 would be wider and would have a smaller peak cross section. The effect is shown in Fig. 54. The data imply that the rate of Z^0 decay to extra invisible species is bounded at about 1% of $\Gamma(Z^0 \to \nu\bar{\nu})$.

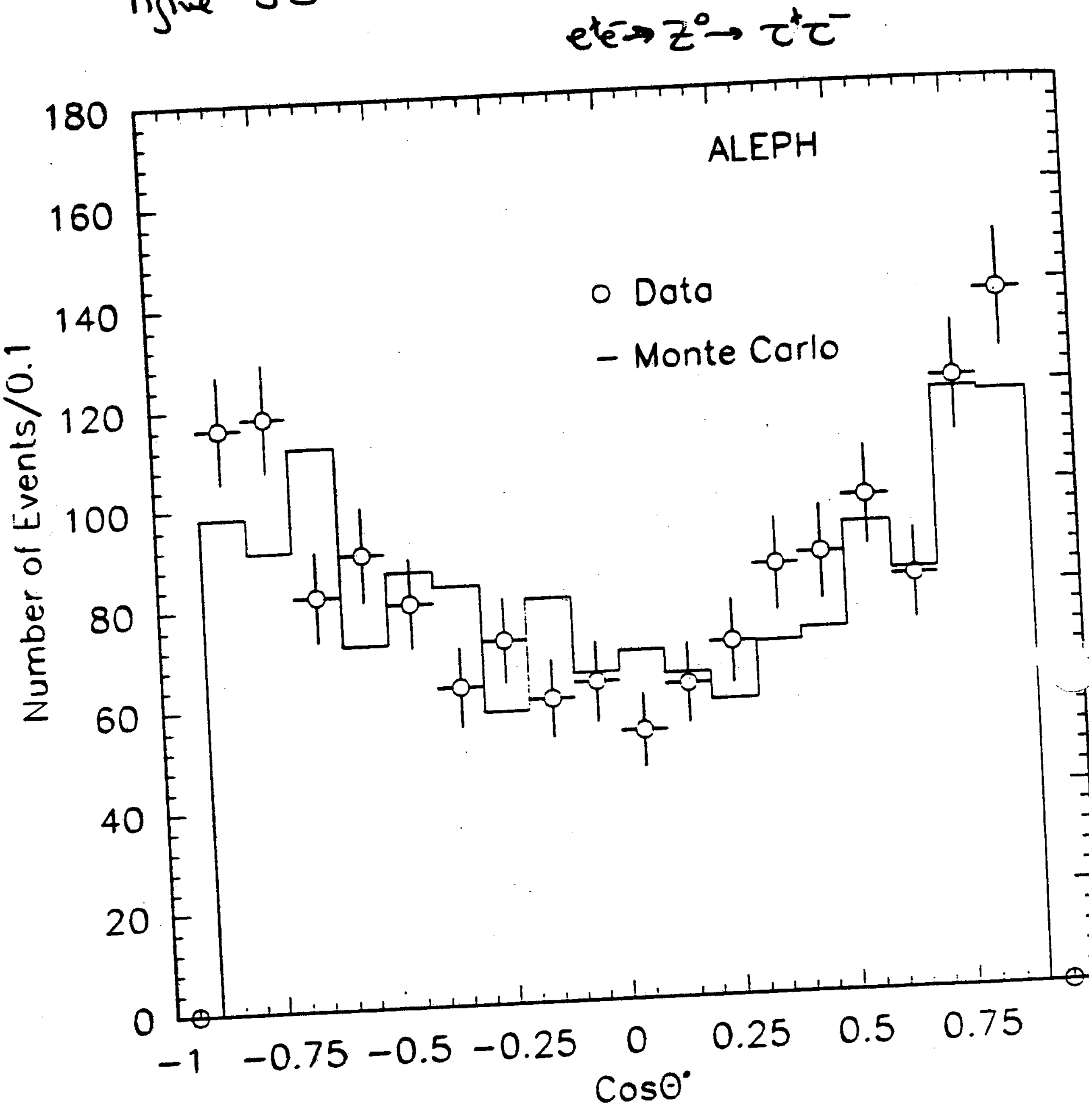
Next, let's discuss the Z^0 couplings to leptons. The lepton angular distribution at the Z^0 is quite symmetrical, in accord with our prediction $A_{FB}(\ell) \approx 2\%$. Fig. 55 shows the angular distribution for $e^+e^- \to \tau^+\tau^-$. But a better way to look at it is that

$$A_{FB}(e^+e^- \to \ell^+\ell^-)$$

is a function of E_{cm} that has a zero near m_Z. Fig. 56 shows measurements of $A_{FB}(e^+e^- \to \ell^+\ell^-)$ at a series of energies across the Z^0 peak, done by the OPAL experiment. The solid line is a fit to the $SU(2) \times U(1)$ model; I'll collect the values of $\sin^2\theta_w$ obtained by the fit later.

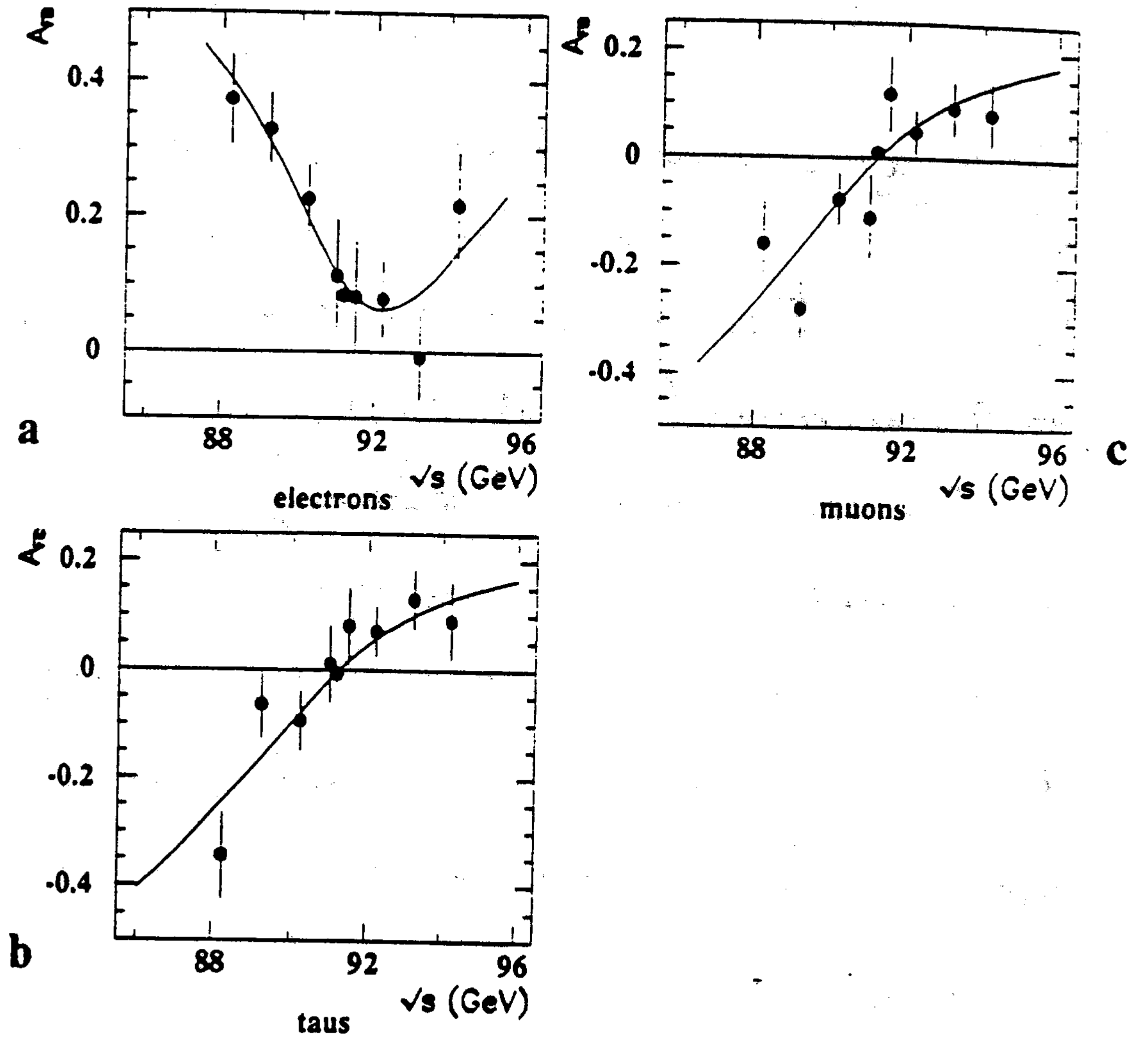
It is also possible to measure A_ℓ, the polarization asymmetry to leptons, directly. The first method to do this is to measure the Z^0 production rate from polarized

Figure 55



D. Decamp et al (ALEPH)
Z Phys. C48, 365 (1990)

Figure 56



G. Alexander et al (OPAL)
Z. Phys. C 52, 175 (1991)

electrons. The linear accelerator at SLAC naturally maintains the electron polarization, so it is possible there to collide electron beams of 80% polarization with positrons. Then a single counting experiment measures.

$$A_e = \frac{\sigma(e_L^- e^+ \to Z^0) - \sigma(e_R^- e^+ \to Z^0)}{\sigma(\quad) + \sigma(\quad)}.$$

This is quite a remarkable effect; you tweak the polarization of a laser which produces the electron beam and then — 5 km away — the counting rate changes. And, one indeed finds the expected asymmetry: $A_e = 0.1542 \pm .0037$.

The second method for determining A_ℓ is to measure the polarization of τ^-'s produced by the Z^0, using the decay distributions as a polarimeter. I explained in Lecture 5 that, in μ^- decay, the fastest electrons are emitted opposite to the μ^- spin direction. The same distributions apply to the decay

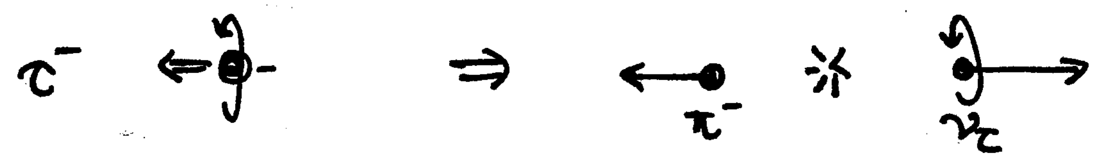
$$\tau^- \to e^- \bar{\nu}_e \nu_\tau \quad , \mu^- \bar{\nu}_\mu \nu_\tau$$

Thus, after a boost:

$$\tau_L^- \to \text{fast } e^-, \mu^- \qquad \tau_R^- \to \text{slow } e^-, \mu^-$$

By fitting the energy distribution of $\tau^- \to e^-$, $\tau^- \to \mu^-$, we can determine the polarization. Another useful

τ decay is $\tau^- \to \pi^- + \nu_\tau$. Since the π^- has spin 0, the ν_τ must carry the τ spin. But since the ν_τ is left-handed:



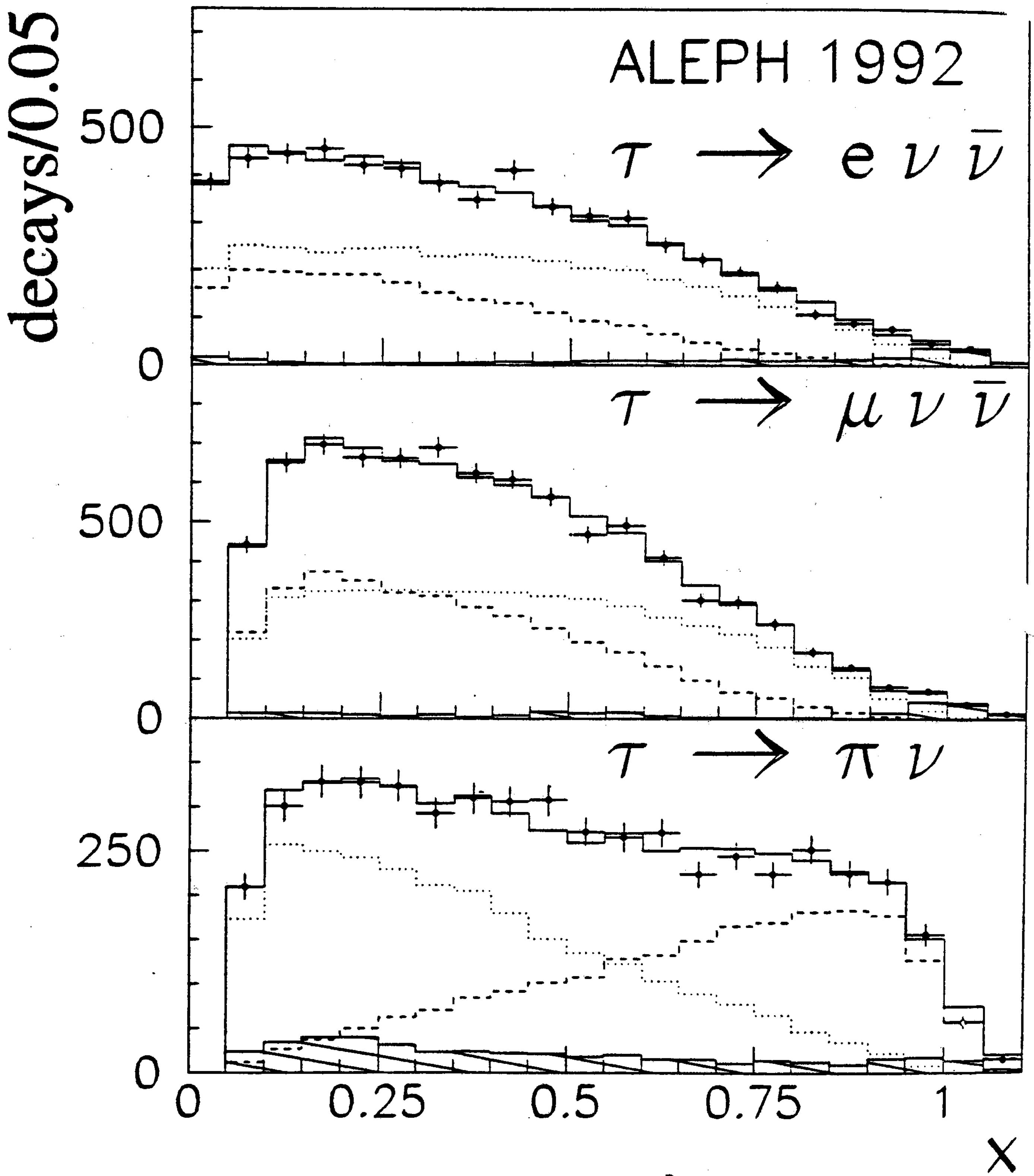
Thus: $\tau_L^- \to$ slow π^- $\qquad$ $\tau_R^- \to$ fast π^-

The energy distributions for the decays $\tau^- \to e^-, \mu^-, \pi^-$ are shown in Fig. 57; the fit shown gives the components due to τ_L^- and τ_R^-. Especially in the bottom figure, you can see the 15% asymmetry favoring τ_L^-. Our theory predicts that forward-produced τ's are lefthanded and backward-produced τ's are right-handed with the same 15% asymmetry. The correlation of τ polarization with $\cos\theta$, as measured by the ALEPH experiment, is shown in Fig. 58.

Another species that is especially amenable to study at the Z^0 is the b quark. The b quark decays by the weak interaction process

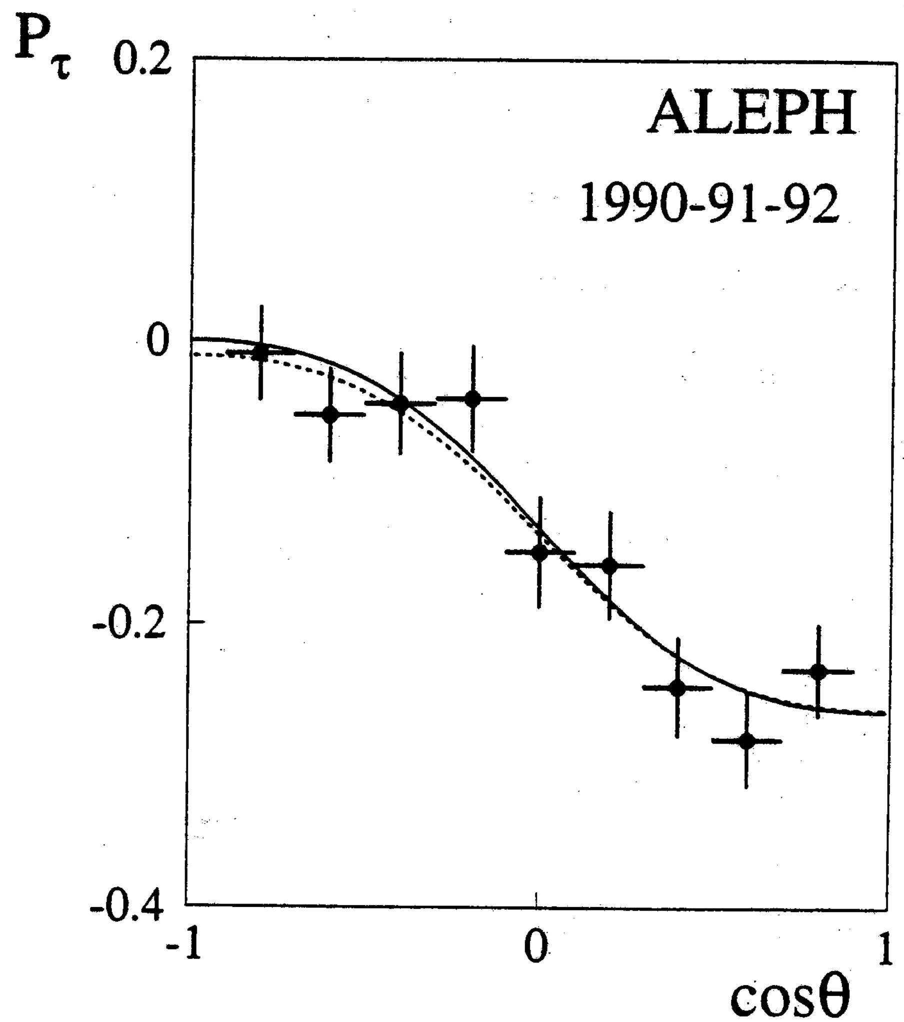
$$b \to c + (W^-)$$
$\hookrightarrow$ leptons, quarks

Figure 57 $\cdots\ \tau_L^-$ $\quad$ -- τ_R^-



D. Buskulic et al (ALEPH)
Z. Phys. C69, 183 (1996)

Figure 58



D Buskulic et al (ALEPH)
Z Phys. C69 183 (1996)

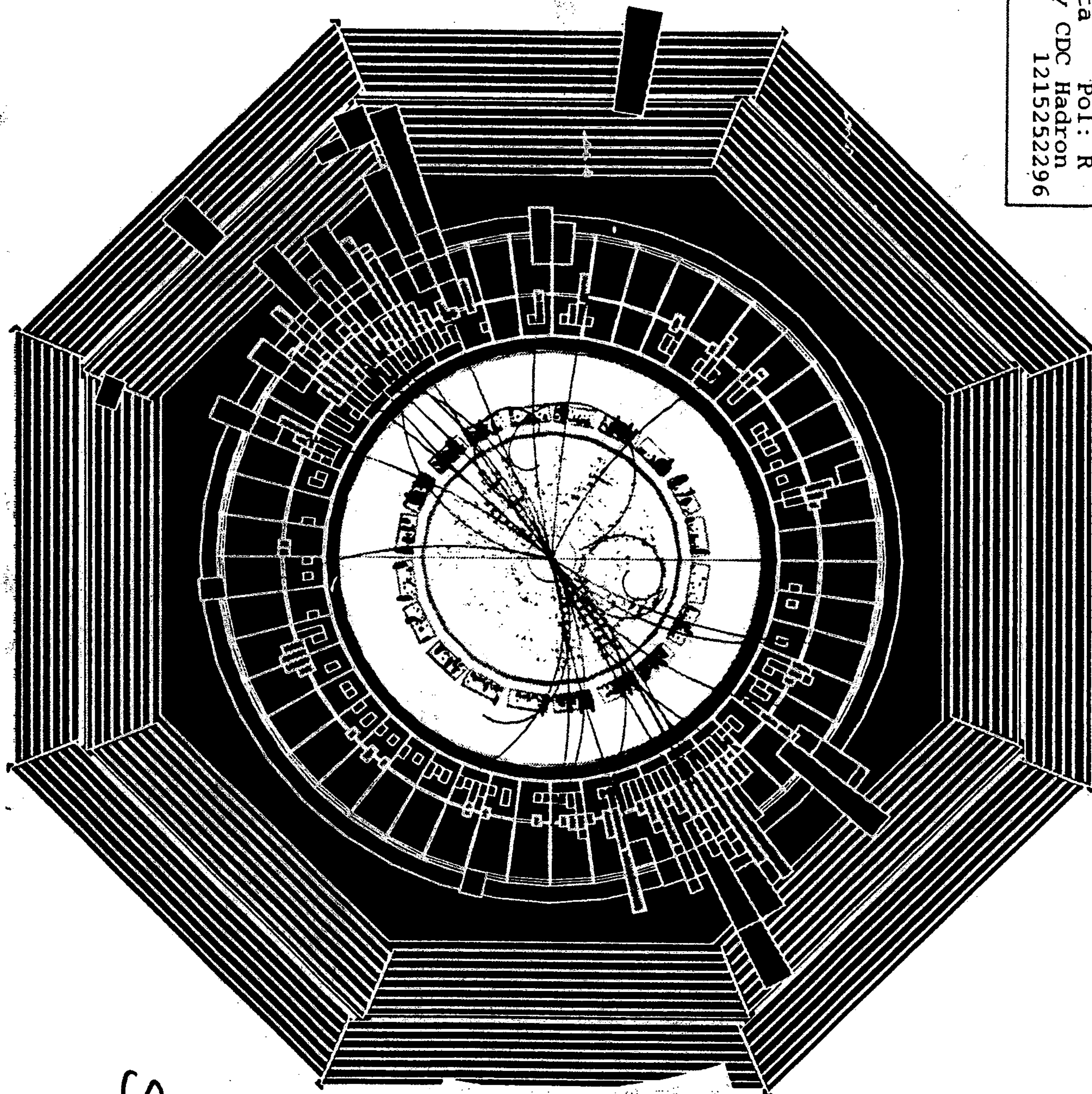
Since b is not the partner of c, the process is hindered and the rate is rather slow; the lifetime of the b is about 1 ps ≈ 0.3 mm. With the Lorentz boost, a typical hadron containing a b quark will travel 1-2 mm from the point of the Z^0 decay before it decays to lighter particles. Figure 59 shows an example of a $Z^0 \to b\bar{b}$ event in the SLD detector. The scale here is meters, so it is hard to see the b decays. However, the SLD includes a precision vertex chamber, consisting of a barrel of CCD's located about 3 cm from the e^+e^- collision axis. This allows me to blow up the center of this picture and show, in Fig. 60, the precise extrapolation of tracks back to the collision point. You see two displaced vertices.

