4 independent gauge transformations α^a, β

$\Rightarrow$ 4 gauge fields W_μ^a, B_μ

couplings : $g \quad g'$

e.g. $D_\mu L = \left(\partial_\mu - ig W_\mu^a T^a - ig' B_\mu \frac{-1}{2}\right) L$

gauge boson mass matrix :

$$\mathcal{L} \sim -\frac{1}{2} V_\mu M^2 V^\mu \qquad V_\mu = \begin{pmatrix} W_1 \\ W_2 \\ W_3 \\ B \end{pmatrix}_\mu$$

$$M^2 = \frac{v^2}{4} \begin{pmatrix} g^2 & & & \\ & g^2 & & \\ & & g^2 & -gg' \\ & & -gg' & g'^2 \end{pmatrix}$$

$$W^\pm = \frac{1}{\sqrt{2}}(W_1 \mp i W_2) \longrightarrow m_W = \frac{gv}{2}$$

diagonalize W^3-B :

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix} \begin{pmatrix} W_{3\mu} \\ B_\mu \end{pmatrix} \iff \begin{pmatrix} W_3 \\ B \end{pmatrix} = \begin{pmatrix} c & s \\ -s & c \end{pmatrix} \begin{pmatrix} Z \\ A \end{pmatrix}$$

$$s_W \equiv \sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}} \qquad c_W = \frac{g}{\sqrt{g^2+g'^2}}$$

$$\Rightarrow \quad M^2_{AZ} = \frac{v^2}{4}(g^2+g'^2)\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$M_Z = \frac{v\sqrt{g^2+g'^2}}{2} = M_W / c_W$$

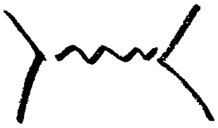
$$M_\gamma = 0$$

γ, Z couplings?

$$D_\mu = \partial_\mu \underbrace{- ig W_\mu^3 \overset{I_3}{\frac{\sigma_3}{2}} - ig' B_\mu Y}_{-igI_3(cZ+sA) - ig'Y(cA-sZ)} + W_\mu^\pm \text{ terms}$$

$$= -iA_\mu \underbrace{\frac{gg'}{\sqrt{g^2+g'^2}}}_{=e}(I_3+\overset{Q_{em}-I_3}{Y}) - iZ_\mu \underbrace{\left(\frac{g^2}{\sqrt{}} I_3 - \frac{g'^2}{\sqrt{}} Y\right)}_{\sqrt{g^2+g'^2}(I_3 - s_W^2 Q_{em})}$$

$$= \partial_\mu - ieQ_{em}A_\mu - i\frac{e}{s_W c_W}(I_3 - s_W^2 Q_{em})Z_\mu$$

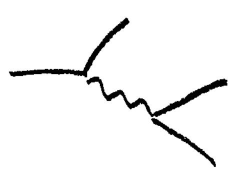
current-current 

$$M = \underbrace{\frac{g^2}{p^2 - M_W^2}}_{\approx -\frac{g^2}{M_W^2}} j_L^{+\mu} j_{L\mu}^{-} + \underbrace{\frac{g^2+g'^2}{p^2 - M_Z^2}}_{\approx -\frac{g^2}{M_W^2}} \frac{1}{2}\left(j_\mu^3 - s_W^2 j_\mu^{em}\right)^2$$

$\approx -\frac{g^2}{M_W^2}$ for $p^2 \ll v^2$

$SU(2)\times U(1)$ electroweak interactions.

precision tests:

- $R^\nu, R^{\bar\nu} \Rightarrow \sin^2\theta_W = 0.233$
- $\frac{m_W}{m_Z} = c_W \Rightarrow \sin^2\theta$
- Z decays, $A_{FB} \Rightarrow \sin^2\theta_W$
- μ-decay  $\sim \frac{g^2}{m_W^2} \Rightarrow \alpha_W$

$$\sin^2\theta_W = \frac{e^2}{g^2} = \left.\frac{\alpha_{em}}{\alpha_W}\right|_{M_Z}$$

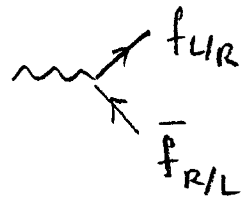
consistent results 10^{-3} precision.