Fig. 61 shows the distribution of measured impact parameters of tracks in Z^0 events observed by SLD. Because of measurement errors, a given track may be observed to originate from in front of or behind the e^+e^- collision point (with the orientation given by the axis of its jet). b decays (and, to a lesser extent, c decays) produce additional

Figure 59

21

Run 33544, EVENT 6476
27-APR-1996 06:05
Source: Run Data Pol: R
Trigger: Energy CDC Hadron
Beam Crossing 1215252296



SLD

[redacted]

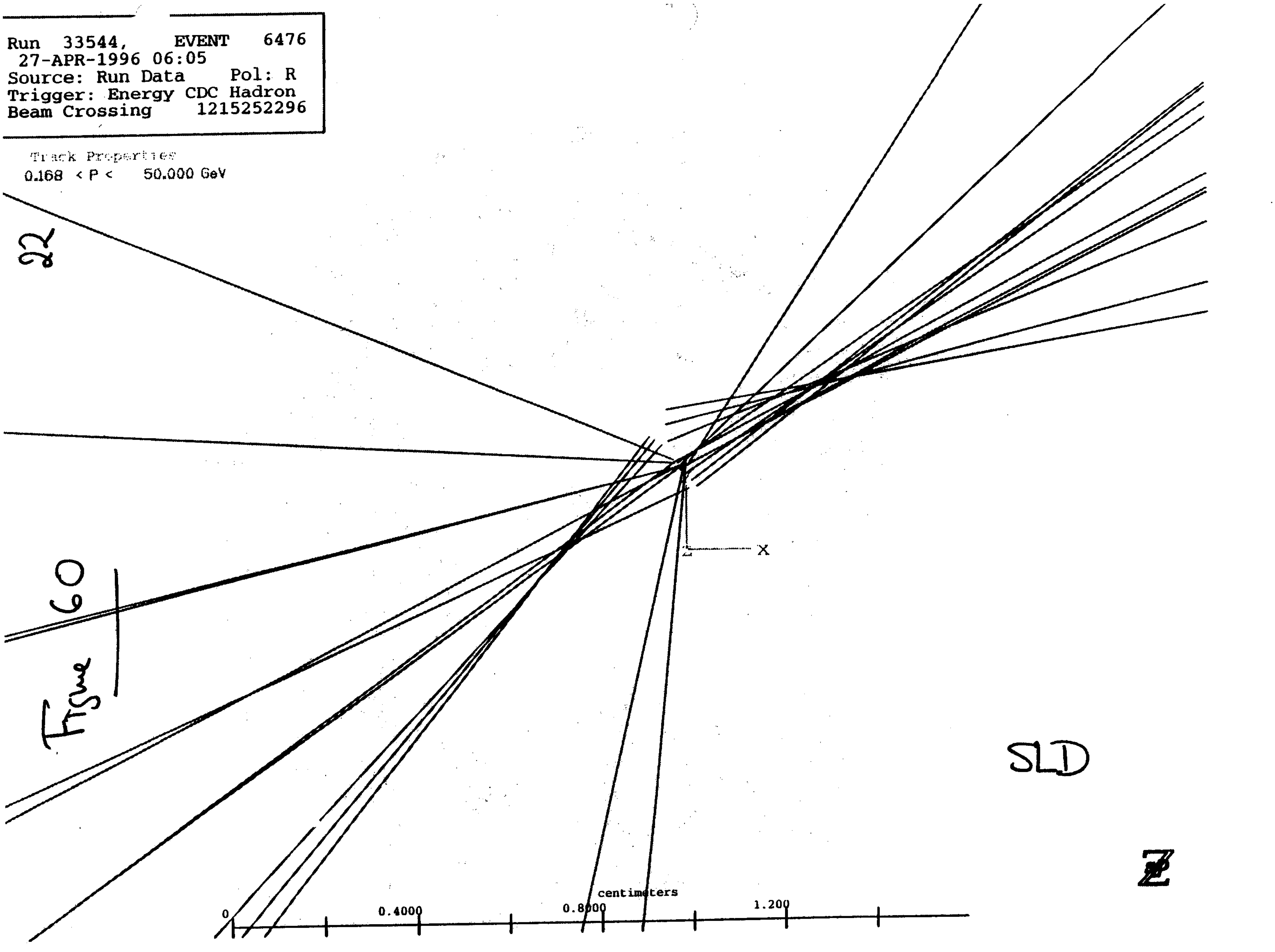
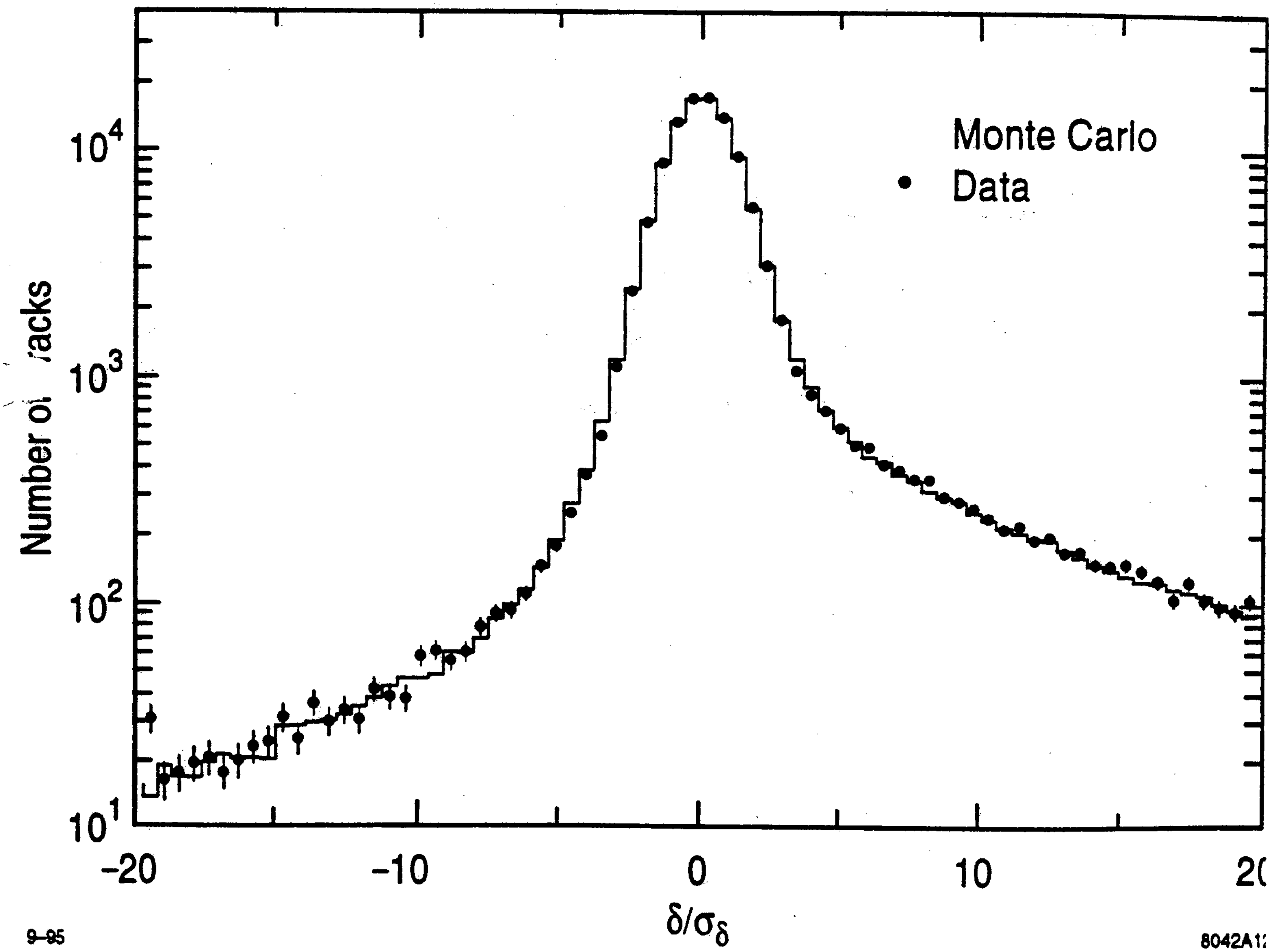

Run 33544, EVENT 6476
27-APR-1996 06:05
Source: Run Data Pol: R
Trigger: Energy CDC Hadron
Beam Crossing 1215252296
Track Properties
0.168 < P < 50.000 GeV
22
Figure 60
Z
X
centimeters
0
0.4000
0.8000
1.200
SLD

Figure 61



SLD

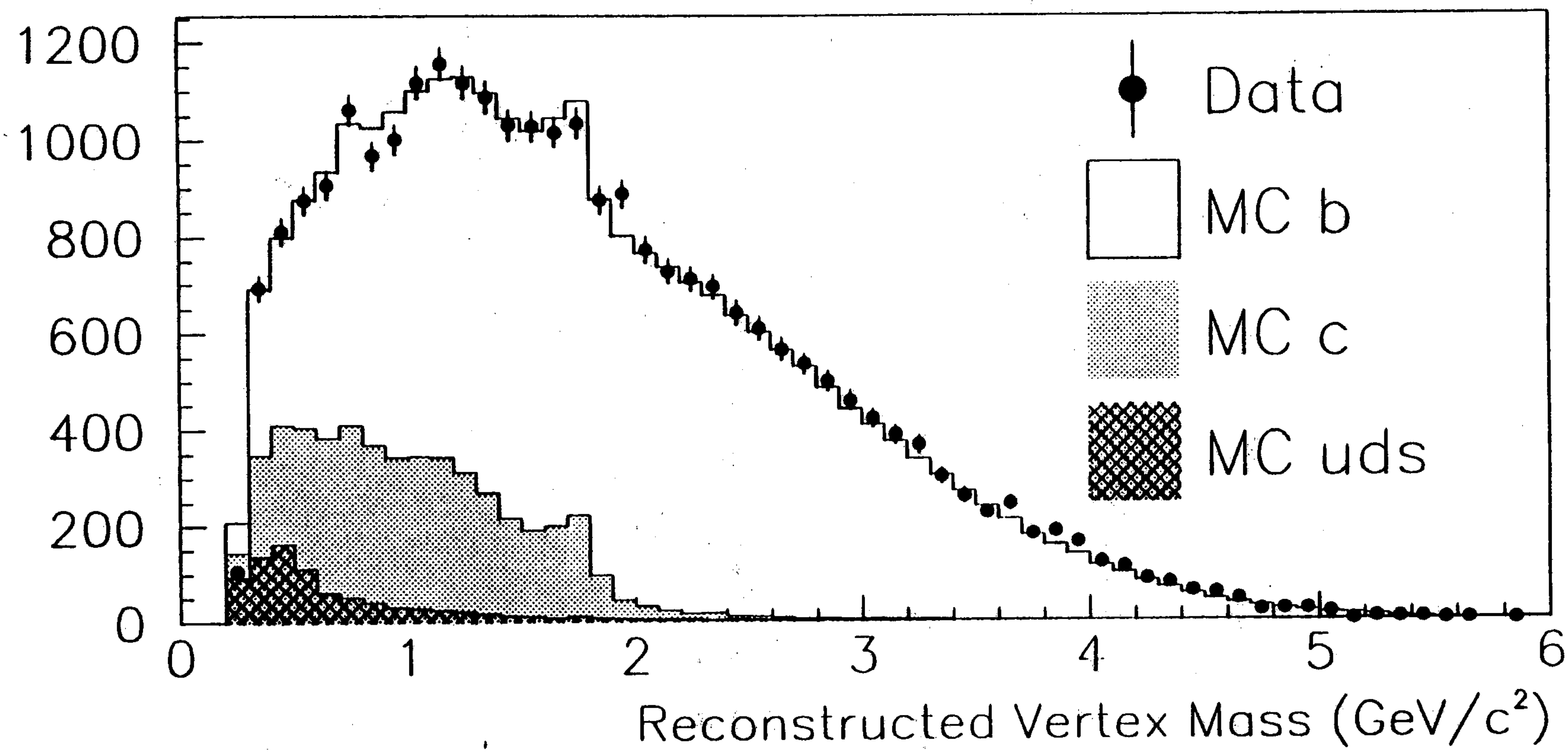
tracks with positive impact parameter. Thus, the left-hand side of Fig. 61 can be used by the experimenters to check that they understand their tracker, and the excess of events on the right-hand side is the signal from decaying heavy quarks.

As a check on this interpretation, one can measure the mass of the combination of particles associated by the precision tracker with a displaced vertex (136). The distribution measured by the SLD is shown in Fig. 62, together with simulation results giving the expected contributions from b, c, and light quarks. By insisting that this measured mass is greater than 2.5 GeV, we obtain a very pure sample of $Z^0 \to b\bar{b}$ events.

We can use these events to determine the branching ratio

$$\frac{\Gamma(Z^0 \to b\bar{b})}{\Gamma(Z^0 \to \text{hadrons})}$$

Those events with decays such as $b \to c e^- \bar{\nu}_e$ or $b \to c \mu^- \bar{\nu}_\mu$ which allow us to distinguish the b-jet from the $\bar{b}$-jet can be used to measure the b forward-backward asymmetry. I'll discuss those results in a moment. First, though, I would like to point out a very beautiful



SLD

Figure 62

effect: Since A_b is very close to 1, the angular distribution for

$$e^-_L e^+ \to Z^0 \to b\bar{b}$$

should be almost precisely $(1+\cos\theta)^2$, and the angular distribution for $e^-_R e^+ \to Z^0 \to b\bar{b}$ should be almost precisely $(1-\cos\theta)^2$. We can test this with the 80% polarized electron beam of the SLC. The results are shown in Fig. 63; in this measurement, the b jet is identified statistically by the charge of the leading particles (negative $\to$ b ; positive $\to$ $\bar{b}$).

Now let me summarize the results of these experiments. Because of the relation

$$A_\ell \approx 8\left(\tfrac{1}{4} - \sin^2\theta_w\right)$$

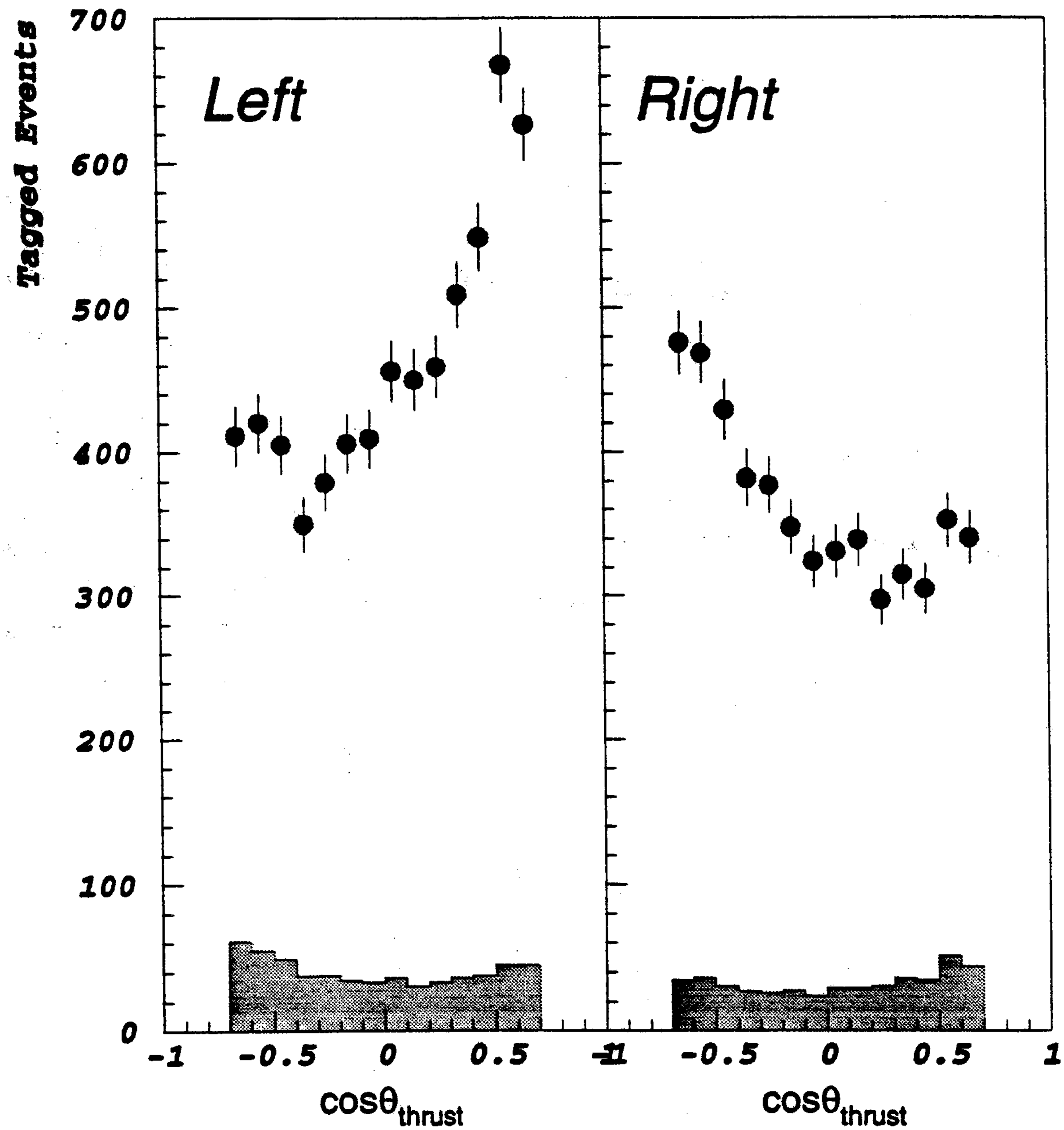
a measurement of A_ℓ gives a very accurate determination of $\sin^2\theta_w$. Because A_b is almost maximal,

$$A_{FB}(b) = \tfrac{3}{4} A_e A_b$$

$$\cong 0.70\, A_e$$

also can be used to determine $\sin^2\theta_w$. In Fig. 64,

Figure 63

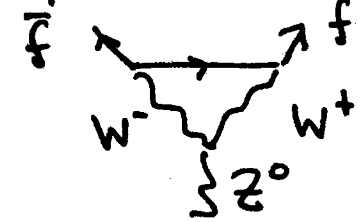


SLD

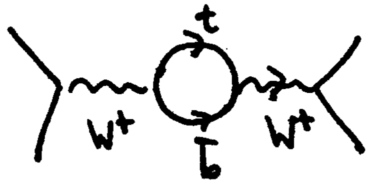
I show a summary of the various techniques used to determine $\sin^2\theta_W$ from polarization and forward-backward asymmetries at the Z^0. The results are consistent and lead to a very accurate value:

$$\langle \sin^2\theta_W \rangle = 0.23129 \pm .00030$$

This number is sufficiently accurate that we should ask whether there is detailed consistency among the various values of $\sin^2\theta_W$ that I have quoted in this lecture. The differences among these values are real, but they are also at the level at which higher-order quantum processes must be considered. For example, the following process, in which a Z^0 fluctuates to a W^+W^- pair, affects the amplitude for Z^0 decay to $f\bar{f}$



A curious contribution is that of the top quark, which appears in diagrams such as



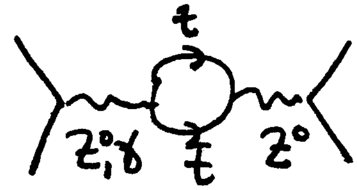
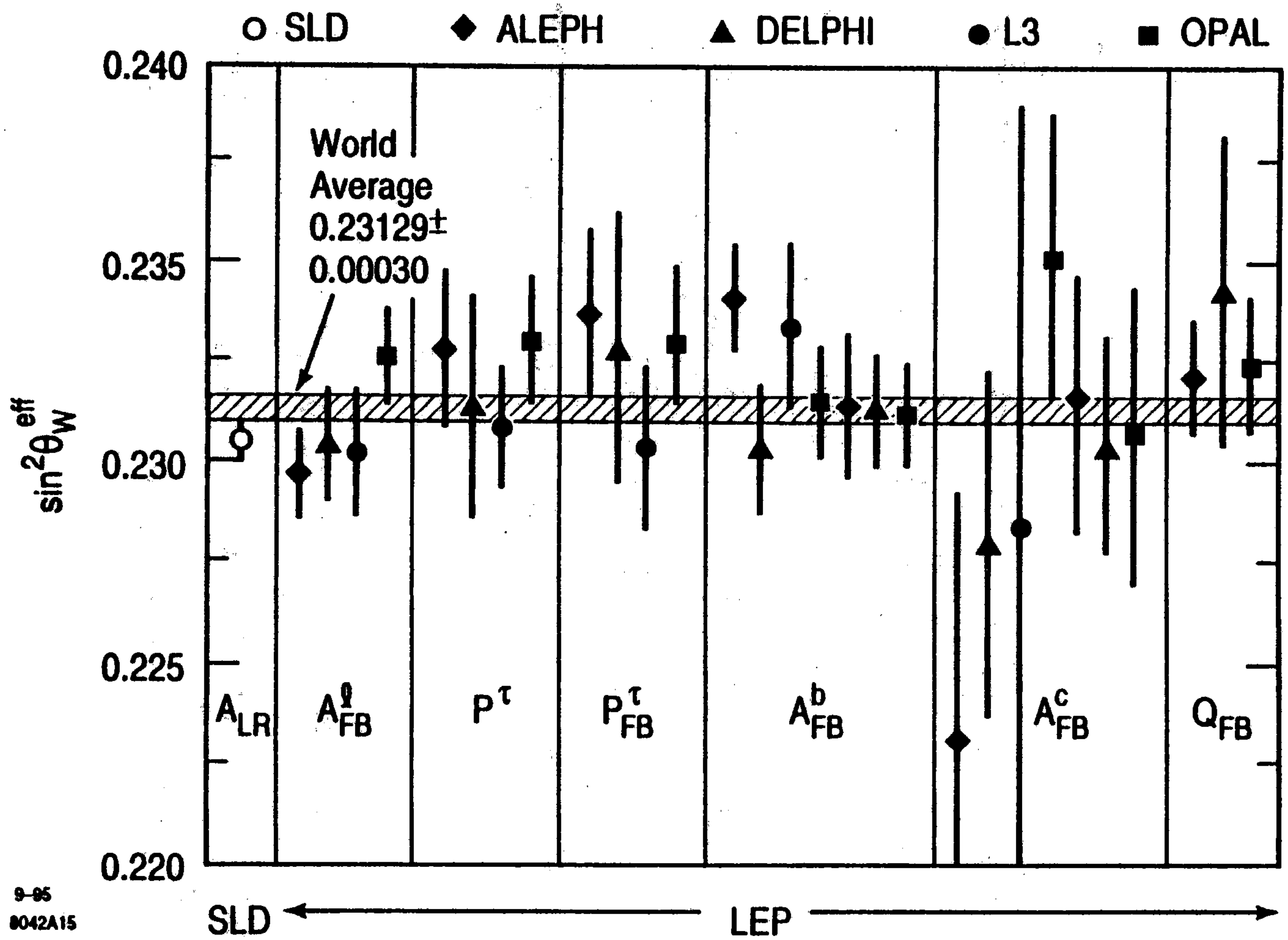


Figure 64



CY Prescott in Proc. of the 1995 Lepton-Photon conference.

Figure 15: Comparison of 30 measurements of $\sin^2 \theta_W^{eff}$ by LEP and SLD.

These diagrams turn out to be of order

$$\frac{\alpha_w}{\pi} \cdot \frac{m_t^2}{m_W^2}$$

and since the top quark is heavy, this correction is an important one. For example, it plays a major role in explaining the low value of $\sin^2\theta_w = 1 - m_W^2/m_Z^2$ on p. 3. The consistency of the whole corpus of weak interaction measurements requires

$$m_t = 170 \pm 20 \text{ GeV}$$

in good accord with the value $m_t \simeq 175$ GeV obtained from the observation of top quark production at Fermilab.

In Fig. 65, I show a comparison of a large number of Z^0 observables to precise calculations in the $SU(2) \times U(1)$ theory. Apparently, this model does give a complete description of the structure of the weak interactions.

Figure 65

P. Langacker and J. Erler hep-ph/9703428

Quantity	Value	Standard Model
M_Z (GeV)	91.1863 ± 0.0020	input
Γ_Z (GeV)	2.4946 ± 0.0027	$2.496 \pm 0.001 \pm 0.001 \pm [0.002]$
$R = \Gamma(\text{had})/\Gamma(\ell\bar{\ell})$	20.778 ± 0.029	$20.76 \pm 0.003 \pm 0.001 \pm [0.02]$
$\sigma_{\text{had}} = \frac{12\pi}{M_Z^2} \frac{\Gamma(e\bar{e})\Gamma(\text{had})}{\Gamma_Z^2}$	41.508 ± 0.056	$41.46 \pm 0.002 \pm 0.002 \pm [0.02]$
$R_b = \Gamma(b\bar{b})/\Gamma(\text{had})$	0.2178 ± 0.0011	$0.2156 \pm 0 \pm 0.0002$
$R_c = \Gamma(c\bar{c})/\Gamma(\text{had})$	0.1715 ± 0.0056	$0.172 \pm 0 \pm 0$
$A_{FB}^{0\ell} = \frac{3}{4}\left(A_\ell^0\right)^2$	0.0174 ± 0.0010	$0.0157 \pm 0.0003 \pm 0.0003$
$A_\tau^0\,(P_\tau)$	0.1401 ± 0.0067	$0.145 \pm 0.001 \pm 0.001$
$A_e^0\,(P_\tau)$	0.1382 ± 0.0076	$0.145 \pm 0.001 \pm 0.001$
$A_{FB}^{0b} = \frac{3}{4} A_e^0 A_b^0$	0.0979 ± 0.0023	$0.101 \pm 0.001 \pm 0.001$
$A_{FB}^{0c} = \frac{3}{4} A_e^0 A_c^0$	0.0735 ± 0.0048	$0.072 \pm 0.001 \pm 0.001$
$\bar{s}_\ell^2 \left(A_{FB}^Q\right)$	0.2320 ± 0.0010	$0.2318 \pm 0.0002 \pm 0.0001$
$A_\ell^0 \left(A_{LR}^0, A_{e,\mu,\tau}^0\right)$ (SLD)	0.1542 ± 0.0037	$0.145 \pm 0.001 \pm 0.001$
A_b^0 (SLD)	0.863 ± 0.049	$0.935 \pm 0 \pm 0$
A_c^0 (SLD)	0.625 ± 0.084	$0.667 \pm 0.001 \pm 0.001$
N_ν	2.989 ± 0.012	3

Lecture 8 : August 6

In the past three lectures, we have made a lengthy study of the weak interactions. By the end of our discussion, we had constructed an explicit Lagrangian for the weak interactions, and we had discussed sensitive tests of this weak interaction theory at the Z^0 resonance.

Now it is time to return to the strong interactions. In lecture 4, we discussed the coupling of the <u>gluon</u>. This discussion, however, left two lingering questions. First, in what sense does the gluon "mediate" the strong interactions? Second, what is the <u>soft component</u> of the strong interactions which is responsible for the binding of quarks into hadrons. Our analysis in lecture 6 raises a third question: If the gluon is a spin-1 particle like γ, W^+, W^-, and Z, can we write its field equations explicitly? It will turn out that the answer to this question also answers the others.

After our discussion in lecture 6, it is natural to propose that the gluons are described by a Yang-Mills theory. This theory would be completely determined if we specify the local symmetry group. Let me propose that we take this symmetry to be the group of local unitary transformations of the <u>color</u> quantum numbers of the quarks. That is, arrange the three color states of a quark into a column vector

as.
$$\begin{pmatrix} u_{red} \\ u_{green} \\ u_{blue} \end{pmatrix}$$

and consider the local symmetry to be

$$\begin{pmatrix} u_r \\ u_g \\ u_b \end{pmatrix} \rightarrow U(x) \begin{pmatrix} u_r \\ u_g \\ u_b \end{pmatrix}$$

(and similarly for all other quarks) where $U(x)$ is a 3x3 unitary matrix.

There is a subtlety here that I should discuss. A 3x3 unitary matrix has the form

$$U = e^{i\alpha^a t^a}$$

where t^a are 3x3 Hermitian matrices. There are 9 3x3 Hermitian matrices, but one of these

$$t^0 = \begin{pmatrix} 1 & & 0 \\ & 1 & \\ 0 & & 1 \end{pmatrix}$$

commutes with all of the others. This implies that the matrices

$$U_0 = e^{i\alpha t^0} = e^{i\alpha} \underset{\sim}{1}$$

commute with all other 3x3 unitary matrices, and it is true: applying U_0 to a vector simply multiplies the vector by $e^{i\alpha}$. The remaining 8 matrices must generate a group by themselves; this is the group of 3x3 unitary matrices with $\det U = 1$. This group is called $SU(3)$. The generators of $SU(3)$ can be characterized as the 8 3x3 Hermitian matrices that satisfy

$$\mathrm{tr}\, t^a = 0$$

All of this is similar to the case of 2x2 unitary matrices. The Pauli sigma matrices σ^a are all the 3 2x2 Hermitian matrices which satisfy $\mathrm{tr}\, \sigma^a = 0$. These matrices generate $SU(2)$.

To fix my conventions, I will normalize the generators of SU(3) using the same convention as for the generators of SU(2) $\tau^a = \sigma^a/2$

$$\mathrm{tr}\, t^a t^b = \tfrac{1}{2}\delta^{ab} \qquad a,b = 1,\ldots,8$$

Now we can immediately write down the Yang-Mills theory of gluons:

$$\mathcal{L} = -\tfrac{1}{4}(F^a_{\mu\nu}F^{\mu\nu a}) + \sum_f (q^\dagger_{fL}\, i\bar{\sigma}\cdot D\, q_{fL} + q^\dagger_{fR}\, i\sigma\cdot D\, q_{fR})$$

where

$$D_\mu = (\partial_\mu - i g_s A^a_\mu t^a)$$

$$F_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f^{abc} A^b_\mu A^c_\nu$$

where f^{abc} are the structure constants of SU(3):

$$[t^a, t^b] = i f^{abc} t^c$$

and g_s is a new coupling constant. If we were not consistently ignoring the quark masses, we could add in these mass terms. This theory is called 'Quantum Chromodynamics' (QCD). Now we can ask, is QCD sufficient to be the theory of the strong interactions.

First of all, the quark-gluon coupling in QCD is exactly that required to explain the experiments on 3-jet events and Bjorken scaling violation that I discussed in Lecture 4. In QCD, the square of the amplitude for gluon emission contains the factor:

$$g_s^2\, t^a t^a$$

where in SU(3) $t^a t^a = \frac{4}{3}$, just as in SU(2) $(\frac{\sigma^a}{2})^2 = \frac{3}{4}$. So, if we just substitute $\alpha_g \to \frac{4}{3}\alpha_s$, the formulae in

Lecture 4 become familiar of QCD.

QCD also contains 3-gluon and 4-gluon couplings:

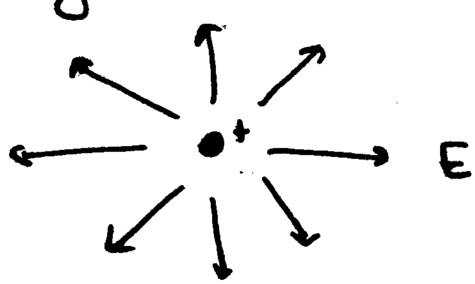
I told you in lecture 4 that these are needed for quantitative agreement with the production rate for multi-jet events in e^+e^- annihilation.

The fact that the gluons of QCD interact with one another tells us ~~that~~ the dynamics of this theory can be more complicated than that of QED. In fact, QCD dynamics has two remarkable features, both of which were discovered in 1973:

The first of these discoveries (chronologically, the second) is due to Wilson. Wilson found a way to study gauge theories in the limit of strong coupling, $g \to \infty$. According to the Maxwell equation

$$\vec{\nabla}\cdot E = \rho$$

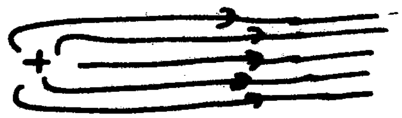
(which is generalized in Yang-Mills theory to $\vec{D}\cdot\vec{E}^a = \rho^a$), electric flux begins and ends on charges. In electrodynamics, the electric flux spreads out evenly



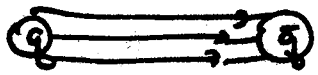
giving rise to the familiar result of Coulomb's law:

$$|\vec{E}| \sim \frac{1}{r^2}$$

However, Wilson showed that in strong-coupling Yang-Mills theory and also in a slightly modified version of electrodynamics at strong coupling) lines of electric flux attract and form tubes of quantized strength. Then the lowest energy configuration of a charge and its electric field is



In strong-coupling QCD, then, a quark-antiquark system would look like



and this system would have energy

$$V(r) \sim (\text{Const}) \cdot |\vec{r}|$$

if the quark and antiquark were separated. At strong coupling, then, it is not possible to separate a quark and an antiquark to arbitrary distances. We can thus understand why we do not see physical charge $\frac{1}{3}$ or $\frac{2}{3}$ particles in Nature.

You might recall that in lecture 2, when I discussed the production of hadrons in e^+e^- annihilation, I made use of a physical picture in which quarks and antiquarks are connected by "strings". Wilson's theory gives a physical explanation of the strings: they are the electric flux tubes of strong-coupling QCD.

The baryons also find a natural place in this picture. A baryon must look like

with 3 strings and a junction

In general, such junctions exist if the system of charges is invariant under the gauge symmetry group. Indeed, the object

$$\epsilon_{ijk}\; q_i\, q_j\, q_k \qquad i,j,k = \text{red, green, blue}$$

is invariant to SU(3):

$$q_i \to U_{i\ell}\, q_\ell \quad \text{gives} \quad \epsilon_{ijk}\, q_i q_j q_k \to \epsilon_{ijk}\, U_{i\ell} U_{jm} U_{kn}\, q_\ell q_m q_n$$
$$= \det U \cdot \epsilon_{\ell m n}\, q_\ell q_m q_n$$

and, for SU(3) matrices, $\det U = 1$.

Now we really have a puzzle. In hard processes, QCD looks like electrodynamics, with gluon emission following the laws of photon emission. The coupling is weak,

$$\alpha_s \sim 0.1$$

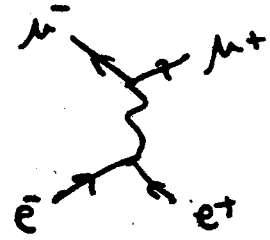
But now I am trying to explain the soft part of the strong interactions by modelling it with QCD with very strong coupling. How could this be consistent?

The solution to this puzzle comes with the second (or first) discovery noted above, the discovery by 't Hooft, Politzer, Gross, and Wilczek of a phenomenon called "asymptotic freedom". Before we discuss this phenomenon, however, I would like to introduce a concept that will be useful here and in many other places throughout the remaining lectures, the concept of a "running coupling constant".

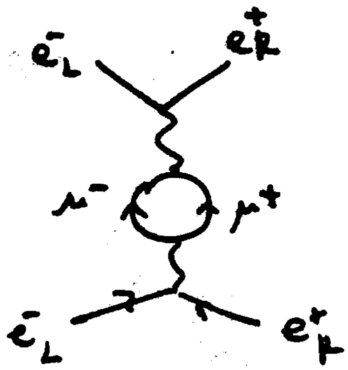
The simplest example of this concept comes in electrodynamics. Let's consider first only a model with electrons and muons. Then in the leading order we have the process $e^+e^- \to \mu^+\mu^-$, which has the cross-section

unpolarized e^-e^+ $\quad \sigma(e^-e^+\to\mu^-\mu^+) = \frac{4\pi\alpha^2}{3E_{cm}^2}$

polarized $e^-_L e^+_R$ $\quad \sigma(e^-_L e^+_R\to\mu^-\mu^+) = \frac{8\pi\alpha^2}{3E_{cm}^2}$



This process leads to a higher-order contribution to the scattering amplitude for $e^-_L e^+_R \to e^-_L e^+_R$:



in which the muon pair re-annihilates into an e^-e^+ pair.

We can compute the amplitude for this process by applying the optical theorem. In nonrelativistic quantum mechanics, this result reads:

$$\mathrm{Im}\, f(\cos\theta=1) = \frac{k}{4\pi}\sigma_{total}.$$

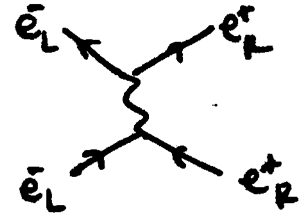
it expresses the fact that, if the Schrödinger wave scatters, it must also create a shadow at $\theta=0$ that removes the probability from the original waveform. In my conventions for relativistic scattering amplitudes (and for zero masses) the optical theorem reads:

$$\mathrm{Im}\, \mathcal{M}(\cos\theta=1) = E_{cm}^2\,\sigma_{tot}.$$

Also, in relativistic quantum mechanics, the theorem is true diagram by diagram, with each process contributing to σ_{tot}, giving the imaginary part of a diagram contributing to forward scattering. In this case, the value of the diagram shown on p.7 is given by

$$\mathrm{Im}\,\mathcal{M}(e^-_L e^+_R \to e^-_L e^+_R) = \frac{8\pi}{3}\alpha^2$$

Now, the value of the diagram



$= e^2(1+\cos\theta) = 2e^2$ at $\cos\theta = 1$.

so $\mathrm{Im}\,\mathcal{M}\big($  $\big) = \mathcal{M}\big($  $\big) \cdot \frac{\alpha}{3}$ / $2e^2$

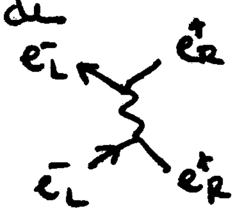
The function of $E_{cm}^2 = q^2$ that satisfies this relation is

$$\mathcal{M} = 2e^2 \cdot \frac{\alpha}{3\pi}\log\left(\frac{-q^2}{4m_\mu^2}\right)$$

at least for $q^2 \gg 4m_\mu^2$

since $\mathrm{Im}\,\lg(-q^2) = i\pi$ for $q^2 > 0$.

Going to still higher-order processes, we should consider



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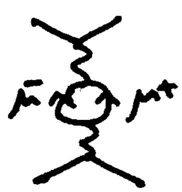

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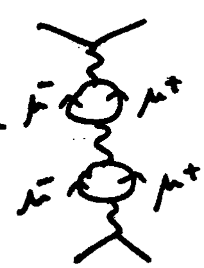

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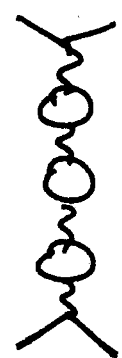


$+ \cdots$

$$= 2e^2\left[1 + \frac{\alpha}{3\pi}\lg\frac{-q^2}{m_\mu^2} + \left(\frac{\alpha}{3\pi}\lg\frac{-q^2}{m_\mu^2}\right)^2 + \cdots\right]$$

$$= 2e^2\,\frac{1}{1 - \frac{\alpha}{3\pi}\lg(-q^2/m_\mu^2)}$$

We can write this as

$$\mathcal{M}(e_L^- e_R^+ \to e_L^- e_R^+) = 2\, e_{\text{eff}}^2(q^2)$$

where the "effective coupling" e_{eff}^2 is given by

$$e_{\text{eff}}^2(q^2) = \frac{e^2}{1 - \frac{e^2}{12\pi^2}\log(q^2/m_\mu^2)}$$

Notice that e_{eff}^2 increases at high energy or large q^2. Another way of expressing e_{eff}^2 is to write it as the solution to the differential equation

$$\frac{d}{d\log q^2}\, e_{\text{eff}}^2 = + \frac{e_{\text{eff}}^4}{12\pi^2}$$

This equation has a fancy name: It is called the Renormalization Group (RG) equation.

The physics of the RG equation is simple and interesting. We know that QED causes $\mu^+\mu^-$ pairs to appear and disappear at any point in space. When these pairs appear, they make the vacuum of space a dielectric medium that can screen electric fields. Thus, the strength of the electric field produced by a charge should decrease as the distance r from the charge increases (or as q^2 decreases). For $r \ll 1/m_\mu$ or $q^2 \gg m_\mu^2$, this process should occur on every distance scale. Thus, the dielectric effect should be expressed by an equation of the form

$$\frac{d}{d\log r}(e_{\text{eff}}^2) = -(\text{const})\cdot(e_{\text{eff}}^2)^2$$

Taking $\frac{d}{d\log r} = -\frac{d}{d\log q} = -2\frac{d}{d\log q^2}$

this is just of the form of the RG equation.