Z-decays @ LEP I

$e^+e^- \longrightarrow Z$ at rest $\longrightarrow f\bar{f}$

$\sqrt{s} = 91\,\text{GeV}$

$$\Gamma_{f\bar{f}} = \frac{\alpha_W}{6 c_W^2} m_Z \left(K_{f_L}^2 + K_{f_R}^2 \right)$$



$$\underbrace{(I_3 - s_W^2 Q)}_{K_f} \frac{g}{c_W}$$

[MeV]	ν_e	e^-	u	d
Γ	167	84	298	384

$\underbrace{298 \quad 384}$ include factor of $3 \cdot 1.03$

3: colors

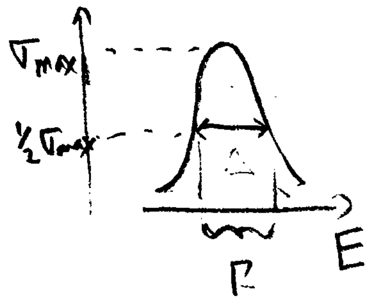
1.03: $(1 + \frac{\alpha_s}{\pi})$ correction

Branching fractions $\quad B_{f\bar{f}} = \dfrac{\Gamma_{f\bar{f}}}{\Gamma_{tot}}$

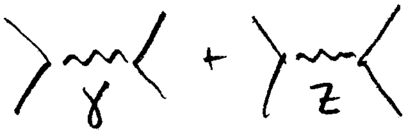
e^-, μ^-, τ^-	3% each
$\Sigma \nu_i$	18%
hadronic	73%

measuring the width?

$$\sigma_{\mu^+\mu^-}(s) \propto \left| \frac{1}{s-M_Z^2+iM_Z\Gamma} \right|^2 = \frac{1}{(s-M_Z^2)^2+M_Z^2\Gamma^2}$$

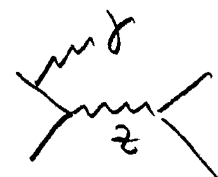


width at ½ maximum.