I have accounted the effect of the μ^- on the running of e^2, but of course all species which are approximately massless at the given value of q^2 will have an effect. The full RG equation for e^2 should then read

$$\frac{d}{d\log q^2} e^2(q^2) = \sum_f \frac{1}{12\pi^2} Q_f^2 \, e^4(q^2)$$

where the sum runs over massless quarks and leptons, with the color factor of 3 for quarks. The solution of this equation is

$$e^2(q^2) = \frac{e_0^2}{1 - \frac{1}{3\pi}\frac{e_0^2}{4\pi}\sum_f Q_f^2 \log(q^2/4m^2)}$$

Taking the inverse and replacing $\alpha = e^2/4\pi$, $\alpha_0 = e_0^2/4\pi = \frac{1}{137}$, we have

$$\frac{1}{\alpha}(q^2) = \frac{1}{\alpha_0} - \frac{1}{3\pi}\sum_f Q_f^2 \log(q^2/4m_f^2)$$

Let's account the contributions to $\alpha(m_Z^2)$:

$$Q_f^2 \log(m_Z^2/4m_f^2) = \begin{cases} e & 22.8 \\ \mu & 12.1 \\ \tau & 6.5 \end{cases}$$

$u+d$	20.4	($m_f \sim 100$ MeV)
s	3.0	($m_f \sim 500$ MeV)
c	9.4	
b	1.5	

so $$\frac{1}{3\pi}\sum_f Q_f^2 \log(m_Z^2/4m_f^2) \cong 8.0$$

and we find

$$\alpha_0 = 1/137. \quad \Rightarrow \quad \alpha(m_Z^2) \approx 1/129$$

In this calculation, the effects of strong interactions on the quark contributions were ignored. When this effect is included carefully, one finds

$$\alpha^{-1}(m_Z^2) = 127.90 \pm .09$$

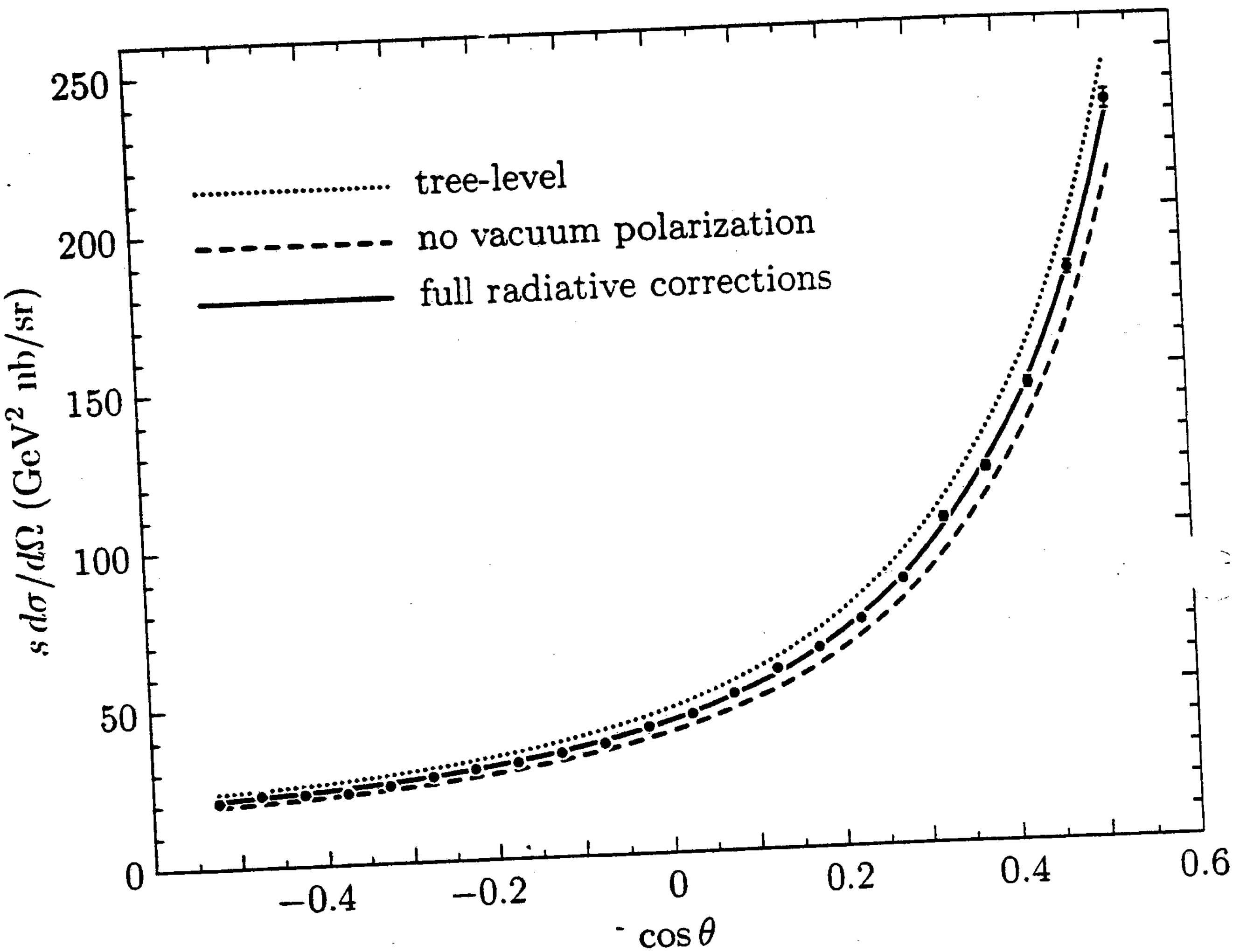
or $\alpha(m_Z^2) \sim 1/128$, as I claimed in the previous lecture.

The running of e^2 can be measured directly. Fig. 66 shows a measurement by the HRS experiment at PEP ($E_{cm} = 29$ GeV) of the cross section for $e^+e^- \to e^+e^-$. The effect of the running of e^2 is the difference between the dashed curve and the solid curve, which is small but significant.

The idea of running coupling constants is very interesting for the problem we have found in QCD. But the QED coupling runs the wrong way: it is strong at short distances and weak at large distances, while for QCD it must be the other way around: In QCD we need a <u>weak</u> coupling at large q^2 and a <u>strong</u> coupling at low q^2. I gave two arguments for the direction of the running in QED, one based on the physics of the dielectric vacuum ("vacuum polarization"), one based on the optical theorem (with the sign coming from $\sigma_{tot.} > 0$). So how could we possibly find an RG equation with the opposite sign?

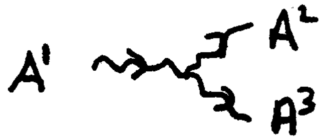
For definiteness, let's think about SU(2) Yang-Mills theory.

Figure 66



M. Derrick et al (HRS)
Phys. Rev. D34, 3286 (1986)

Then we have three vector bosons A_μ^a $a=1,2,3$. For SU(2), $f^{abc} = \epsilon^{abc}$, so there is a 3-boson coupling



Thus, there is a vacuum polarization effect, which gives a contribution

$$\frac{d}{d\log q^2}\, g_s^2(q^2) = +\,(\text{const})\,(g_s^4(q^2))$$

with a positive sign. However, there is also another effect which is special to theories with local gauge invariance.

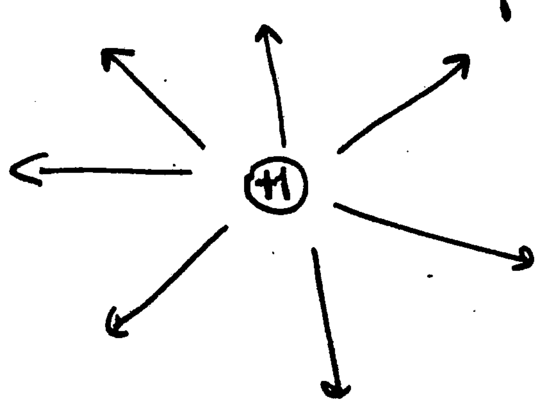
Let's study the generalized Gauss's law:

$$\vec{D}\cdot\vec{E}^a = \rho^a \qquad a=1,2,3$$

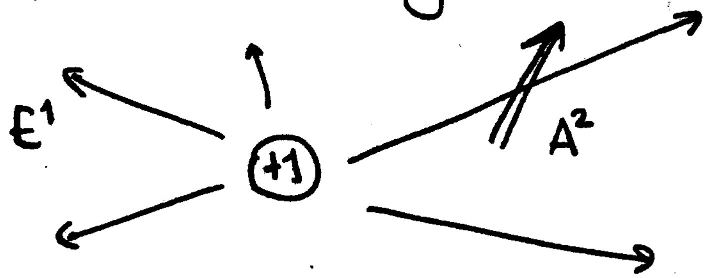
now $\vec{D}\cdot E^a = \vec{\nabla}\cdot\vec{E}^a + g_s\,\epsilon^{abc}\vec{A}^b\cdot\vec{E}^c$, so we can write this equation as

$$\vec{\nabla}\cdot\vec{E}^a = \rho^a - g_s\,\epsilon^{abc}\vec{A}^b\cdot\vec{E}^c$$

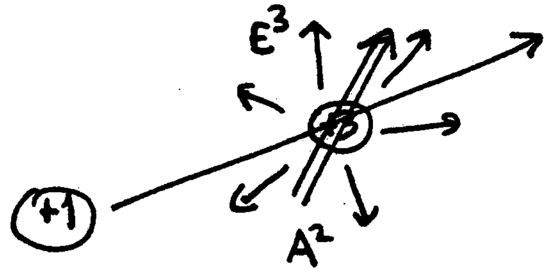
Then an explicit charge is a source of $\vec{E}^a$, but there is also a nonlinear effect in which $\vec{A}\cdot\vec{E}$ is a source of $\vec{E}$. With this in mind, put a charge of type $+1=a$ into the vacuum. It creates an $\vec{E}$ field of type 1:



Since quantum fluctuations of the $\vec{A}^a$ fields can appear and disappear in the vacuum, let's assume that an $\vec{A}^2$ field appears in some randomly chosen direction

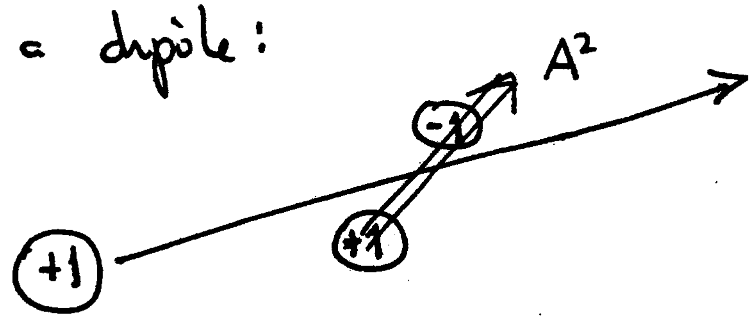


According to Gauss' law, this creates a source for $\vec{E}^3$
$(\epsilon^{321} = -1)$



Now we have $\vec{A}^2 \cdot \vec{E}^3 \neq 0$, so we have sources for E^1.
In fact, we get a dipole:

$\varepsilon^{123} = +1$



and the dipole points the wrong way! This dipole is just the opposite of that in a dielectric and tends to increase the coupling constant at large distances. It turns out that this new effect is 12 times as large as the effect of vacuum polarization. Thus, one finds an RG equation with a negative coefficient. The same effect appears in any Yang-Mills theory based on a non-Abelian group. For SU(N) Yang-Mills theory:

$$\frac{d}{d\log q^2}\, g^2(q^2) = -\frac{1}{(4\pi)^2}\left(\frac{11}{3}N\right)\left(g^2(q^2)\right)^2$$

The vacuum polarization due to quarks has an effect of the sign of vacuum polarization in QED. Thus, for QCD, we have

$$\frac{d}{d\log q^2}\, g^2(q^2) = -\frac{1}{(4\pi)^2}\left(11-\frac{2}{3}n_f\right) g^4(q^2)$$

where n_f is the number of quark flavors that are approximately massless at the given q^2.

The solution of the RG equation is

$$\alpha_s(q^2) = \frac{\alpha_s(M^2)}{1+\frac{(11-2/3 n_f)}{4\pi}\alpha_s(M^2)\log(-q^2/M^2)}$$

where $\alpha_s(M^2)$ is an initial or reference value of q^2. As q^2 decreases from M^2, $\alpha_s(q^2)$ increases. As q^2 increases, $\alpha_s(q^2)$ decreases; that is why this effect is called "asymptotic freedom".

Let me give some numerical examples, taking $n_f = 5$ for simplicity. I will show in a moment that

$$\alpha_s(m_Z^2) \approx 0.12$$

then we can compute at $q^2 = (10\text{ GeV})^2$ $\quad \alpha_s = 0.18$

at $q^2 = (1\text{ GeV})^2$ $\quad \alpha_s = 0.35$

and, formally $\alpha_s \to \infty$ for $q^2 \sim (100\text{ MeV})^2$

Now we can visualize the true structure of an event of $e^+e^- \to q\bar{q}$. The largest momentum transfers correspond to the shortest distances and the shortest times. First the photon materializes into $q\bar{q}$. Then the gluon of largest $p_\perp$ is emitted. Then more and more gluons of lower $p_\perp$ are emitted; as $p_\perp$ decreases, the effective value of α_s increases

Finally, as $p_\perp$ goes below about 1 GeV, we go over into Wilson's strong-coupling regime in which the many gluons form into color strings between the sources. The final state then contains jets of hadrons.

We see then that QCD with a running α_s can account for both the hard and the soft components of the strong interactions.

Finally, I would like to discuss precision tests of QCD. At large q^2, where QCD is weakly coupled, we can compute the dependence of QCD effects on α_s

By measuring these effects, we can determine the value of $\alpha_s(q^2)$ at the q^2 of the experiment. If QCD is correct, all of these determinations should fall on the curve $\alpha_s(q^2)$ which gives the solution to the RG equation.

A recent compilation of α_s measurements is shown in Fig. 67. This figure includes measurements of α_s from the following effects:

① The rate of $e^+e^- \to 3$ jets and multijets ("Event shapes")

② The rate of violation of Bjorken scaling ("DIS")

These two effects were discussed in lecture 4

③ The total cross-section for $e^+e^- \to$ hadrons ("R_ℓ", "R_τ")

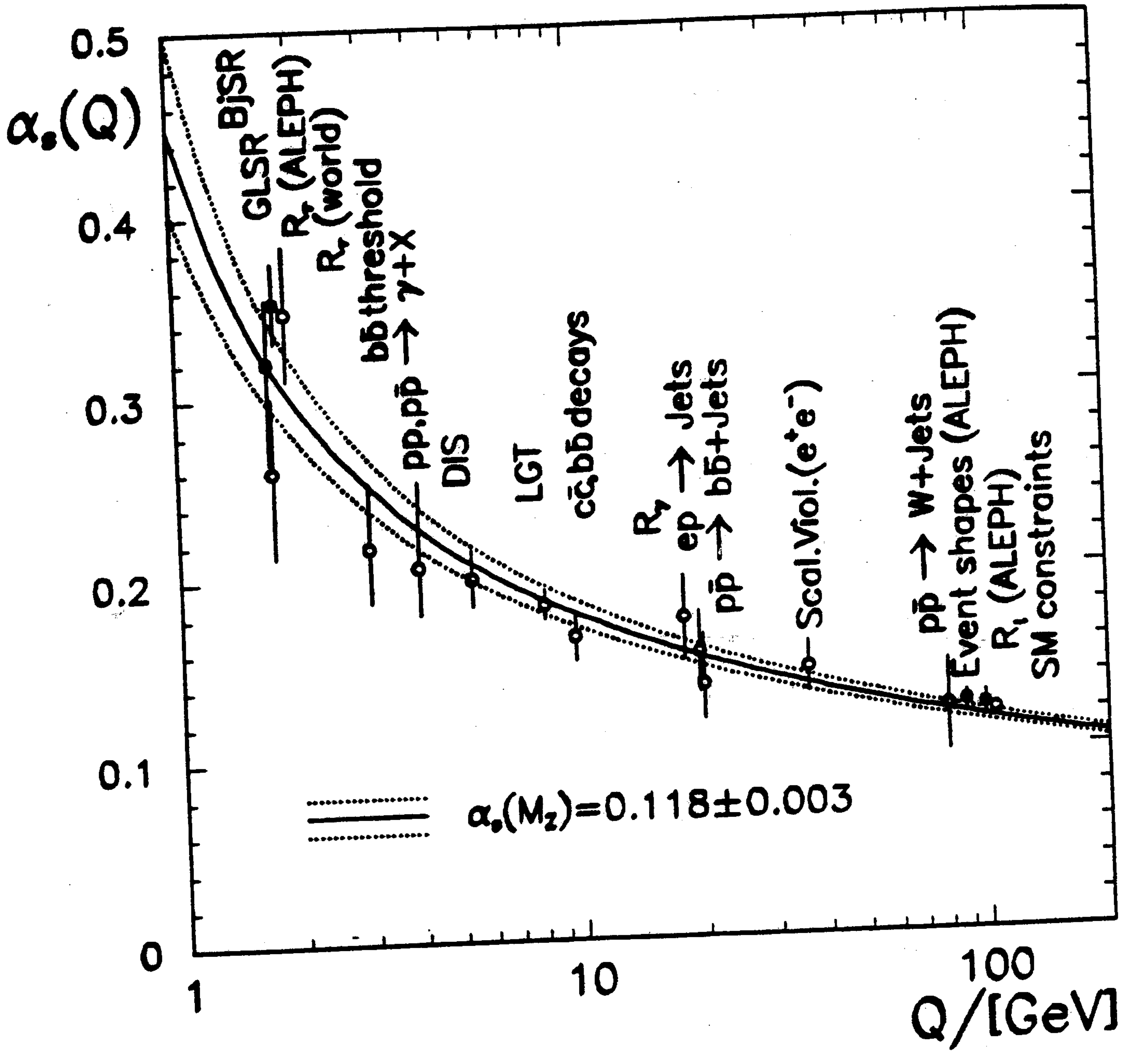
The process of gluon radiation increases the cross section for $e^+e^- \to$ hadrons. However, the diagram

interferes with the lowest-order diagram to decrease the cross-section. The net effect is a correction:

$$\sigma(e^+e^- \to \text{hadrons}) = \frac{4\pi\alpha^2}{3E_{cm}^2} \cdot 3 \sum_f Q_f^2 \left(1 + \frac{\alpha_s(E_{cm}^2)}{\pi} + \cdots\right)$$

The factor $3(1 + \frac{\alpha_s}{\pi} + \cdots) \approx 3.1$ appeared in our discussion of the Z^0 width in lecture 7. The α_s-dependent

Figure 67



R. Barate et al (ALEPH)
Phys. Repts. (to appear)

correction term is actually best measured in the lepton/hadron ratio in Z^0 decays (R_ℓ) and in the hadron/lepton ratio in τ decay (R_τ). The measurements give $\alpha_s(q^2)$ at $q^2 = m_Z^2$ and $q^2 = m_\tau^2$, covering a large dynamic range in q^2.

(4) Sum rules for deep inelastic scattering ("GLSR", "BjSR")

By a similar effect, the cross-sections in deep inelastic scattering which measure sum rules such as

$$\int_0^1 dx \left[f_u(x) + f_d(x) - f_{\bar{u}}(x) - f_{\bar{d}}(x) \right] = 3$$

are decreased by the factor

$$\left(1 - \frac{\alpha_s(Q^2)}{\pi} + \cdots\right) \approx 0.9 \quad \text{at} \quad Q^2 \sim (2\,\text{GeV})^2$$

(5) Numerical calculations of the $b\bar{b}$ bound state spectrum ("LGT")

By writing QCD on a discretized space-time, it is possible to compute the level spacing in the $b\bar{b}$ system numerically in terms of an underlying value of α_s. Comparing the results to experiment determines this α_s.

(6) The rate of jet production in $p\bar{p}$, pp, and ep collisions.

You see from the figure that all of these determinations of α_s are consistent with a common solution of the

RG equation, which corresponds to

$$\alpha_s(M_Z^2) = 0.118 \pm .003$$

Thus, while our knowledge of the strong interactions does not yet rival our understanding of the weak interactions, the idea that the strong interactions are given by a Yang-Mills theory of the group SU(3) is verified in very stringent experimental tests.

Lecture 9: August 6

In the first eight lectures of this series, I have described the structure of the strong, weak, and electromagnetic interactions. I have argued to you that these interactions result from a Yang-Mills theory with the symmetry $SU(3) \times SU(2) \times U(1)$ which acts on the quarks and leptons. And I have shown you a very substantial amount of experimental evidence which supports this conclusion.

I hope you are impressed by how much we understand about the interactions of particle physics. And I hope you are intrigued that much of this experimental knowledge has come quite recently, in particular, as a result of the LEP experiments. But the very clarity of the picture I have described brings into focus a list of new questions: Where did the symmetry group of the Standard Model come from? Why are there just these quarks and leptons and (apparently) no more? Where did the W and Z masses come from? Where did the quark and lepton masses come from, and why do these span a range of 6 orders of magnitude? The list goes on and on. In these last three lectures, I would like to discuss some tentative answers to a few of these questions. For the theories that I will present from here on, there is no definite experimental evidence. So you should treat these theories only as suggestions which need to be tested in the future, and for which you might look for alternatives.

The first problem I would like to discuss is the origin of the quark and lepton masses. In Lecture 6, when I wrote down the Lagrangian of weak-interaction theory, I completely avoided the issue of fermion masses. I did this because these mass terms pose a serious problem for the model. I told you that the mass term in a spin-½ Lagrangian is a coupling of the left- and right-handed helicity components.

$$\delta\mathcal{L} = -m(\psi_R^{\dagger}\psi_L + \psi_L^{\dagger}\psi_R)$$

But I also told you that the left-handed fermions have $I = ½$ while the right-handed fermions have $I = 0$ under the weak-interaction SU(2) symmetry. Thus, the standard mass term, e.g.

$$\delta\mathcal{L} = -m_e(e_R^{\dagger}e_L + e_L^{\dagger}e_R)$$

is an $I = ½$ object; it cannot be invariant under the local (even the global) SU(2) symmetry.

Here is a solution to this problem. Let's assume that there exists a boson field ϕ with the quantum numbers

$$I = ½ \qquad Y = +½$$

That is ϕ is a 2-component field with electric charge assignments

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \qquad \phi^0, \phi^+ \text{ complex}$$

which transforms as a spinor under SU(2). Then $\phi^\dagger \cdot L$,
$L = \binom{\nu_e}{e}_L$ would be an invariant under SU(2), and

$$\delta\mathcal{L} = -\lambda_e (e_R^\dagger \phi^\dagger \cdot L + h.c.)$$

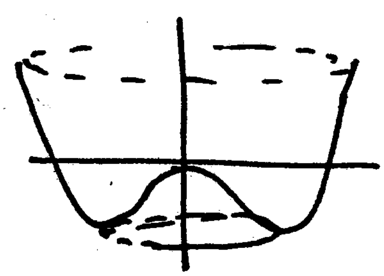
would be a term with $Y = +1 - \frac{1}{2} - \frac{1}{2} = 0$. So if we introduce ϕ into the theory, we are allowed to write this term in the Lagrangian. ϕ is called the "Higgs field".

This $\delta\mathcal{L}$ term could turn into an electron mass term if ϕ could somehow pick out the electron from the neutrino part of L. This can happen as follows: Assume that ϕ has an associate potential energy:

$$\mathcal{L}_\phi = \partial_\mu \phi^\dagger \partial^\mu \phi - V(\phi)$$

such that V forces $|\phi|$ to be nonzero. V does not have to violate SU(2) symmetry; a form such as

$$V(\phi) = - A\,\phi^\dagger\phi + B(\phi^\dagger\phi)^2$$



is quite sufficient. If $V(\phi)$ is invariant to SU(2) but the minimum lies at $\phi \neq 0$, there must be a continuous family of minima, since if ϕ_0 is a minimum, any SU(2) transform of this $\quad \phi_0' = e^{i\vec{\alpha}\cdot\vec{\sigma}/2}\phi_0$

must also be a minimum.

The full energy of the ϕ system is

$$E = \int d^3x \left\{ \vec{\nabla}\phi^\dagger \cdot \vec{\nabla}\phi + V(\phi) \right\}$$

Since it costs energy for ϕ to change from place to place, it must settle into <u>one</u> minimum everywhere in space. This chooses an <u>orientation</u> in the SU(2) group which was not present in the equations of motion. We write

$$\langle \phi(x) \rangle = \phi_0 \quad \text{the "vacuum expectation value of } \phi\text{"}$$

Since any spinor ξ can be rotated into any other one with the same value of $\xi^\dagger\xi$, we can choose coordinates so that

$$\langle \phi(x) \rangle = \phi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$$

The idea that minimizing the energy picks out a direction with respect to a symmetry of the equations of motion is a familiar one in solid-state physics. Think, for example, about a ferromagnet. Each iron atom has a free spin, which could in principle point in any direction. At high temperatures, the magnet exists in a disordered state with no overall spin orientation. However, below a certain temperature (T_c, the Curie temperature) the spins align with one another throughout the magnet and choose a direction for the overall magnetization. The Higgs field ϕ should have an analogous behavior. At very high temperature (perhaps in an early era in the universe) this field would be disordered. When the ambient temperature in the universe becomes sufficiently low, the Higgs field would have frozen throughout space into a definite orientation. This situation, in which the equations of motion are invariant under a symmetry but the state of minimum energy is asymmetrical, is called "spontaneous symmetry breaking".

Returning to the electron mass term, if we replace $\phi(x)$ by its vacuum expectation value in our standard coordinates

$$\langle\phi^\dagger\rangle\cdot L = \frac{1}{\sqrt{2}} v\, e_L^-$$

then

$$\delta\mathcal{L} = -\lambda_e\left(e_R^\dagger \phi^\dagger\cdot L + L^\dagger\cdot\phi\, e_R\right)$$

$$\rightarrow -\frac{\lambda_e v}{\sqrt{2}}\left(e_R^\dagger e_L + e_L^\dagger e_R\right)$$

and we obtain the electron mass term with $m_e = \frac{\lambda_e}{\sqrt{2}} v$.

We can give mass to all of the quarks and leptons in this way. Consider the $SU(3)\times SU(2)\times U(1)$-invariant terms:

$$\delta\mathcal{L} = -\lambda_e\, e_R^\dagger \phi^\dagger\cdot L - \lambda_d\, d_R^\dagger \phi^\dagger\cdot Q + \lambda_u\, u_R^\dagger \epsilon^{ab}\phi_a Q_b + \text{h.c.}$$

where $L = \begin{pmatrix}\nu_e\\ e^-\end{pmatrix}_L$ $Q = \begin{pmatrix}u\\ d\end{pmatrix}_L$, and in the last term I have

contracted ϕ_a and Q_b with the SU(2) invariant symbol ε^{ab}.
Replacing ϕ_a by $\langle\phi\rangle = \frac{1}{\sqrt{2}}\binom{0}{v}$, we find

$$\delta\mathcal{L} = -\left(\frac{\lambda_e v}{\sqrt{2}}\right) e_R^\dagger e_L - \left(\frac{\lambda_d v}{\sqrt{2}}\right) d_R^\dagger d_L - \left(\frac{\lambda_u v}{\sqrt{2}}\right) u_R^\dagger u_L + \text{h.c.}$$

The quantities in parentheses may be associated with m_e, m_d, m_u.
If we choose not to include in the theory a right-handed neutrino (which is in any event an object with $I = Y = 0$) then we generate zero masses for the neutrinos.

The introduction of ϕ has one more interesting consequence. The kinetic energy term for ϕ is

$$\mathcal{L}_k = D_\mu\phi^\dagger D^\mu\phi$$

where $D_\mu = \left(\partial_\mu - ig A_\mu^a \frac{\sigma^a}{2} - ig' B_\mu \cdot \frac{1}{2}\right)$ since ϕ has $Y = \frac{1}{2}$
Replacing ϕ by its vacuum expectation value, we find:

$$\mathcal{L}_L = \frac{1}{2}(0\ v)\left[g A_\mu^a \frac{\sigma^a}{2} + g' B_\mu \cdot \frac{1}{2}\right]\left[g A^{\mu b}\frac{\sigma^b}{2} + g' B \frac{1}{2}\right]\binom{0}{v}$$

now

$$A_\mu^a\sigma^a A^{\mu b}\sigma^b = A_\mu^a A^{\mu a} \qquad (\sigma^a)^2 = 1$$

and the cross term between A and B involves

$$(0\ v)\ \sigma^a \binom{0}{v} = \begin{cases} 0 & a = 1,2 \\ -v^2 & a = 3 \end{cases}$$

then

$$\mathcal{L}_K = \frac{1}{2}\left(\frac{g^2 v^2}{4}\left[(A_\mu^1)^2 + (A_\mu^2)^2\right] + \frac{(-gA_\mu^3 + g' B)^2 v^2}{4}\right)$$

This is a mass term for A^a_μ B. The mass matrix is read off as

$$\mathcal{L} = \frac{1}{2}\,(A^1 A^2 A^3 B)\,(\underset{\sim}{m}^2)\begin{pmatrix} A^1 \\ A^2 \\ A^3 \\ B \end{pmatrix}$$

and we find

$$m^2 = \frac{v^2}{4}\begin{pmatrix} g^2 & & & \\ & g^2 & & \\ & & g^2 & -gg' \\ & & -gg' & (g')^2 \end{pmatrix}$$

This is exactly the mass term that we postulated at the end of lecture 6!

This brings us back to the formula

$$m_W = \frac{g}{2}\,v$$

Since we now know the value of g $\left(\frac{g^2}{4\pi} = \frac{1}{29.5}\right)$ we can compute v, and find

$$v = 246.\ \text{GeV}$$

The idea of spontaneous symmetry breaking thus allows us to write locally symmetric terms which give mass to $W^+ W^- Z^0$ and all of the fermions of the Standard Model. If the agent of spontaneous symmetry breaking is the Higgs field ϕ, we can obtain all of these masses in a simple way from just this field. However, this explanation leaves much to be desired. We do not explain the values of the quark and lepton masses; each of these is proportional to a new adjustable constant λ_f. (At best, we explain why all of these masses are less than v.) And we do not explain why ϕ acquires a vacuum expectation value. To build a ferromagnet, we need iron atoms with exchange interactions among their d-electrons. Presumably some similarly complicated physical mechanism is needed to drive ϕ away from the symmetrical configuration $\langle\phi\rangle = 0$.

Even the explanation for the W and Z masses is less than meets the eye. The mass matrix on p.7 is the most general one satisfying three conditions:

(1) The terms quadratic in A^a_μ are symmetric among $a = 1, 2, 3$.

(2) The fields A^a_μ B_μ couple proportional to their coupling constants g, g'.

(3) The matrix has a zero eigenvalue.

Condition (1) is satisfied by the Higgs model, but presumably also by many other models. Condition (2) follows from the structure of D_μ and is true in any model in which the W and Z masses come from spontaneous symmetry breaking. Condition (3) can be shown to be equivalent to the condition that $\langle\phi(x)\rangle$ leaves one gauge symmetry unbroken. It is not hard to see that this is so: Consider SU(2)×U(1) transformations

$$e^{i\alpha^a\sigma^a/2}\, e^{i\beta\cdot Y}$$

with $Y = \frac{1}{2}$ for ϕ, $\alpha^3 = \beta = \gamma$, $\alpha^1, \alpha^2 = 0$. The resulting matrices

$$\begin{pmatrix} e^{i\gamma} & 0 \\ 0 & 1 \end{pmatrix} \qquad \text{(which form a U(1) group)}$$

leave $\langle \phi \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$ unchanged. Note that the upper component of ϕ, ϕ^+, has charge $+1$ under this transformation. This unbroken $U(1)$ group is the local symmetry group of electromagnetism. Because it is not spontaneously broken, one vector boson — the photon — must remain massless.

There are many models in which $SU(2) \times U(1)$ is spontaneously broken to $U(1)$. In some of these there are several bosonic Higgs fields. In others, there is no elementary Higgs fields. For example, the symmetry might be broken by a condensation of fermion pairs in the ground state, as in the theory of superconductivity.

I'll come back to the question of the origin of spontaneous symmetry breaking in Lectures 10 and 11. Now I would like to address another question: Where did the group $SU(3) \times SU(2) \times U(1)$ come from? Here there is a very beautiful suggestion, due to Georgi and Glashow and Pati and Salam: The symmetry group of the Standard Model is the result of spontaneous symmetry breaking of a larger group which unifies all of the Standard Model gauge bosons.

Georgi and Glashow* proposed that the "grand unification" symmetry group is $SU(5)$. Here is how it works. The generators of $SU(5)$ are 5×5 Hermitian matrices T^a satisfying

$$\mathrm{tr}\, T^a = 0 \qquad \mathrm{tr}\,[T^a T^b] = \tfrac{1}{2}\delta^{ab}$$

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* Everyone should read their paper: Phys. Rev. Letters, 32, 438 (1974).

Introduce a field Φ which is a 5×5 Hermitian matrix with the transformation law

$$\Phi(x) \to \mathcal{U}(x)\,\Phi(x)\,\mathcal{U}^\dagger(x) \qquad \mathcal{U} = e^{i\alpha^a T^a}$$

The infinitesimal form of this transformation is

$$\Phi \to (1+i\alpha^a T^a)\,\Phi\,(1-i\alpha^a T^a)$$

or

$$\Phi \to \Phi + i\alpha^a[T^a,\Phi] + O(\alpha^2)$$

Now let Φ obtain the vacuum expectation value

$$\langle\Phi\rangle = \begin{array}{c} \left(\begin{array}{ccc|cc} A & & & & \\ & A & & & \\ & & A & & \\ \hline & & & B & \\ & & & & B \end{array}\right) \end{array} \begin{array}{l} \left.\begin{array}{c} \\ \\ \\ \end{array}\right] \text{3 rows} \\ \left.\begin{array}{c} \\ \\ \end{array}\right] \text{2 rows} \end{array}$$

By the principle discussed on p.8, the generators of SU(5) which leave ~~this~~ expectation value invariant will lead to massless vector bosons, while the generators of SU(5) that transform $\langle\Phi\rangle$ will yield massive vector bosons. According to the transformation law above, the generators giving massless bosons will be those that commute with $\langle\Phi\rangle$. These are:

$$\left(\begin{array}{c|c} t^a & 0 \\ \hline 0 & 0 \end{array}\right) \qquad \left(\begin{array}{c|c} & \\ \hline & \sigma^a/2 \end{array}\right) \qquad \sqrt{\tfrac{3}{5}}\left(\begin{array}{ccc|cc} -\tfrac{1}{3} & & & & \\ & -\tfrac{1}{3} & & & \\ & & -\tfrac{1}{3} & & \\ \hline & & & +\tfrac{1}{2} & \\ & & & & +\tfrac{1}{2} \end{array}\right)$$

t^a a 3×3 generator of SU(3)

$\sigma^a/2$ a 2×2 generator of SU(2)

commutes w. the others and generates a U(1)

For reasons that will be clear in a moment, I will call the last of these matrices

$$\sqrt{\tfrac{3}{5}}\, Y \qquad\qquad Y = \text{hypercharge}$$

the factor is chosen so that

$$\mathrm{tr}\left(\sqrt{\tfrac{3}{5}}Y\right)^2 = \tfrac{3}{5}\left[\tfrac{1}{9}\cdot 3 + \tfrac{1}{4}\cdot 2\right] = \tfrac{3}{5}\cdot\left[\tfrac{1}{3}+\tfrac{1}{2}\right] = \tfrac{1}{2}$$

in accord with our normalization convention.

It is natural to put fermions into the 5-dimensional representation of SU(5). But I hope it is alright if I choose instead

$$\psi_L \quad \text{in} \quad \bar{5} \quad (\text{anti-}5) \quad \text{of } SU(5)$$

then

$$\psi_L = \begin{pmatrix} \bar{d}_1 \\ \bar{d}_2 \\ \bar{d}_3 \\ \hline L_1 \\ L_2 \end{pmatrix}_L \begin{array}{l} \left.\right] \text{triplet of fields w. color } \bar{3},\ I=0 \\ \left.\right] \text{2 fields forming } I=\tfrac{1}{2},\ \text{color } 1 \end{array}$$

To find the hypercharge of ψ_L, apply $-Y$; then

$$\bar{d}_i \text{ has } Y = +\tfrac{1}{3} \qquad\qquad L_i \text{ has } Y = -\tfrac{1}{2}$$

The $\bar{d}_L$ are the anti<u>particles</u> of right-handed fields

$$d_R \quad \text{w. color } 3,\ I=0 \quad Y = -\tfrac{1}{3}$$

The L_i have

$$L_{iL} \quad \text{w. no color} \quad I=\tfrac{1}{2},\ Y=-\tfrac{1}{2}$$

If we were to choose more fermions in the 5 of SU(5), we would obtain a left-right symmetric model. The real world is not left-right symmetric. So we need to put the other fermions into a representation that transforms differently under SU(5). The magic choice is

$$\Psi_L^{ij} \qquad i,j = 1-5 \qquad \underline{\text{antisymmetric}}$$

The components of Ψ_L are:

$$\Psi_L = \left(\begin{array}{ccc|cc} 0 & \bar{u}_3 & -\bar{u}_2 & u_1 & d_1 \\ -\bar{u}_3 & 0 & \bar{u}_1 & u_2 & d_2 \\ u_2 & -\bar{u}_1 & 0 & u_3 & d_3 \\ \hline -u_1 & -u_2 & -u_3 & 0 & \bar{e} \\ -d_1 & -d_2 & -d_3 & -\bar{e} & 0 \end{array}\right)_L$$

$\begin{pmatrix} u_i \\ d_i \end{pmatrix}_L$ form a color 3 $I = \frac{1}{2}$

$\bar{e}_L$ has no color and has $I = (\frac{1}{2} \times \frac{1}{2})_{\text{antisymmetric}} \Rightarrow I = 0$

$\bar{u}_{iL}$ has $I = 0$ and (it turns out) color $\bar{3}$

To find the hypercharges, apply Y on both indices; then

$$Y(\bar{u}) = -\frac{1}{3} - \frac{1}{3} = -\frac{2}{3} \qquad Y(u,d) = -\frac{1}{3} + \frac{1}{2} = \frac{1}{6}$$

$$Y(\bar{e}) = \frac{1}{2} + \frac{1}{2} = 1$$

then $\begin{pmatrix} u \\ d \end{pmatrix}_L$ has color 3, $I = \frac{1}{2}$, $Y = \frac{1}{6}$

$\bar{u}_L$ is the antiparticle of u_R w. color 3, $I=0$, $Y = \frac{2}{3}$

$\bar{e}_L$ is the antiparticle of e_R^- w. no color, $I=0$, $Y=-1$

The SU(5) gauge theory with

$$\psi_{Li} + \Psi^{ij}_L$$

contains exactly one multiplet of fermions (one "generation")

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad e^-_R \quad \begin{pmatrix} u \\ d \end{pmatrix}_L \quad u_R \quad d_R$$

and their antiparticles, coupling correctly to $SU(3)\times SU(2)\times U(1)$. By introducing three of these multiplets, we would account for the Standard Model.

SU(5) grand unification makes a very strong prediction about the relations among the Standard Model coupling constants. The SU(5) covariant derivative is

$$D_\mu = (\partial_\mu - i g_5 A^a_\mu T^a)$$

$$= \partial_\mu - i g_5 A^a_\mu \begin{pmatrix} t^a & \\ & \end{pmatrix} - i g_5 A^i_\mu \begin{pmatrix} & \\ & \sigma^i/2 \end{pmatrix}$$

$$- i g_5 \sqrt{\tfrac{3}{5}}\, A^Y_\mu\, Y - (\text{massive bosons})$$

We should identify the gauge bosons in this formula with those of the Standard Model. Then we can read off:

$$g_s = g_w = g_5 \qquad g' = \sqrt{\tfrac{3}{5}}\, g_5$$

This gives the unfortunate relations

$$\frac{\alpha_w}{\alpha_s} = 1 \qquad \sin^2\theta_w = \frac{(g')^2}{g^2 + (g')^2} = \frac{3}{8}$$

So it would seem that the model makes no sense: g_s is predicted to be too small, and g' is predicted to be too large.

But, we have to remember that g_s, g, g' are running coupling constants. g_s becomes weaker at short distances. g' is a $U(1)$ coupling and so becomes larger at short distances. So in principle the relations at the bottom of p. 13 could be valid at some very small distance scale. If it is so, that is where we should find the grand unification.

Let's analyze this quantitatively. Write

$$g_1 = \sqrt{\tfrac{5}{3}}\, g' \quad , \quad g_2 = g \quad , \quad g_3 = g_s$$

The best current values are: all at $q^2 = m_Z^2$

$$\alpha_1 = 1/58.98 \pm .08 \qquad \leftarrow \text{from } \alpha_w, \sin^2\theta_w$$

$$\alpha_2 = 1/29.60 \pm .04$$

$$\alpha_3 = 1/8.47 \pm .22$$

The couplings obey RG equations

$$\frac{d}{d\log q^2}\, g_i^2(q^2) = -\frac{1}{(4\pi)^2}\, b_i \left(g_i^2(q^2)\right)^2$$

where

$$b_3 = 11 - \tfrac{4}{3} n_g \qquad n_g = \#\text{ generations} = 2 \cdot \#q$$

$$b_2 = \tfrac{22}{3} - \tfrac{4}{3} n_g$$

$$b_1 = \quad - \tfrac{4}{3} n_g$$

the first term in the coefficient b_i is $\frac{11}{3}N$ (see p.15 of lecture 8). The second term is the contribution of fermions; this must be SU(5) symmetric because the fermions fill SU(5) multiplets. If we assume that no other particles contribute to the vacuum polarization up to the grand unification scale, we can integrate these equations and see if the g_i^2 converge. Since the solution to the RG equation is

$$g_i^2(q^2) = \frac{g_i^2(M^2)}{\left(1 + (b_i/16\pi^2)\, g_i^2(M)\log q^2/M^2\right)}$$

or

$$\alpha_i^{-1}(q^2) = \alpha_i^{-1}(M^2) + \frac{b_i}{4\pi}\log\frac{q^2}{M^2}$$

it is simplest to plot $\alpha_i^{-1}(q^2)$. This plot is shown in Fig. 68.