in reality: fit $\sigma(s)$ to $\Big|$  $\Big|^2$

+ corrections

most important correction: "radiative return" at $\sqrt{s} > 91\,\text{GeV}$



$\boxed{\text{PLOT 53}}$

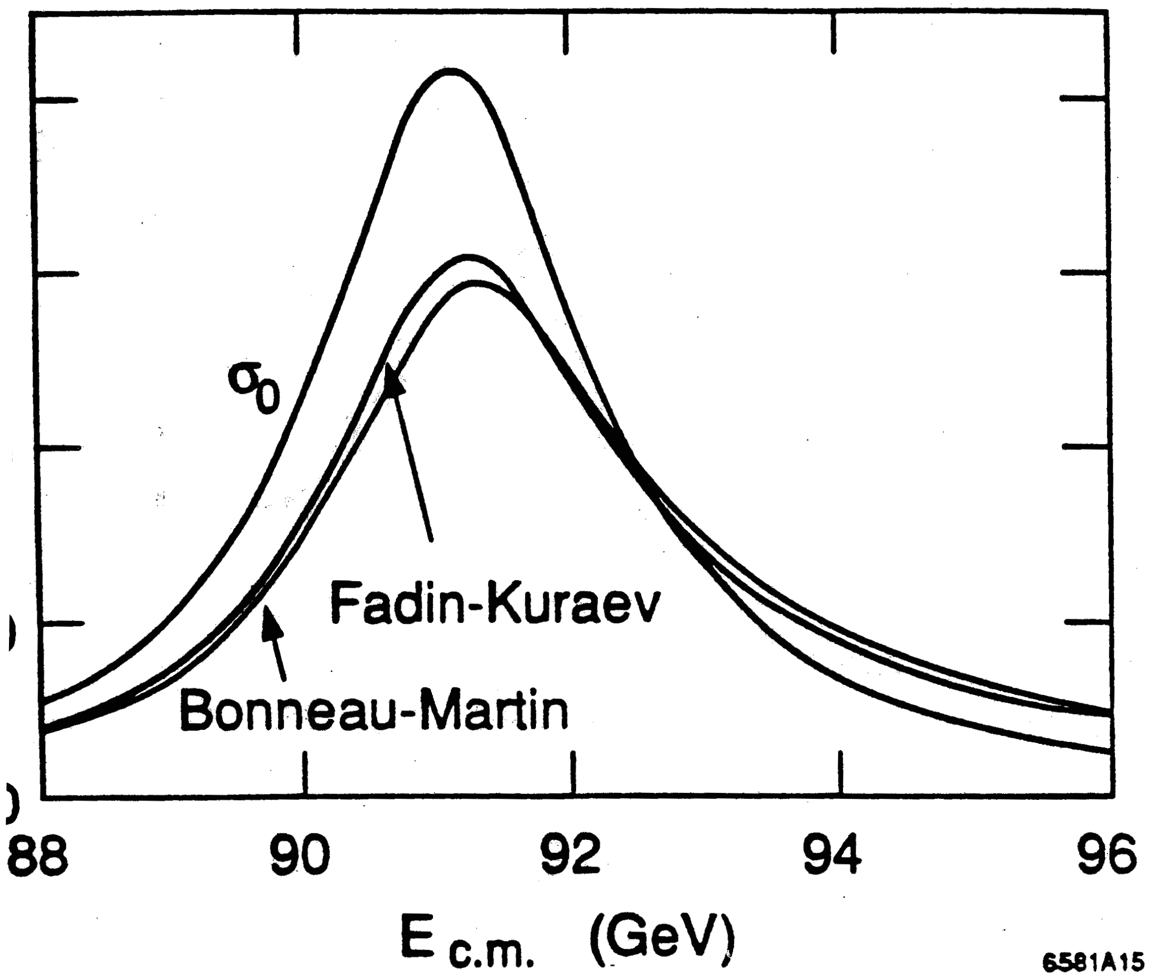
LEP: $\Gamma_{tot} = 2.4946 \pm 0.0027$

$= \Gamma_{visible} + \Gamma_{invisible}$

↳ interpret as ν's $\Rightarrow N_\nu = 2.99 \pm 0.01$

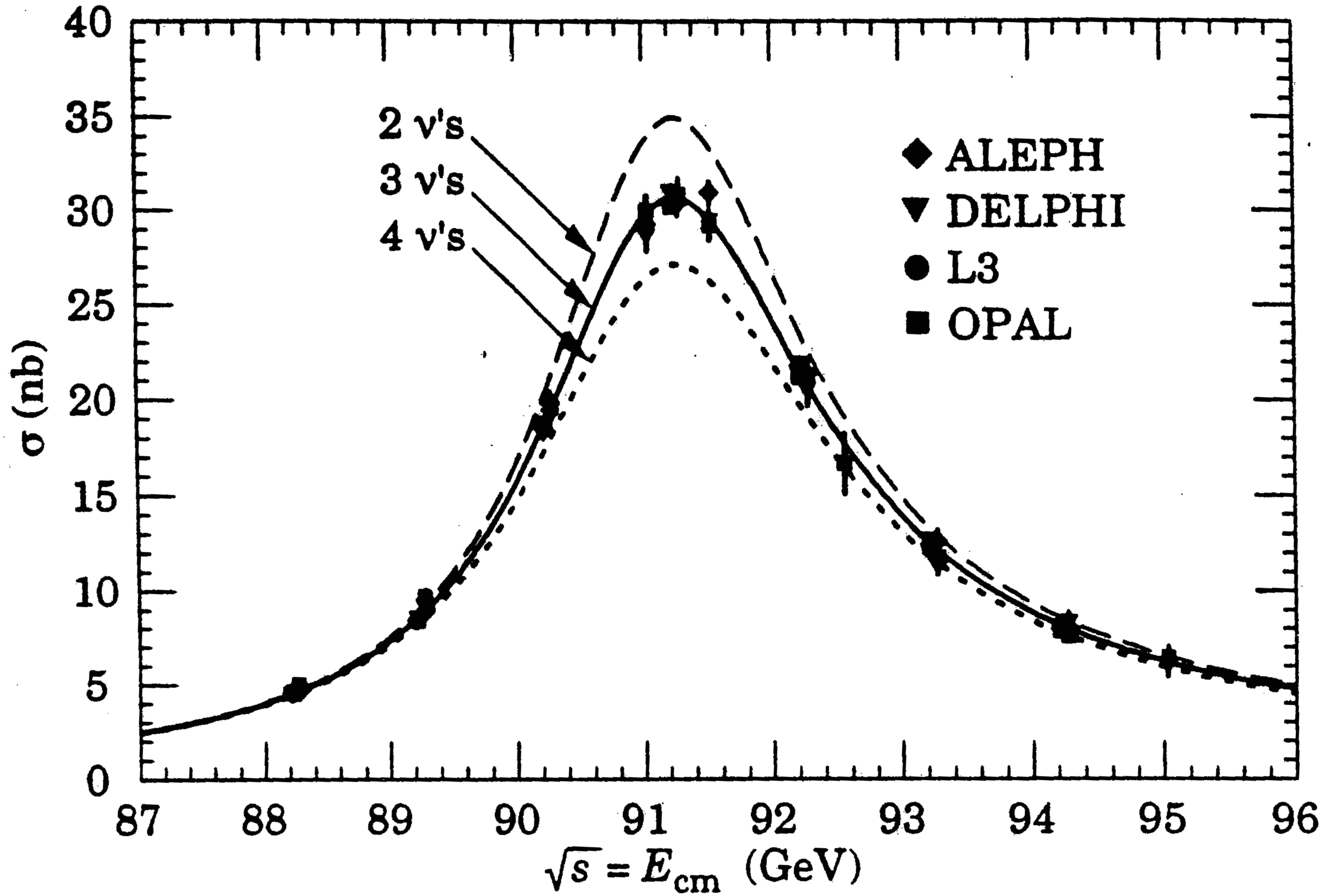