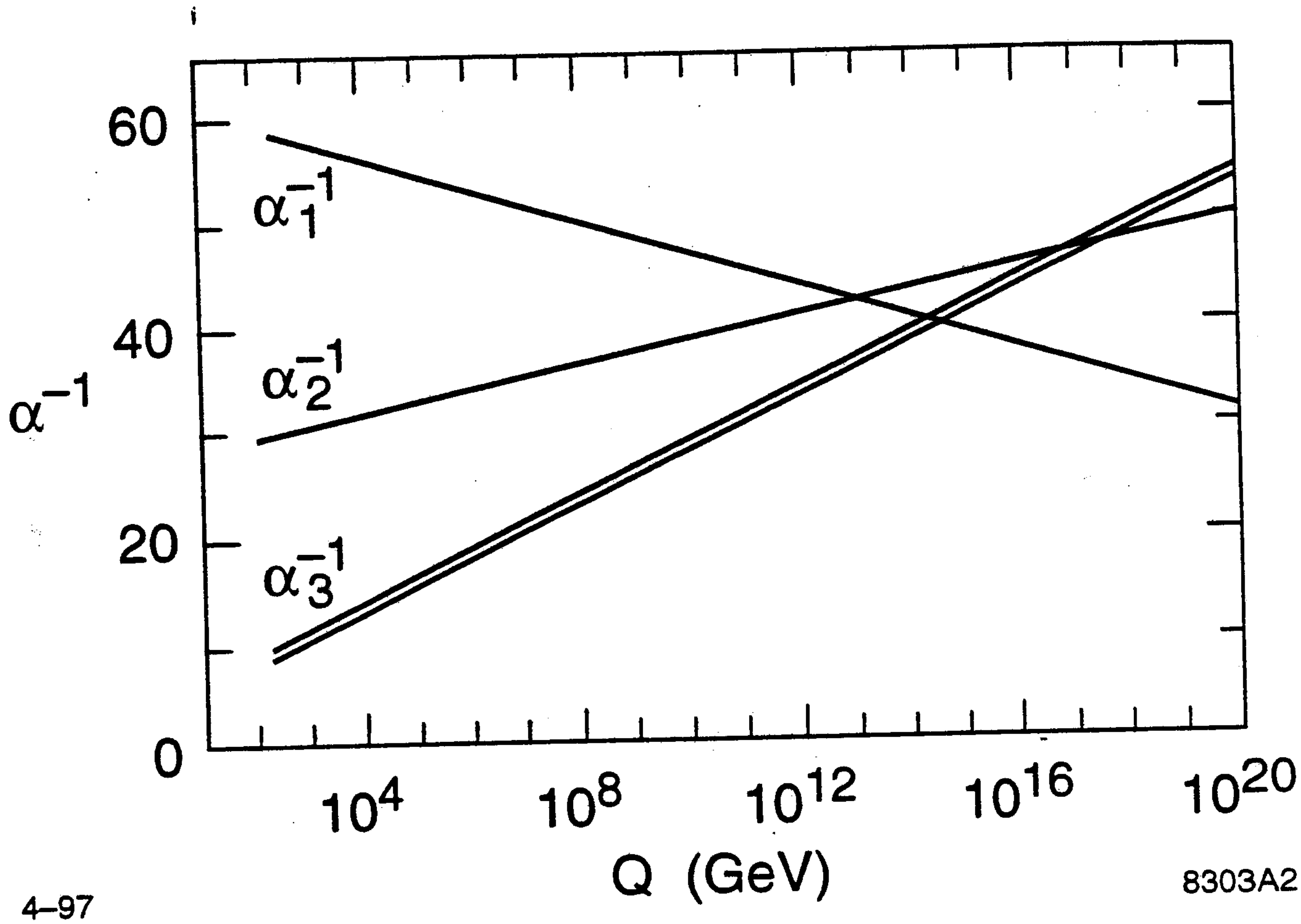
After all of this buildup, the result is somewhat disappointing. The three couplings $\alpha_1, \alpha_2, \alpha_3$ do not come close to converging. They do not even come close until

$$Q \sim 10^{14}\ \text{GeV}$$

which puts any possible grand unification at a very inaccessible energy.

If we try to rescue grand unification by adding new species between accessible energies ($Q \sim 100$ GeV) and very high Q, we would like a criterion for the necessity of the

Figure 68


60
40
20
0
α^{-1}
α_1^{-1}
α_2^{-1}
α_3^{-1}
10^4
10^8
10^{12}
10^{16}
10^{20}
Q (GeV)
4–97
8303A2

coupling constants. We can obtain one as follows: We have three equations for the $\alpha_i^{-1}(q^2)$. If the couplings meet, these depend on two unknowns, $\alpha^{-1}(M_U^2)$ and M_U^2, where M_U is the meeting point. If we eliminate these unknowns, we get a relation for α_3 in terms of α_1 and α_2:

$$\alpha_3^{-1} = (1+B)\,\alpha_2^{-1} - B\,\alpha_1^{-1} \qquad \text{at any } q^2$$

where
$$B = \frac{b_3 - b_2}{b_2 - b_1}$$

From the values of the $\alpha_i(m_Z^2)$ on p. 14, one finds

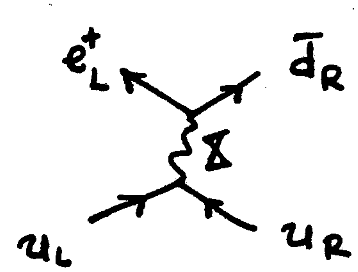
$$B = 0.719 \pm .008 \pm .03$$

where the second error is my guess about the expected size of higher-order corrections not included in this simple analysis. On the other hand, the Standard Model gives (from p. 14)

$$B = \tfrac{1}{2}$$

plus a 3% contribution from the Higgs field.

There is another, more direct way to test grand unification. If quarks and leptons are in the same SU(5) multiplet, it ought to be possible to make transitions between quarks and leptons. For example, the SU(5) model contains the process



mediated by one of the bosons which becomes heavy when Φ acquires a vacuum expectation value. This process allows

$$p(uud) \longrightarrow e^+ + \pi^0 \; (\bar{d}d)$$

so it allows the proton to decay!

We know that protons do not decay very often, but we also know that the unification mass M_U is very large. So we might estimate the rate of proton decay as

$$\Gamma(p \to e^+\pi^0) \sim \frac{\alpha_U}{4\pi} \cdot \frac{1}{M_U{}^4} \cdot m_p^5$$

where α_U is the unification coupling, M_U^{-4} appears for the same reason as we found m_W^{-4} in Lecture 5, and m_p^5 is needed to give this expression the correct dimensions. Putting

$$\alpha_U \sim \frac{1}{40} \qquad M_U \sim 10^{15}\,\text{GeV}$$

we find

$$\Gamma(p \to e^+\pi^0) \sim 1.4 \times 10^{-63}\,\text{GeV}$$

$$= 2 \times 10^{-39}/\text{sec}$$

$$= 7 \times 10^{-32}/\text{yr}$$

and we would expect a proton lifetime of order 10^{31} yr.

This long a lifetime is a challenge to measure, but it is not impossible. 1 kg of water (1 liter) contains 6×10^{26} protons and neutrons. A cube 10 m. on a side contains 6×10^{32} nucleons. So make such a cube, fill it with water, and watch it *very* carefully for a year. In the 1980's two groups, the IMB and Kamiokande experiments, made even larger cubes in underground caverns. They used the water as a Cherenkov detector for high energy particles. They saw *many* events due to cosmic-ray muons and neutrinos, but no nucleon decay. The resulting limit is

$$\Gamma(p\to e^+\pi^0) < 1/(5.5\times10^{32}\,\text{yr}) \qquad (90\%\ \text{conf.})$$

with less strong limits on many other nucleon decay modes.

This is an amazing conclusion. The Georgi-Glashow SU(5) grand unified theory has as its basic scale

$$M_U \sim 10^{14}-10^{15}\ \text{GeV}$$

Yet I have shown that it is testable experimentally, and that in fact it *fails* two significant tests.

Still, the idea of grand unification is a very beautiful one, and perhaps we shouldn't give it up yet. I'll discuss it further in the next lecture.

Lecture 10: August 11

In the previous lecture, I discussed the fact that the masses of quarks, leptons, and gauge bosons arise from spontaneous breaking of $SU(2) \times U(1)$ symmetry. I introduced the Higgs field ϕ as the agent of this symmetry-breaking. This led to a consistent but not very attractive theory. In particular, the theory of a single Higgs field does not explain *why* $\langle \phi \rangle \neq 0$, it just *assumes* this.

There are many more models of spontaneous $SU(2) \times U(1)$ symmetry breaking. Some of these involve only elementary scalar fields. Others postulate new fermions and even new strong interactions at energies a few times the scale

$$v = 246 \text{ GeV}$$

Many people believe that the field that breaks $SU(2) \times U(1)$ is a composite of more elementary constituents bound by new interactions.

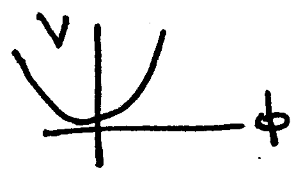
I do not have time in these lectures to review the whole spectrum of models of $SU(2) \times U(1)$ breaking, and I am not sure that that would be appropriate anyway. Instead, I will focus on one particular class of models that I think are quite interesting. The main idea of these models is to embrace the idea that the Higgs boson is an elementary field. To constrain the properties of the Higgs, I will introduce symmetries that relate the Higgs field to other fields in the model. This lecture will mainly be concerned with formal aspects of these symmetries, which I hope you will find intriguing. In the next lecture, I will discuss the implications of these symmetries for elementary particle physics.

I hope you have already noticed that there is an important difference between the Higgs scalar field and the quark, lepton, and vector boson fields of the Standard Model. The Standard Model fields can only obtain mass by the spontaneous breaking of $SU(2) \times U(1)$. However, the Higgs field can always have a mass term:

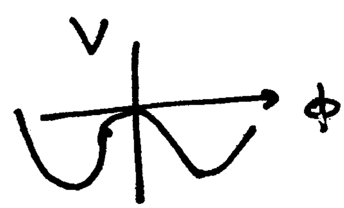
$$\delta \mathcal{L} = -\mu^2 \phi^\dagger \phi$$

this term is $SU(2) \times U(1)$ invariant even though ϕ has $I = \frac{1}{2}$.

If μ^2 is positive, we get a stable potential



If μ^2 is negative, we get a potential with spontaneous symmetry breaking



and either choice is consistent with all symmetries. But maybe if we could add a symmetry that forbid the mass term for ϕ — so that ϕ could obtain a mass only by spontaneous symmetry breaking — this symmetry might then give us insight into the form of the potential for ϕ.

One way to achieve this is to add a symmetry that turns ϕ into a fermion field. Such a symmetry, the turns bosons into fermions and fermions into bosons, is called a supersymmetry. (The obvious alternative, to turn ϕ into a vector field, has many problems which are solved only if the theory also has a supersymmetry.)

Let's try to write a reasonable form for a supersymmetry transformation. The smallest fermionic system is a massless left-handed fermion field (ψ_L) Lagrangian,

$$\mathcal{L} = \psi_L^{\dagger} i \bar{\sigma}^{\mu} \partial_{\mu} \psi_L$$

which describes a left-handed fermion and its right-handed antiparticle. Since there are two particles here, we should connect this system with a system of two bosons, and so we need a complex field ϕ

$$\mathcal{L} = \partial^{\mu} \phi^{*} \partial_{\mu} \phi$$

Before I describe the symmetries of this system, I'd better tell you an important fact about fermion fields. The classical boson field is a number and obeys, straightforwardly

$$\phi(x)\,\phi(y) = \phi(y)\,\phi(x).$$

However, the classical fermion field must take values in a field of anticommuting numbers

$$\psi(x)\,\psi(y) = -\psi(y)\,\psi(x)$$

this property is needed so that the quantum particles will obey the Pauli principle of antisymmetrized wavefunctions

$$|x,y\rangle = -|y,x\rangle$$

As an illustration of these use of this property, let me prove the equivalence of left-handed and right-handed fermion fields.

Let

$$c = -i\sigma^2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \qquad c^\dagger = -c \qquad cc^\dagger = 1$$

This matrix has the property that

$$c\,\vec{\sigma}\,c^\dagger = -(\vec{\sigma})^T$$

thus

$$c\,\sigma^\mu c^\dagger = +(\bar{\sigma}^\mu)^T \qquad c\,\bar{\sigma}^\mu c^\dagger = +(\sigma^\mu)^T$$

Let

$$\phi_R = c\,\psi_L^* \qquad \phi_R^\dagger = -\psi_L^T c = \psi_L^T c^\dagger$$

then

$$S = \int d^4x\, \phi_R^\dagger\, i\sigma^\mu \partial_\mu \phi_R = \int d^4x\, \psi_L^T c^\dagger\, i\sigma^\mu c\, \partial_\mu \psi_L^*$$

$$= \int d^4x\, \psi_L^T\, i(\bar{\sigma}^\mu)^T \partial_\mu \psi_L^* \qquad \text{integrate by parts}$$

$$= \int d^4x \left(-\partial_\mu \psi_L^T\, (i\bar{\sigma}^\mu)^T \psi_L^*\right) \qquad \text{reverse order of } \psi_L \text{ and } \psi_L^*$$

$$= \int d^4x\, \psi_L^\dagger\, i\bar{\sigma}^\mu \partial_\mu \psi_L$$

The minus sign from interchanging fermion fields is needed for this consistency. We also now have a formula for converts from left-handed to right-handed fields or vice versa if that should be convenient.

Now I would like to propose a symmetry of the system

$$\mathcal{L} = \partial^\mu \phi^\dagger \partial_\mu \phi + \psi_L^\dagger i \bar{\sigma}^\mu \partial_\mu \psi_L$$

In infinitesimal form

$$\delta_\xi \phi = \sqrt{2}\, \xi_R^\dagger \psi_L$$

$$\delta_\xi \psi_L = -\sqrt{2} i\, \sigma^\mu \partial_\mu \phi\, \xi_R$$

The extra derivative in the second equation is needed because the ψ kinetic term has one less derivative. Comparing the two formula, we see that

$$\partial_\mu \cdot \xi_R \cdot \xi_R^\dagger \sim \text{dimensionless}$$

or

$$\xi_R \sim (\text{cm})^{1/2}$$

Also, ξ_R is an anticommuting parameter.

To see that δ_ξ is a symmetry, compute directly:

$$\delta_\xi \mathcal{L} = \partial^\mu(\sqrt{2}\,\psi_L^\dagger \xi_R)\partial_\mu \phi + \psi_L^\dagger i \bar{\sigma}^\mu \partial_\mu(-i\sqrt{2}\sigma^\nu \partial_\nu \phi\, \xi_R)$$
$$+ (\text{terms w. } \xi_R^\dagger)$$

$$= \sqrt{2}\,\psi_L^\dagger \xi_R(-\partial^\mu \partial_\mu \phi) + \psi_L^\dagger [\bar{\sigma}\cdot\partial\, \sigma\cdot\partial\, \phi]\xi_R$$
$$+ (\text{terms w. } \xi_R^\dagger)$$

Recall that $(\bar{\sigma}^\mu \partial_\mu)(\sigma^\nu \partial_\nu) = \partial^\mu \partial_\mu$; then the two terms cancel and $\underline{\delta_\xi \mathcal{L} = 0}$.

This construction looks like something very special to the case of free fields, but it turns out to be the tip of a complex and profound phenomenon. To see how to generalize this construction, let's compute the algebra of supersymmetry transformations. We have discussed in lecture 6 that a standard group of continuous symmetries is generated by a Lie algebra of operators T^a with closed commutation relations

$$[T^a, T^b] = i f^{abc} T^c$$

For supersymmetry, we should try to find the rest of the algebra by computing, for supersymmetry transformations ξ, η

$$[\delta_\xi, \delta_\eta]$$

Acting on $\phi(x)$:

$$\begin{aligned}
[\delta_\xi, \delta_\eta]\phi &= \delta_\xi \delta_\eta \phi - (\xi \leftrightarrow \eta) \\
&= \delta_\xi (\sqrt{2}\, \eta_R^\dagger \psi_L) - (\xi \leftrightarrow \eta) \\
&= \sqrt{2}\, \eta_R^\dagger (-\sqrt{2}\, i\, \sigma^\mu \partial_\mu \phi\, \xi_R) - (\xi \leftrightarrow \eta) \\
&= -2i(\eta_R^\dagger \sigma^\mu \xi_R - \xi_R^\dagger \sigma^\mu \eta_R)\, \partial_\mu \phi
\end{aligned}$$

write:

$$a^\mu(\xi, \eta) = -2i\,(\eta_R^\dagger \sigma^\mu \xi_R - \xi_R^\dagger \sigma^\mu \eta_R)$$

Then

$$[\delta_\xi, \delta_\eta]\phi = a^\mu \partial_\mu \phi$$

A somewhat more difficult calculation gives the result

$$[\delta_\xi, \delta_\eta]\psi_L = a^\mu \partial_\mu \psi_L \quad + \text{(terms = 0 by the eqs. of motion)}$$

The right-hand side of these equations is an infinitesimal translation. In quantum mechanics, the operators which generate translations in space and time are $(H, \vec{P}) = P^\mu$ e.g.

$$e^{-iHa^0}\,\psi(t) = \psi(t+a^0)$$

or $$(1 - iHa^0 + \cdots)\,\psi(t) = \psi(t) + a^0\partial_0\psi(t) + \cdots$$

Representing δ_ξ and δ_η also by operators: $\delta_\xi = \xi_R^\dagger Q + Q^\dagger \xi_R$ we find that our commutation relations take the form

$$\{Q_\alpha, Q_\beta^\dagger\} = 2\,\sigma^\mu_{\alpha\beta}\,P_\mu \qquad \text{and} \qquad \{Q_\alpha, Q_\beta\} = 0$$

(the commutator turns into an anticommutator by the manipulation

$$-\,Q^\dagger\cdot\eta_R\,\xi_R^\dagger Q = +\,\xi_{R\alpha}^\dagger\,\eta_{R\beta}\,Q_\beta^\dagger Q_\alpha$$

because ξ, η are anticommuting numbers.) Evaluating this explicitly for $\alpha, \beta = 1$

$$Q_1 Q_1^\dagger + Q_1^\dagger Q_1 = 2\,(H - P^3) \quad , \quad Q_1^2 = 0$$

This equation says that the generators of supersymmetry $Q_\alpha, Q_\alpha^\dagger$ are essentially square roots of energy-momentum. In fact, this implies that the structure of supersymmetry is to replace the algebra of space-time translations by a larger algebra which includes transformations that carry spin-½.

Would we find this same structure in more general theories with fermion-boson symmetry? In fact, we must. If Q is a symmetry which converts fermions to bosons and vice versa, it must carry spin-½. So we would then have a structure

$$[Q_\alpha, H] = [Q_\alpha^+, H] = 0 \qquad \alpha = 1,2$$

Consider the operator

$$\{Q_\alpha, Q_\beta^+\}$$

This object transforms as a 4-vector under Lorentz transformations, and it commutes with H; explicitly

$$\{Q_\alpha, Q_\beta^+\} = 2\sigma^\mu_{\alpha\beta} R_\mu \qquad [R_\mu, H] = 0$$

Further, $R_\mu \neq 0$, as.

$$\langle A|(Q_1 Q_1^+ + Q_1^+ Q_1)|A\rangle = \| Q_1^+|A\rangle\|^2 + \|Q_1|A\rangle\|^2 > 0$$

So what can R_μ be? If $R_\mu \neq P_\mu$, it must be another conserved 4-vector. But, by a theorem of Coleman and Mandula, this is impossible in an interacting quantum field theory. I'll illustrate this theorem by considering the elastic scattering of scalar particles. For scalar particle states $|p_i\rangle$

$$\langle p_i| P^\mu |p_i\rangle = p^\mu$$

$$\langle p_i| R^\mu |p_i\rangle = r_i p^\mu \qquad \text{since } p^\mu \text{ is the only available 4-vector}$$

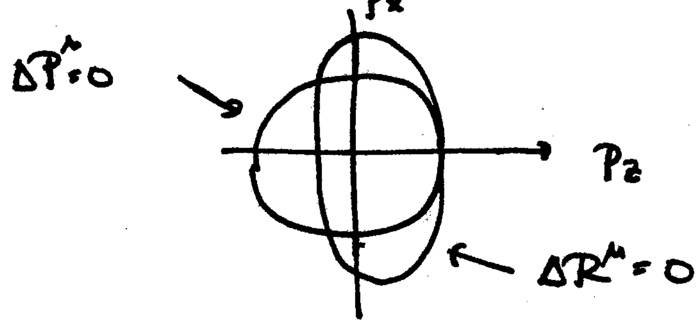
and r_i depends on the species unless $R^\mu = P^\mu$. Momentum conservation implies

$$p_1^\mu + p_2^\mu = p_1'^\mu + p_2'^\mu$$

The vectors on the right lie on a circle in the CM frame. The additional requirement of R^μ conservation would be

$$r_1 p_1^\mu + r_2 p_2^\mu = r_1 p_1'^\mu + r_2 p_2'^\mu .$$

The solutions to this equation lie on an ellipse, e.g.



These conditions coincide, at best, at a few discrete scattering angles. Then, if P^μ and R^μ are distinct and both conserved, the scattering amplitude must vanish at a generic scattering angle.

This is an amazing conclusion. We started out trying to write a theory in which the Higgs boson would be related by symmetry to some fermion. I have now argued to you that this is possible only if the symmetry algebra includes the total energy-momentum of everything. Then every particle and field in the theory must participate in supersymmetry.

To see this most clearly, look back at the relation for $\{Q_1, Q_1^\dagger\}$. We can choose states to be eigenstates of H, P^3; $H - P^3 > 0$ (except for massless 1-particle states moving precisely in the 3 direction). Define

$$a = \frac{Q_1}{\sqrt{2(H-P^3)}} \qquad a^\dagger = \frac{Q_1^\dagger}{\sqrt{2(H-P^3)}}$$

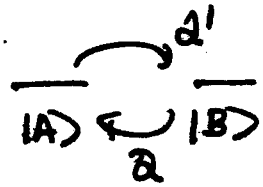
then

$$a a^\dagger + a^\dagger a = 1 \qquad\qquad a^2 = (a^\dagger)^2 = 0$$

These operators act on 2-level systems

$$a|A\rangle = 0 \qquad |B\rangle = a^\dagger|A\rangle \qquad |A\rangle = a|B\rangle \qquad a^\dagger|B\rangle = 0$$

or



Q_1 and $Q_1^\dagger$ change J^3 by ½ unit. Thus, all states in the theory pair up into pairs that differ in J^3 by ½. Thus, if we have a spin-zero particle ϕ, it must have a partner of spin-½. A spin 1 particle must also have a partner of spin-½ or spin-3/2. Thus, we can make the Standard Model supersymmetric only by adding:

for the Higgs field ϕ $\rightarrow$ a fermion $\tilde{h}_\alpha$ w. $I = \frac{1}{2}$ $Y = \frac{1}{2}$

for each species of quark + lepton ψ_L $\rightarrow$ a complex scalar field $\tilde{\psi}$ w. the same I, Y, color

for each vector boson A^a_μ $\rightarrow$ a fermion λ^a_L w. the same global transformation properties

and even,

for the graviton $h_{\mu\nu}$ (spin 2) $\rightarrow$ a spin 3/2 particle $\psi_{\mu\alpha}$, the gravitino.

In general relativity theory, the graviton is the gauge particle associated with local translation symmetry (general covariance). Then the gravitino must be the gauge particle of local supersymmetry and general relativity must also become generalized to a fermionic theory, called supergravity.

Given all of these constraints, it is amazing that there are any nontrivial supersymmetric theories. But these theories do exist, and they can be constructed systematically.

The simplest such theories contain only complex boson fields ϕ_j

and fermion fields ψ_{Lj}. Let $W(\phi)$ (the "superpotential")
be an analytic function of the ϕ_j (depending on ϕ_j, not ϕ_j^*)
Then the following Lagrangian is supersymmetric

$$\mathcal{L} = \partial^\mu \phi_j^* \partial_\mu \phi_j + \psi_{Lj}^\dagger i \bar\sigma^\mu \partial_\mu \psi_{Lj} - \left| \frac{\partial W}{\partial \phi_j} \right|^2$$

$$+ \frac{1}{2} \psi_{Lj}^T c \, \psi_{Lk} \frac{\partial^2 W}{\partial \phi_j \partial \phi_k}$$

This is the "Wess-Zumino model". Taking $W = m\phi_1\phi_2$, we get $(\text{masses})^2 = m^2$ for the boson fields ϕ_1 ϕ_2 and a mass m for ψ_{L1}, $\psi_{R2}^\dagger = -\psi_{L2}^T c$. This theory can be made locally symmetric by adding A_μ^a λ^a in a definite way.

Now we have a large and complex but, I hope, intriguing structure which gives a supersymmetric generalization of the standard model. Now, does this theory solve any of the problems of the Standard Model? I'll discuss that issue in the next lecture.

Lecture 11: August 11

In the previous lecture, I discussed the addition of supersymmetry to the Standard Model. We asked, modestly, to find a symmetry that related the Higgs field to some fermion field. We found that this led inevitably to a large structure that involved all of the particles in the theory. Now let's study the consequences of this structure.

As a preface to this, I would like to discuss the consequence of supersymmetry for mass generation in the standard model. In lecture 9, I noted that the $W^\pm$ and Z^0 masses come naturally from the kinetic energy terms of the Higgs field ϕ, and the quark and lepton masses come from fermion-fermion-Higgs interactions:

$$\delta\mathcal{L} = -\lambda_e e_R^\dagger \phi^\dagger \cdot L - \lambda_d d_R^\dagger \phi^\dagger \cdot Q + \lambda_u u_R^\dagger \epsilon^{ab} \phi_a Q_b + h.c.$$

In a supersymmetric theory, the right-handed fields will be replaced by the fields of their antiparticles, e.g.

$$e_R^\dagger \to -(\bar{e})_L^T C$$

But even so, this $\delta\mathcal{L}$ will not do! As we discussed at the end of the previous lecture, in a supersymmetric theory, fermion-fermion-boson interactions must have the form

$$\psi_{1L}^T C\, \psi_{2L} \frac{\partial W}{\partial\phi_1 \partial\phi_2}$$

where W is a function of the ϕ_j but not of the $\phi_j^\dagger$. So we must rewrite this Higgs interaction as:

$$\delta\mathcal{L} = \lambda_e \bar{e}_L^T C\, \epsilon^{ab}\phi_{1a}L_b + \lambda_d \bar{d}_L^T C\, \epsilon^{ab}\phi_{1a}Q_b - \lambda_u \bar{u}_L^T C\, \epsilon^{ab}\phi_{2a}Q_b$$

$$+ \text{h.c.}$$

where ϕ_2 has $I=\frac{1}{2}$ $Y=\frac{1}{2}$ as before and I have introduced a new Higgs field ϕ_1 with $I=\frac{1}{2}$ $Y=-\frac{1}{2}$. So we need two Higgs supermultiplets. (Maybe this is a step backwards.) If

$$\langle\phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} v_1 \\ 0 \end{pmatrix} \qquad \langle\phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 \end{pmatrix}$$

then all of the vector bosons and spin-$\frac{1}{2}$ particles get mass, and all of their supersymmetry partners get the same masses. That last conclusion is not good; if there were a scalar particle with $Q=\pm 1$ degenerate with the electron, we would know about it. To raise the masses of the supersymmetry partner particles, we will need to spontaneously break supersymmetry. I'll discuss that topic in a moment.

First, though, I would like to discuss the consequence of supersymmetry for grand unification. In lecture 9 I explained that grand unification was a beautiful idea ~~that~~ didn't work out quantitatively. I should have pointed out also that grand unification practically requires supersymmetry. In the Standard Model, we had an argument that the maximum value of a quark, lepton, a vector boson mass was about

$$v \sim 246\ \text{GeV}.$$

However, this argument did not apply to the Higgs boson. Since grand unification involves the enormous scale $m_U > 10^{15}$ GeV, there is no reason why the Higgs fields should not have

$$m_\phi^2 \sim m_U^2$$

Since what we want is

$$m_\phi^2 \sim -\langle v^2 \rangle = -(10^{-26})\, m_U^2$$

we really do need a symmetry to suppress m_ϕ.

But supersymmetry has another implication for grand unification. Since the superpartners are supposed to be light compared to m_U, we need to include their effects in the RG equations for the coupling constants. Recall that in QCD

$$\frac{d}{d\log q^2}\, g_s^2(q^2) = -\frac{1}{(4\pi)^2}\Big(\underbrace{\tfrac{11}{3}N}_{SU(N)\ \text{gluons}} - \underbrace{\tfrac{2}{3}n_f}_{n_f\ \text{quark flavors.}}\Big)\, g_s^4$$

In a supersymmetric version of QCD, we must add the effects of the partners of the gluons λ_L^a (called "gluinos") and the scalar partners of the quarks (called squarks). These two contributions have the standard sign of vacuum polarization and equal

$$-\frac{1}{(4\pi)^2}\Big(\underbrace{-\tfrac{2}{3}N}_{\text{gluinos}} - \underbrace{\tfrac{1}{3}n_f}_{\text{squarks.}}\Big)$$

In all

$$\frac{d}{d\log q^2}\, g_s^2 = -\frac{1}{(4\pi)^2}\,(3N - n_f)\, g_s^4$$

Then (with $N_a = 0, 2, 3$ for $U(1), SU(2), SU(3)$ and $n_g = 2n_f$ the number of generations) the RG coefficients from lecture 9 become

$$b_3 = 9 - 2n_g$$
$$b_2 = 6 - 2n_g - \tfrac{1}{2} n_h$$
$$b_1 = \quad - 2n_g - \tfrac{3}{10} n_h$$

where n_h is the number of Higgs multiplets = 2 at least. For $n_h = 2$ we find

$$B = \frac{b_3 - b_2}{b_2 - b_1} = \frac{5}{7} = 0.714$$

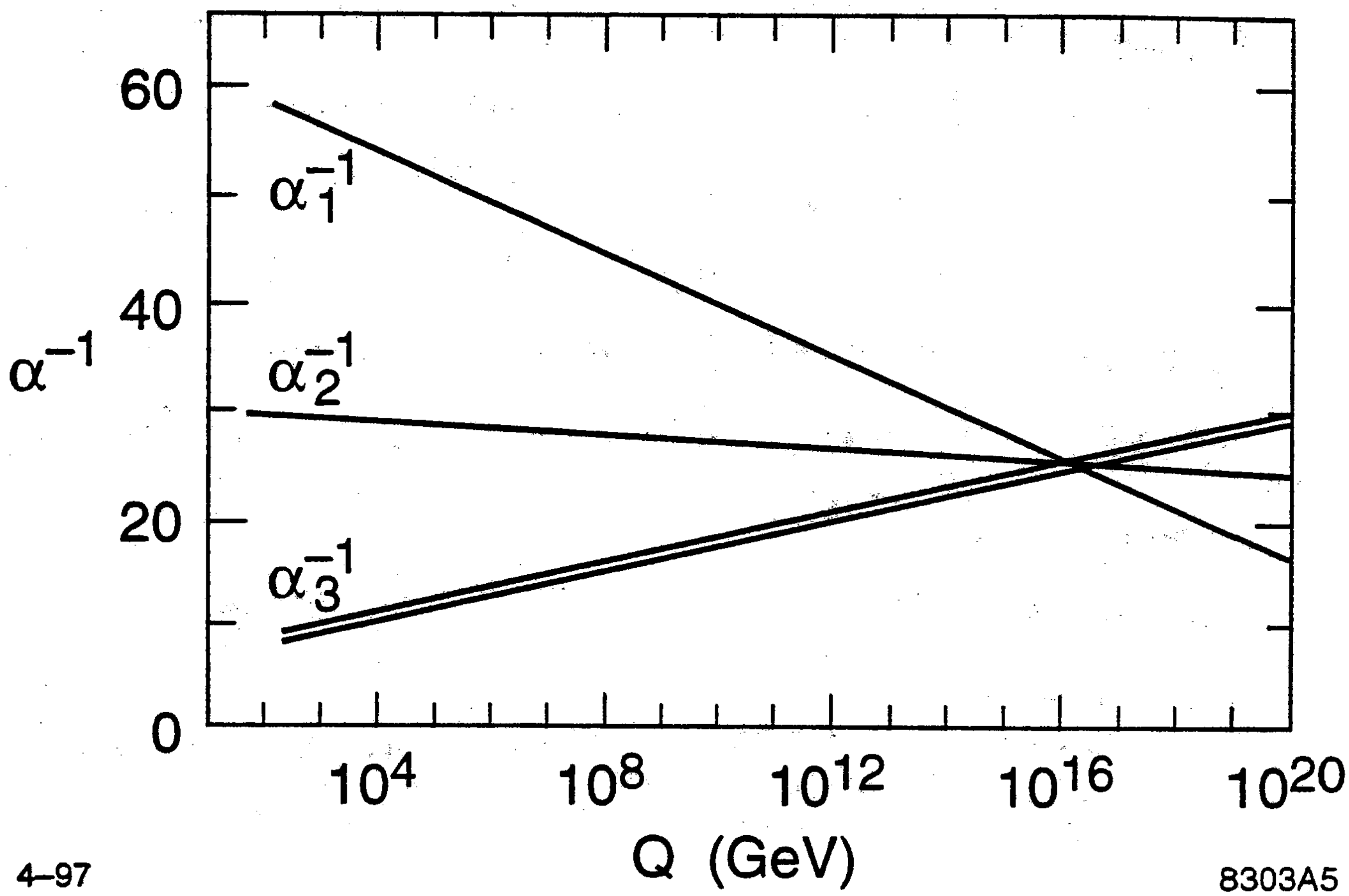
which is almost exactly what experiment requires for a consistent grand unification. If we simply integrate the new RG equations from the known coupling constant values at the scale m_Z^2, we find the picture shown in Fig. 69. The couplings meet in a consistent unification at $m_U \sim 2 \times 10^{16}$ GeV, $\alpha_U \sim 1/27$. This value of m_U is also high enough to avoid a problem with too high a rate of $p \to e^+\pi^0$. (In fact, in supersymmetric models, the dominant decay channel of the proton is supposed to involve a different mode, $p \to \nu K^+$, with a rate

$$\Gamma(p \to \nu K^+) \sim (10^{32} - 10^{34}\, \text{yrs})^{-1}$$

This decay, and other modes of proton decay, are now being looked for by the Super-Kamiokande experiment.)

Is this support for grand unification evidence for supersymmetry? I'll leave that up to you. Let's return to the question of the masses of superparticles.

Figure 69



60
40
20
0
α^{-1}
α_1^{-1}
α_2^{-1}
α_3^{-1}
10^4
10^8
10^{12}
10^{16}
10^{20}
Q (GeV)
4–97
8303A5

There is not yet any direct observational evidence for the supersymmetric partners of quarks, leptons, and vector bosons. This means that, to build a supersymmetric model, we must find a way to give large masses to the superpartner fields. As I mentioned above, these can arise through spontaneous supersymmetry breaking. I don't want to go into any particular model of supersymmetry breaking, but I would like to discuss the consequences of models, through a discussion of the following obvious question: Why is it ~~that~~ the standard model particles are light and their partners are heavy, rather than the other way around?

There is an interesting answer to this question: The masses of the quarks, leptons, and gauge bosons are forbidden by SU(2)×U(1) symmetry. The masses of their partners are forbidden only by supersymmetry. Thus, we can imagine the following chain of cause and effect, coming down from high energy scales or up from short distances:

(supersymmetry breaking) → (masses for scalar partners (including Higgs bosons))

↓

(SU(2)×U(1) breaking)

We still have to explain why the Higgs $(\text{mass})^2$ is <u>negative</u>; I'll come to that in a moment. But if we can arrange this, it is quite reasonable to build a model with

$$\text{squark, slepton, Higgs } (\text{masses})^2 \sim v^2$$
$$\text{quark, lepton masses} \sim \lambda v \ll v$$

Where does the value v come from. It turns out that if any interaction in Nature breaks supersymmetry spontaneously, this interaction can generate scalar mass terms for any supermultiplet with which it couples. By default, everything couples to everything else through gravity. So we get a formula for the mass induced by spontaneous supersymmetry breaking:

$$m_S \sim \frac{M_{SSB}^2}{M_M} \sim v$$

where M_{SSB} is the mass scale of the supersymmetry breaking interactions and M_M is the mass of the particles that connect these interactions to the particles of the Standard Model. The default is

$$M_M = M_{Planck} = 10^{19}\,\text{GeV} \qquad M_{SSB} = 10^{11}\,\text{GeV}$$

However, many other choices are possible. In principle, every theory of supersymmetry breaking leads to a spectrum of squark, slepton, and gauge boson partner ("gaugino") masses with distinctive features that we could observe experimentally.

There is, however, an interesting complication in the theory of the superparticle masses. These masses obey RG equations, just like the coupling constants. We should think of the masses as being generated at the large scale M_M, and then we must integrate RG equations to find their values at the physical point $q^2 \sim m^2 \sim v^2$.

The RG equations for the gaugino masses are especially simple:

$$\frac{d}{d\log q^2}\, m_i(q^2) = -\frac{1}{(4\pi)^2}\, b_i\, g_i^2\, m_i \qquad (b_i \text{ as on p.4})$$

where $i = 1,2,3$, g_i^2 are the grand unified couplings, and m_i are the masses of the corresponding gauginos. If the masses of the U(1), SU(2), and SU(3) boson partners are grand unified, at low energies they obey the relations

$$\frac{m_1}{\alpha_1} = \frac{m_2}{\alpha_2} = \frac{m_3}{\alpha_3}$$

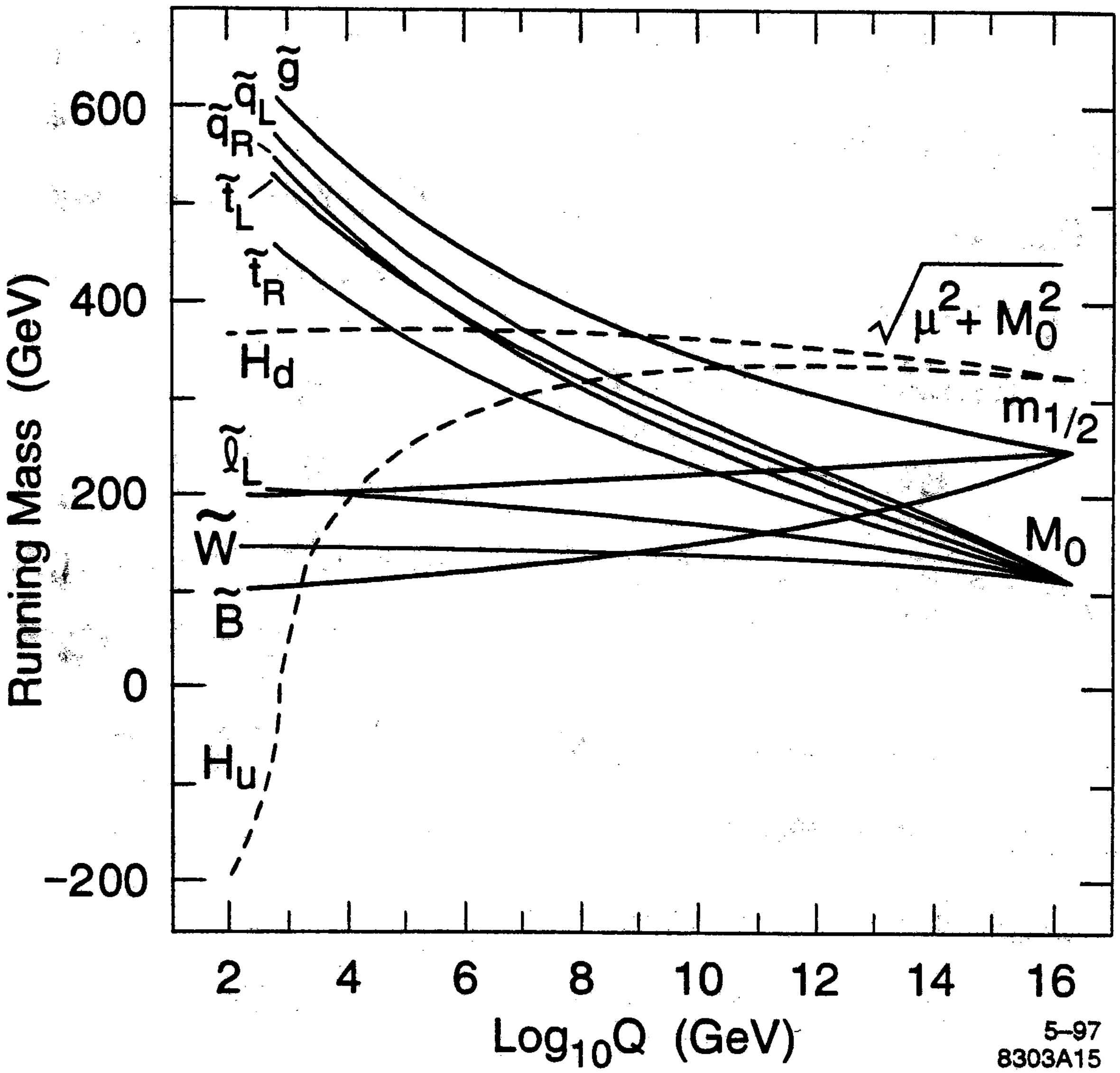
so the gluino mass should be about 4 times the W-partner ("wino") mass. The squark and slepton masses obey more complicated equations, but with the same general trend: Particles with color become heavier by a factor of 3 or so than particles without color. A particular solution to the complete supersymmetry RG equations — assuming that all of the squark and slepton masses are equal to m_0 — is shown in Fig. 70.

This picture contains a surprise: the $(\text{mass})^2$ of one of the Higgs bosons is driven negative by the RG evolution. The origin of this effect is a term

$$\frac{d}{d\log q^2}\, m^2_{\phi_2} = +\frac{1}{(4\pi)^2}\,\frac{3}{2}\,\lambda_t^2\,(m^2_{\phi_2} + m^2_{t_L} + m^2_{t_R})$$

involving the top-Higgs coupling λ_t and the masses of the Higgs and the top squark fields. The same effect pushes down the masses of

Figure 70



G. L. Kane, C. Kolda, L. Roszkowski, and J. D. Wells,

Phys. Rev. D49, 6173 (1994)

the top squarks, as you see in the figure. The effect is large because λ_t is large, that is, because the top quark is heavy. Again, the heaviness of the top quark plus the RG equations of supersymmetry gives a natural explanation for the instability to $SU(2) \times U(1)$ breaking. [I wish I knew a simple explanation for the <u>sign</u> of this effect. I don't.]

Can we go to higher energies with our accelerators and find superparticles? It seems very reasonable. The two most favorable reactions are:

$$e^+e^- \to \tilde{f}\,\bar{\tilde{f}} \qquad f = \text{squark } (\tilde{q}),\ \text{slepton } (\tilde{\ell}),\ \text{wino } (\tilde{w}^+),\ \text{etc.}$$

$$pp \to gg \to \tilde{c}\,\bar{\tilde{c}} \qquad c = \text{colored superparticle (gluino } (\tilde{g}) \text{ or squark } (\tilde{q}))$$

If the Higgs mass of order v sets the scale of supersymmetry-breaking masses, we need

$$E_{cm}(e^+e^-) \sim 500 \text{ GeV}$$

$$E_{cm}(gg) \sim 1500 \text{ GeV} \quad \Leftarrow \quad E_{cm}(pp) \sim 10 \text{ TeV}$$

We may be lucky and find these particles sooner.* But the next generation of colliders — the LHC at 14 TeV and the e^+e^- linear collider at 1 TeV — should certainly get there.

What will these particles look like when they appear? There are many possibilities for their decay patterns, but there

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* Perhaps this week, at LEP 2!

is the simplest. Consider the quantity:

$$R = (-1)^{2S + 3B + L}$$

where S = spin, B = baryon no. ($3B$ = quark number), L = lepton no. If angular momentum, B, and L are conserved, R is conserved. Now note that R assigns

$R = +1$ to all quarks, leptons, gauge bosons, Higgs bosons

$R = -1$ to their supersymmetric partners with $|\Delta S| = \frac{1}{2}$

Thus, if R is conserved, the lightest supersymmetric particle is stable. In the simplest models, this stable particle is the $U(1)$ gaugino $\tilde{b}^0$. (often called $\tilde{\chi}_1^0$). The gas of these particles left-over from supersymmetric reactions in early cosmology is a good candidate for the cosmological "dark matter."