$\boxed{\text{PLOT 54}}$

Radiative Return



Z width, number of neutrinos

$\sigma(e^+e^- \to \text{hadrons})$



Beam polarization: SLC (SLAC)

$$A_e = \frac{\sigma(e^-_L e^+ \to Z) - \sigma(e^-_R e^+ \to Z)}{\text{"} \quad + \quad \text{"}}$$

e^- polarization preserved in LINAC. $\Rightarrow$ polarize at production

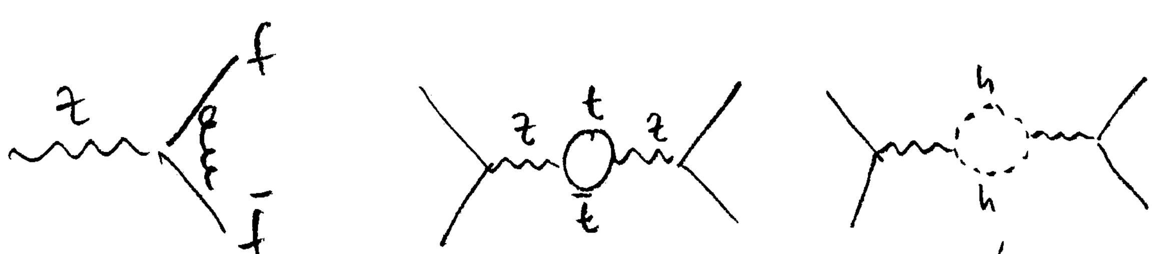
can get 80% polarization.

$$A_e = 0.1542 \pm 0.0037 \Longrightarrow S_W^2 = 0.23129 \pm 0.00030$$

$$\simeq 8\left(\tfrac{1}{4} - S^2_{\theta_W}\right)$$

$\boxed{\text{PLOT 64}}$ S_W^2 from all LEP & SLD

Sensitive to quantum corrections

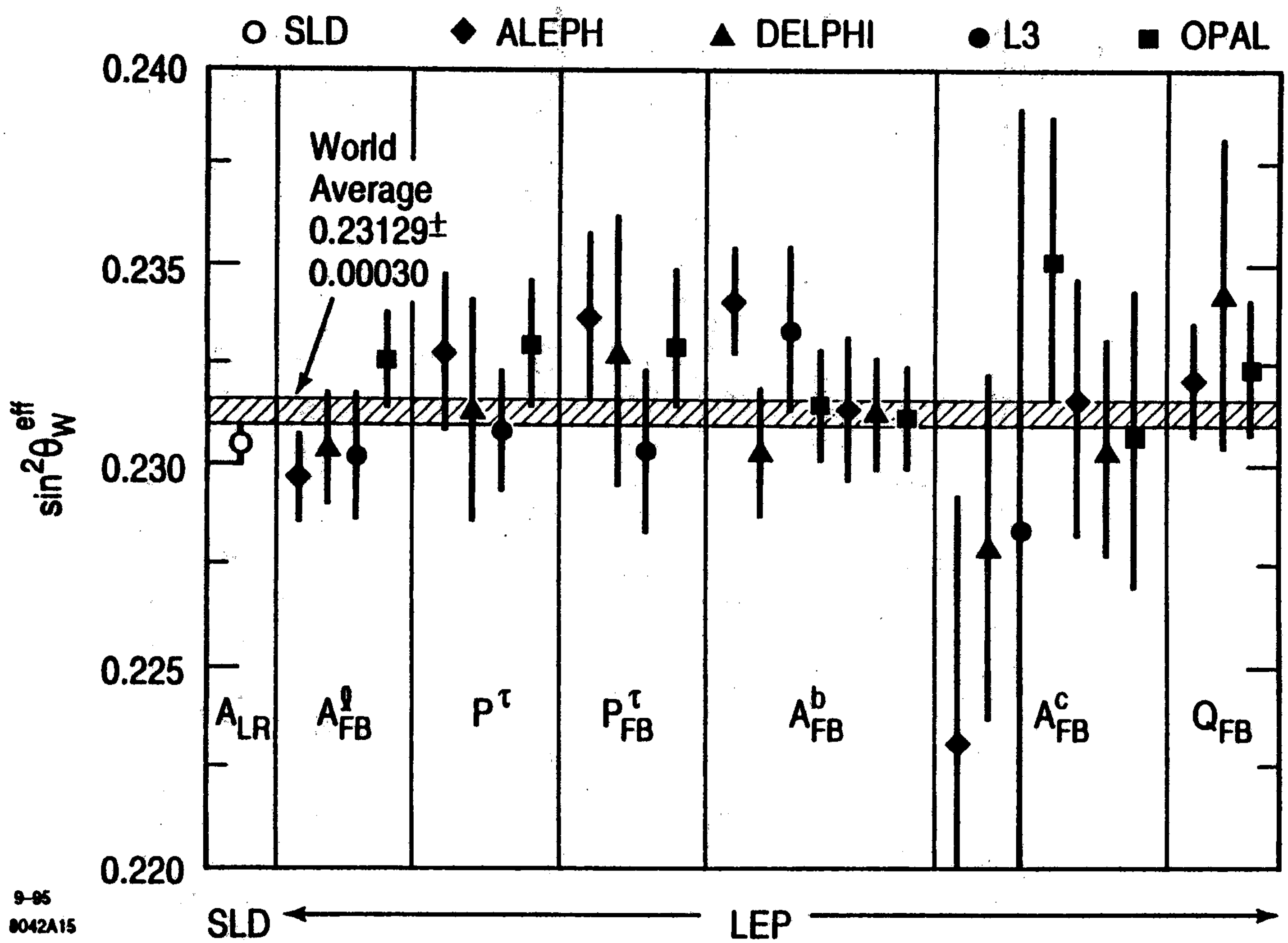


$\approx \log(m_h/m_Z)$

precision fit LEP: $m_h \approx 80 \pm 50$ GeV

$\boxed{\text{PLOT 65}}$ $\boxed{\text{PLOT PEW Higgs fit}}$

$\sin^2(\theta_W)$ experimental status



CY Prescott in Proc. of the 1995 Lepton-Photon conference.