Heavier superparticles decay to the $\tilde{b}^0$ plus quarks, leptons, or gauge bosons. For example, the W partner may decay by

$$\tilde{w}^+ \to q\bar{q}\,\tilde{b}^0 \quad \text{or} \quad \ell^+\nu\,\tilde{b}^0.$$

The $\tilde{b}^0$ is stable, neutral, and very penetrating, so it is not seen by typical high-energy particle detectors. The decay of the $\tilde{w}^+$ and other superpartners would then seem to have missing energy and momentum.

At the LHC, typical supersymmetry reactions are complex. e.g.

$$gg \to \tilde{g}\tilde{g} \to q\bar{q}\,\tilde{w}^0 \to q\bar{q}\,\tilde{b}^0$$

$$\tilde{g} \to q\bar{q}\,\tilde{h}^0 \to l^+l^-\,\tilde{b}^0$$

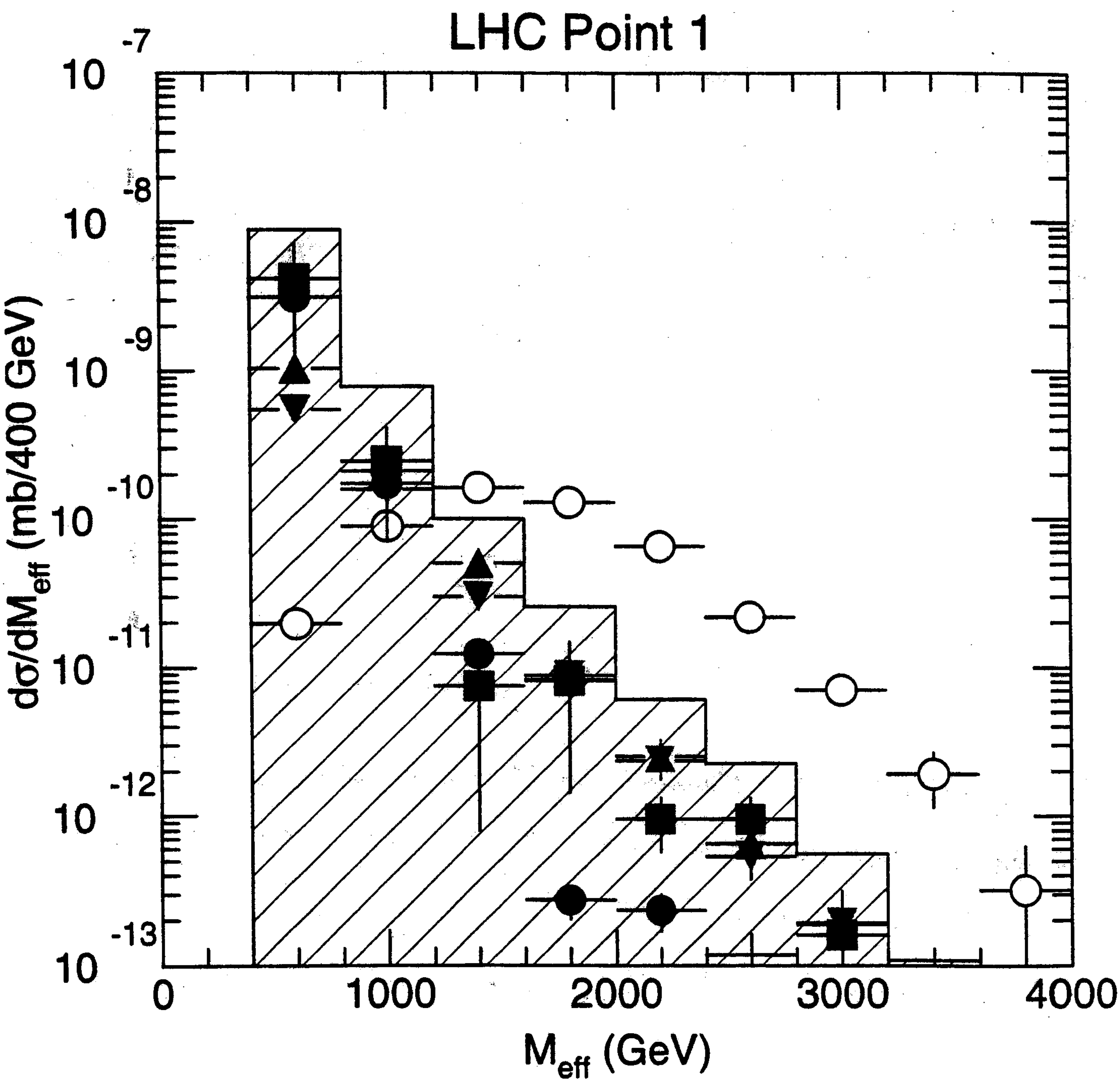
But these events are quite characteristic, mainly missing energy plus many high transverse momentum jets. In the variable

$$M_{eff} = (E_T)_{missing} + \sum_{\substack{\text{four hardest}\\\text{jets}}} (E_{Ti})$$

supersymmetry events stand out well above standard model events (with missing energy from $W^{\pm} \to l^{\pm}\nu$, for example). A typical simulation result is shown in Fig. 71.

In fact, it is possible not only to measure the general characteristics of supersymmetry reactions, but also to reconstruct these new particles and measure their masses precisely. If supersymmetry is indeed correct, these masses come from supersymmetry breaking, and so the mass spectrum gives clues about some very deep scale in Nature, perhaps even the scale of grand unification or gravity. So it will be important to determine these masses as well as we can. For this reason, there has already been quite a bit of work on simulations of the mass measurements at the next generation of colliders. Let me show you a few of the results.

Figure 71



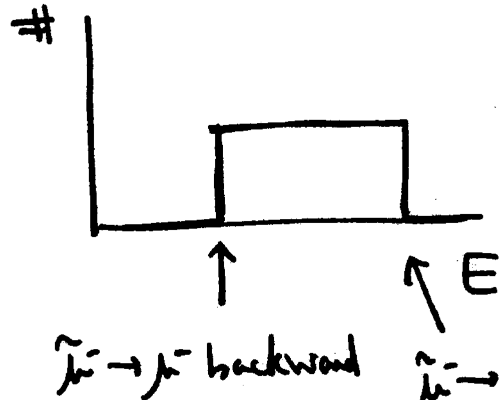
I. Hinchliffe, et al (ATLAS)

Phys. Rev. D55, 5520 (1997)

First, let's consider e^+e^- reactions. A very simple process is

$$e^+e^- \to \tilde{\mu}^+\tilde{\mu}^- \qquad \tilde{\mu}^- \to \mu^- + \tilde{b}^0$$

The final state contains two muons and two (missing) $\tilde{b}^0$'s. Now, the $\tilde{\mu}^-$ is a scalar (spin 0) particle. Thus, it decays isotropically. The boost of an isotropic momentum distribution is a distribution flat in energy. Thus, the muons should come out in an energy distribution



The endpoints are defined by the kinematics; by measuring these endpoints, we can solve for $m(\tilde{\mu}^-)$ and $m(\tilde{b}^0)$. Simulation results are shown in Fig. 72; they lead to a 1% measurement of these masses. Fig. 73 shows a similar analysis of the decay process

$$e^+e^- \to \tilde{\nu}\,\bar{\tilde{\nu}} \qquad \tilde{\nu} \to e^- + \tilde{w}^+$$

($\tilde{w}^+$ is called $\tilde{\chi}_1^+$ in this figure.)

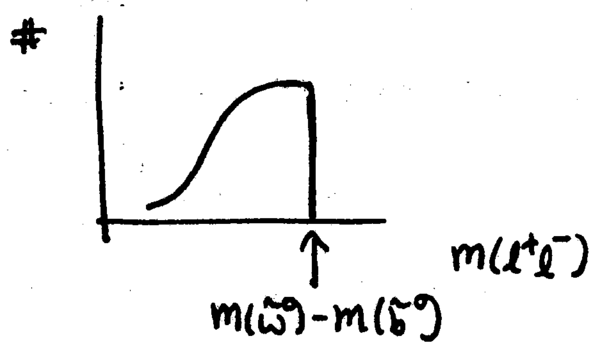
Despite the complexity of supersymmetry reactions at the

LHC, it may also be possible to do precise mass reconstructions there. One of the American groups in ATLAS has studied a particular supersymmetric model with the decay chain

$$\begin{aligned}\tilde{g} &\to \tilde{b} + \bar{b} \\ &\quad\ \ \hookrightarrow b + \tilde{w}^0 \\ &\qquad\qquad\ \hookrightarrow \ell^+\ell^- + \tilde{b}^0\end{aligned}$$

$\tilde{b}$ = b-squark

The $\ell^+\ell^-$ pair has a characteristic mass spectrum going up to $m(\ell^+\ell^-) = m(\tilde{w}^0) - m(\tilde{b}^0)$. In fact, it has a sharp cutoff at this point



Taking advantage of the high luminosity of the LHC, we select events with $\ell^+\ell^-$ near the endpoint. Then, the $\tilde{b}^0$ is approximately at rest in the frame of the $\ell^+\ell^-$ pair. If we know the mass of the $\tilde{b}^0$ (I've just told you how to measure it.) we can add

$$p^\mu(\ell^+\ell^-) + p^\mu(\tilde{b}^0) = p^\mu(\tilde{w}^0)$$

Now we have fully accounted the missing energy-momentum.

By adding jets with b-tags, we should reconstruct the $\tilde{b}$ and then the $\tilde{g}$. The scatter plot of events in the plane of $m(\tilde{b})$ vs. $(m(\tilde{g}) - m(\tilde{b}))$ is shown in Fig. 74; the projections of the data onto the axes are given in Fig. 75. Again, these are 1% measurements.

By presenting these very detailed results, I am not trying to say that the future of particle physics is written in stone. Indeed, it is quite the reverse. This theory of what lies beyond the standard model is one among many, and experimental results must decide what model is realized. What I have tried to show, for this particular example, is that accelerator experiments really will provide the information we need, not only to probe models of the next scale of physics, but to work out that theory with precision and to find clues to even deeper levels in Nature. In the first lectures of this series, I described to you the detailed process of the discovery and verification of the Standard Model. That is now history. But the mysteries of spontaneous symmetry breaking do point to a new scale in physics and to a new era of discovery in the

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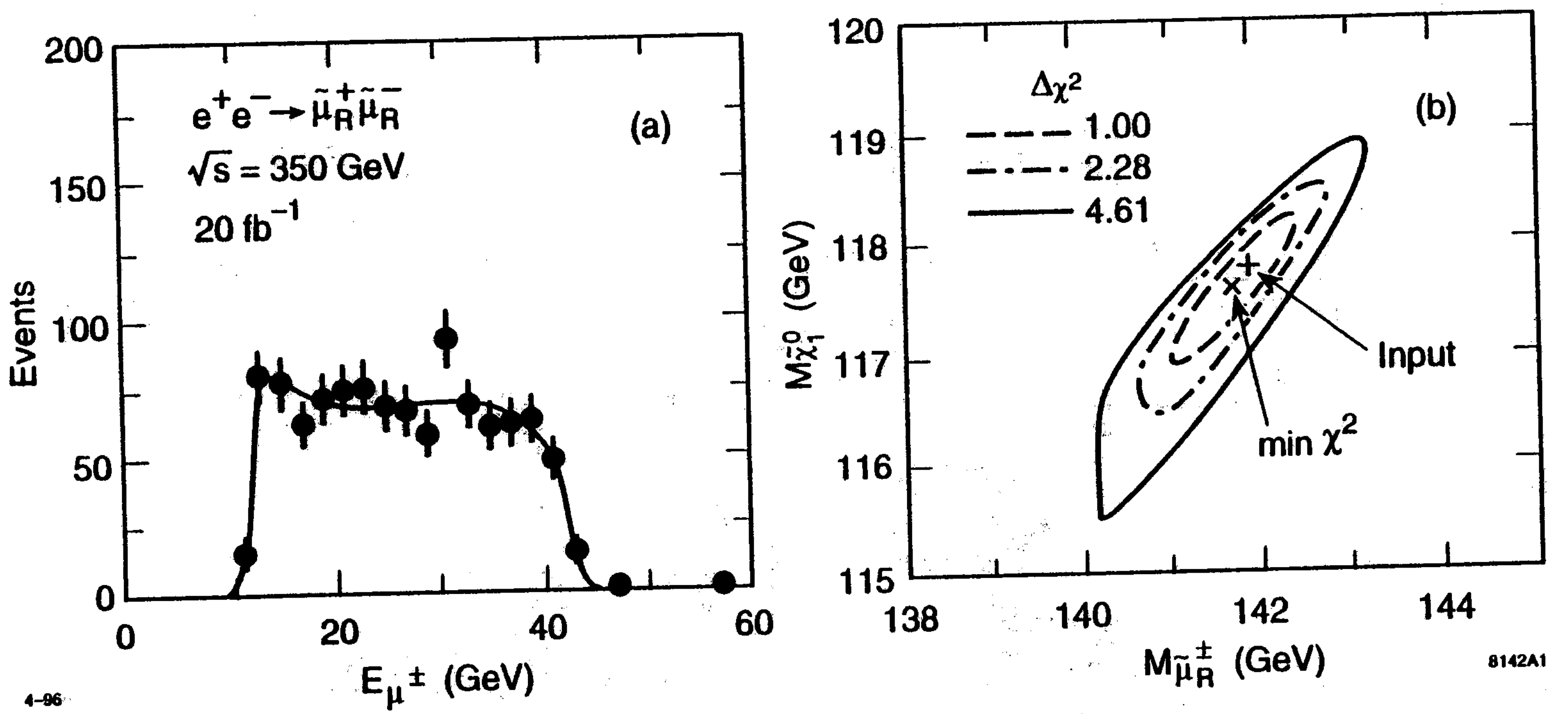
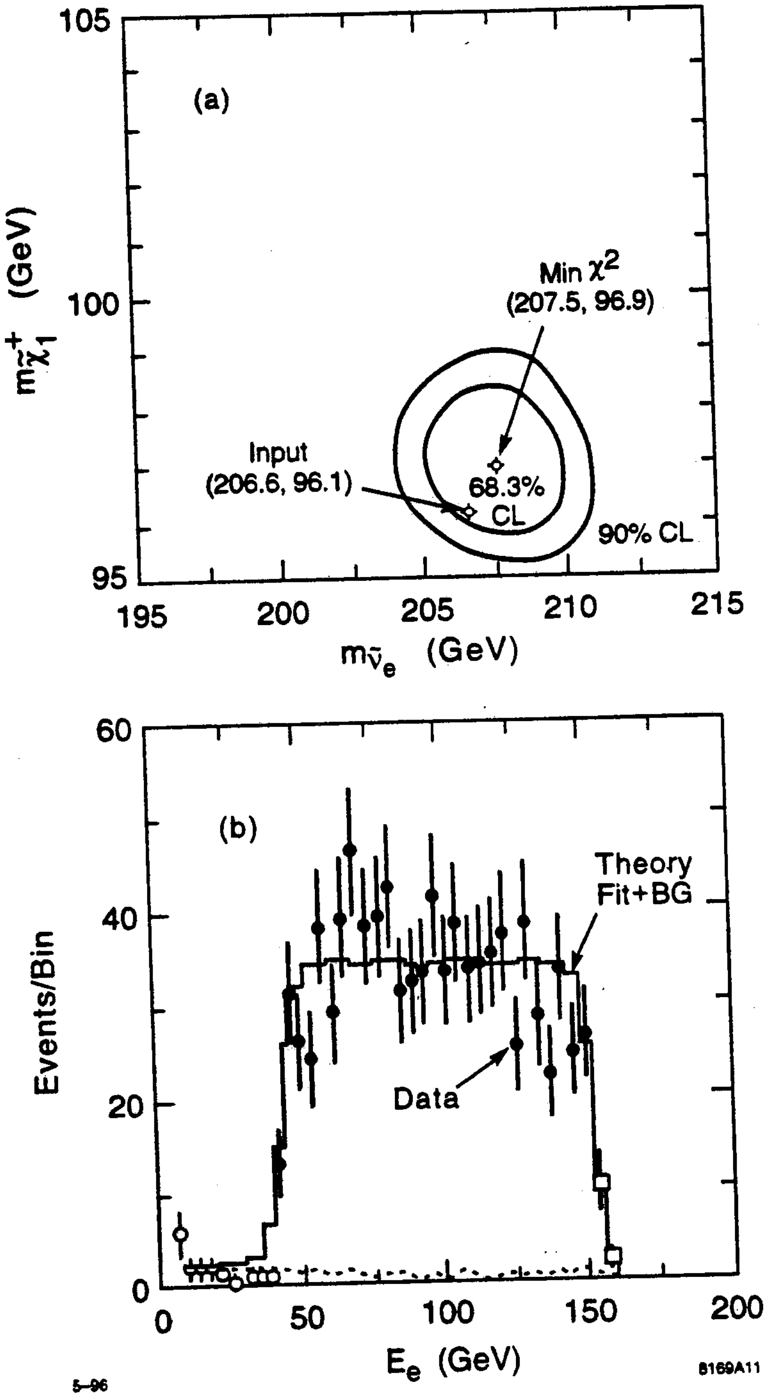


T. Tsukamoto et. al (JLC)
Phys. Rev. D51, 3153 (1995)

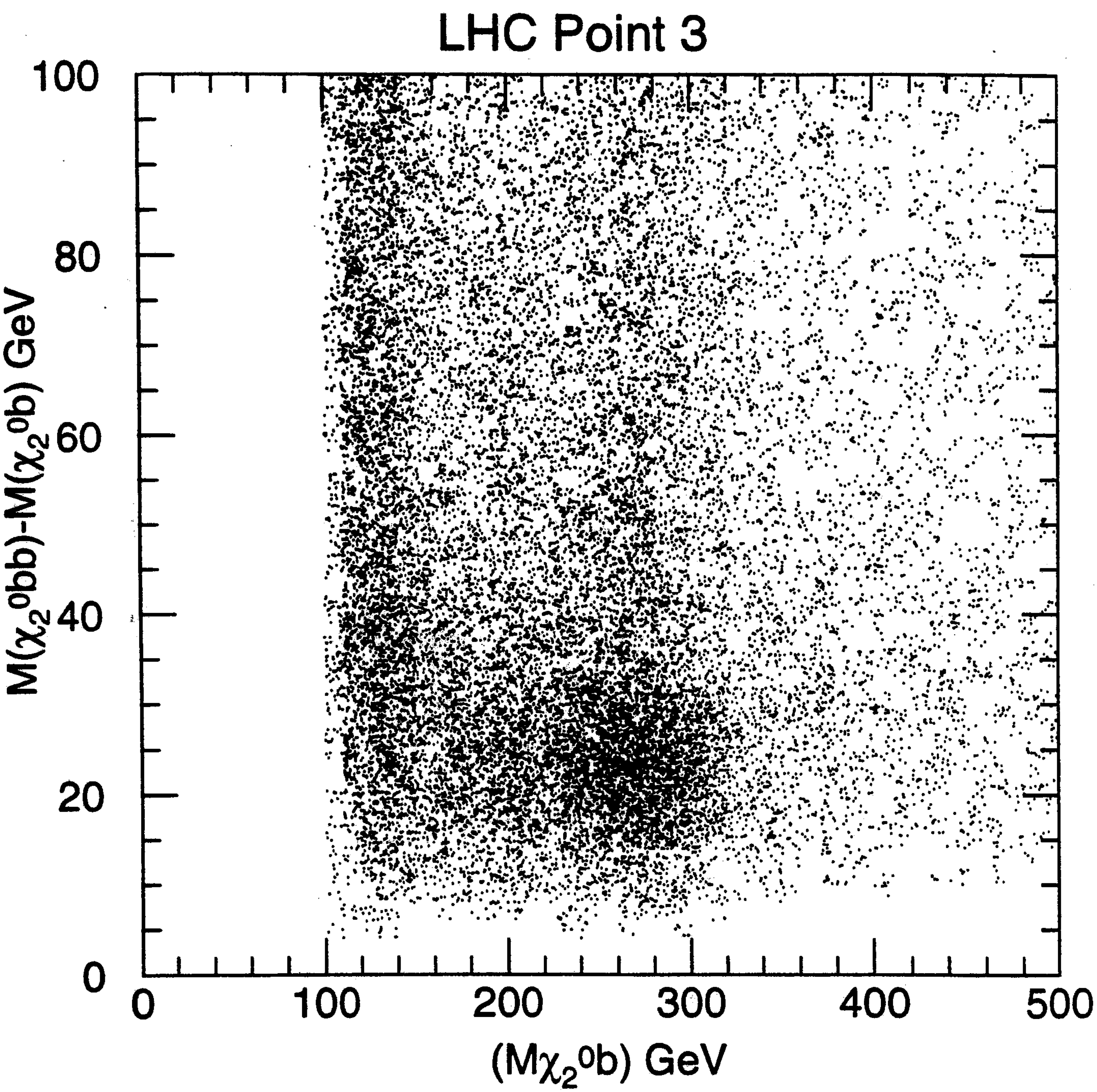
Figure 72

Figure 73



from S. Kuhlman et al. (NLC - PTR)

Figure 74



I. Hinchliffe et al (ATLAS)

Phys. Rev. D55, 5520 (1997)