Figure 15: Comparison of 30 measurements of $\sin^2 \theta_W^{eff}$ by LEP and SLD.

Precision electro-weak experimental status

P. Langacker and J. Erler   hep-ph/9703428

Quantity	Value	Standard Model
M_Z (GeV)	91.1863 ± 0.0020	input
Γ_Z (GeV)	2.4946 ± 0.0027	$2.496 \pm 0.001 \pm 0.001 \pm [0.002]$
$R = \Gamma(\text{had})/\Gamma(\ell\bar{\ell})$	20.778 ± 0.029	$20.76 \pm 0.003 \pm 0.001 \pm [0.02]$
$\sigma_{\text{had}} = \frac{12\pi}{M_Z^2} \frac{\Gamma(e\bar{e})\Gamma(\text{had})}{\Gamma_Z^2}$	41.508 ± 0.056	$41.46 \pm 0.002 \pm 0.002 \pm [0.02]$
$R_b = \Gamma(b\bar{b})/\Gamma(\text{had})$	0.2178 ± 0.0011	$0.2156 \pm 0 \pm 0.0002$
$R_c = \Gamma(c\bar{c})/\Gamma(\text{had})$	0.1715 ± 0.0056	$0.172 \pm 0 \pm 0$
$A_{FB}^{0\ell} = \frac{3}{4}\left(A_\ell^0\right)^2$	0.0174 ± 0.0010	$0.0157 \pm 0.0003 \pm 0.0003$
$A_\tau^0\,(P_\tau)$	0.1401 ± 0.0067	$0.145 \pm 0.001 \pm 0.001$
$A_e^0\,(P_\tau)$	0.1382 ± 0.0076	$0.145 \pm 0.001 \pm 0.001$
$A_{FB}^{0b} = \frac{3}{4} A_e^0 A_b^0$	0.0979 ± 0.0023	$0.101 \pm 0.001 \pm 0.001$
$A_{FB}^{0c} = \frac{3}{4} A_e^0 A_c^0$	0.0735 ± 0.0048	$0.072 \pm 0.001 \pm 0.001$
$\bar{s}_\ell^2\left(A_{FB}^Q\right)$	0.2320 ± 0.0010	$0.2318 \pm 0.0002 \pm 0.0001$
$A_\ell^0\left(A_{LR}^0, A_{e,\mu,\tau}^0\right)$ (SLD)	0.1542 ± 0.0037	$0.145 \pm 0.001 \pm 0.001$
A_b^0 (SLD)	0.863 ± 0.049	$0.935 \pm 0 \pm 0$
A_c^0 (SLD)	0.625 ± 0.084	$0.667 \pm 0.001 \pm 0.001$
N_ν	2.989 ± 0.012	3

Table 10.5: Principal Z pole observables and their SM predictions (*cf.* Table 10.4). The first $\bar{s}_\ell^2$ is the effective weak mixing angle extracted from the hadronic charge asymmetry at LEP 1 [14], the second is the combined value from the Tevatron [279], and the third is from the LHC [280–283]. The values of A_e are (i) from A_{LR} for hadronic final states [284]; (ii) from A_{LR} for leptonic final states and from polarized Bhabba scattering [285]; and (iii) from the angular distribution of the τ polarization at LEP 1 [14]. The A_τ values are from SLD [285] and the total τ polarization, respectively. Note that the SM errors in Γ_Z, the R_ℓ, and $\sigma_{\rm had}$ are largely dominated by the uncertainty in α_s.

Quantity	Value	Standard Model	Pull
M_Z [GeV]	91.1876 ± 0.0021	91.1882 ± 0.0020	-0.3
Γ_Z [GeV]	2.4955 ± 0.0023	2.4941 ± 0.0009	0.6
$\sigma_{\rm had}$ [nb]	41.481 ± 0.033	41.482 ± 0.008	0.0
R_e	20.804 ± 0.050	20.736 ± 0.010	1.4
R_μ	20.784 ± 0.034	20.736 ± 0.010	1.4
R_τ	20.764 ± 0.045	20.781 ± 0.010	-0.4
R_b	0.21629 ± 0.00066	0.21582 ± 0.00002	0.7
R_c	0.1721 ± 0.0030	0.17221 ± 0.00003	0.0
$A_{FB}^{(0,e)}$	0.0145 ± 0.0025	0.01617 ± 0.00007	-0.7
$A_{FB}^{(0,\mu)}$	0.0169 ± 0.0013		0.6
$A_{FB}^{(0,\tau)}$	0.0188 ± 0.0017		1.5
$A_{FB}^{(0,b)}$	0.0996 ± 0.0016	0.1029 ± 0.0002	-2.0
$A_{FB}^{(0,c)}$	0.0707 ± 0.0035	0.0735 ± 0.0002	-0.8
$A_{FB}^{(0,s)}$	0.0976 ± 0.0114	0.1030 ± 0.0002	-0.4
$\bar{s}_\ell^2$	0.2324 ± 0.0012	0.23155 ± 0.00004	0.7
	0.23148 ± 0.00033		-0.2
	0.23129 ± 0.00033		-0.8
A_e	0.15138 ± 0.00216	0.1468 ± 0.0003	2.1
	0.1544 ± 0.0060		1.3
	0.1498 ± 0.0049		0.6
A_μ	0.142 ± 0.015		-0.3
A_τ	0.136 ± 0.015		-0.7
	0.1439 ± 0.0043		-0.7
A_b	0.923 ± 0.020	0.9347	-0.6
A_c	0.670 ± 0.027	0.6677 ± 0.0001	0.1
A_s	0.895 ± 0.091	0.9356	-0.4

the reference axis of the b quark asymmetry [292], increased the extracted[14] $A_{FB}^{(0,b)}$ by about 0.2 σ. The correlation between $A_{FB}^{(0,b)}$ and $A_{FB}^{(0,c)}$ amounts to 15%.

In addition, SLD extracted the final-state couplings A_b, A_c [14], A_s [288], A_τ, and A_μ [285],

[14] We are grateful to Werner Bernreuther and Long Chen for the re-calculation of their result employing the more appropriate $\overline{\rm MS}$ mixing angle, $\widehat{s}_Z^2$, instead of the on-shell quantity, s_W^2.

EW physics follows from

$$\frac{g^2}{p^2-M_W^2}\, j^+_\mu j^{-\mu} + \frac{g^2+g'^2}{p^2-M_Z^2}\,\frac{\left(j^3_\mu - s_W^2\, j^{em}_\mu\right)^2}{2} + \frac{e^2}{p^2}\frac{\left(j^{em}_\mu\right)^2}{2}$$

extremely successful experimentally.

BUT: 1. W, Z masses break gauge invariance!

$$W^a_\mu \to W^a_\mu + \partial_\mu \alpha^a + \ldots$$

$$-\frac{1}{2} M^2 W^a_\mu W^{a\mu} \quad \text{not invariant}$$

2. no fermion masses

QED: $m\bar{\psi}\psi = m\,\psi_L^\dagger \psi_R + \text{h.c.}$

SM: $L_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L,\ e_R$

$$m\, e_L^\dagger e_R \qquad \text{not allowed!}$$

$\Longrightarrow$ Higgs mechanism.

try to write electron mass

$$L_L^\dagger e_R = 2^*_{-1/2}$$

$2^*_{+1/2}\ 1_{-1}$ $\qquad \uparrow$ SU(2) $\uparrow$ $U(1)_Y$

$\Longrightarrow$ can write invariant term with field of charge $2_{1/2}$:

"Higgs doublet" $\qquad \phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$

$$\lambda_e L_L^\dagger \phi e_R = \lambda_e (\nu_L^\dagger \phi^+ + e_L^\dagger \phi^0) e_R$$

$\Rightarrow$ e-mass iff $\phi^0 = \frac{v}{\sqrt{2}} \Rightarrow m_e = \lambda_e \frac{v}{\sqrt{2}}$

$m_\nu = 0$

What is ϕ? scalar field

$$\mathcal{L} \sim D_\mu \phi^\dagger D^\mu \phi - V(\phi^\dagger \phi)$$

SU(2)×U(1) invariant