non-abelian covariant derivative

$$\phi \to U\phi \equiv \phi' \qquad U = e^{i\alpha(x)} = e^{i\alpha^a(x) T^a}$$

$\partial_\mu \phi^\dagger \partial^\mu \phi$ not invariant

$D_\mu \phi^\dagger D^\mu \phi$ invariant

define: $D_\mu = \partial_\mu - igA_\mu$ and $D_\mu \phi \to U D_\mu \phi = U D_\mu U^\dagger \underbrace{U\phi}_{\phi'}$

$$U D_\mu U^\dagger = \partial_\mu + U\,\partial_\mu(U^\dagger) - ig\, U A_\mu U^\dagger$$

$$\Rightarrow A_\mu \to A_\mu' = U A_\mu U^\dagger + \frac{i}{g} U \partial_\mu U^\dagger$$

$$\approx (1+i\alpha) A_\mu (1-i\alpha) + \frac{i}{g}(1+i\alpha)\,\partial_\mu(1-i\alpha)$$

$$\approx A_\mu + i[\alpha, A_\mu] + \frac{1}{g}\partial_\mu \alpha$$

A_μ traceless, hermitian $\Rightarrow$ can expand $A_\mu = A_\mu^a T^a$

$$A_\mu^a \to A_\mu^a + \frac{1}{g}\partial_\mu \alpha^a + f^{abc} A_\mu^b \alpha^c$$

Kinetic term: $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu] = F_{\mu\nu}^a T^a$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c$$

Note: $\frac{1}{-ig}[D_\mu, D_\nu]\,\phi = \frac{1}{-ig} F_{\mu\nu}\,\phi$

but $[D_\mu, D_\nu]\phi \to [U D_\mu U^+, U D_\nu U^+]\phi'$

$$= U[D_\mu, D_\nu]U^+\phi' = F'_{\mu\nu}\phi'$$

$$\Rightarrow F_{\mu\nu} \to U F_{\mu\nu} U^+$$

$$\Rightarrow \mathcal{L} = \underbrace{-\frac{1}{2}\,\mathrm{tr}\, F_{\mu\nu}F^{\mu\nu}}_{= -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a}} + D_\mu\phi^+ D^\mu\phi - V(\phi^+\phi)$$

SU(N) gauge invariant Lagrangian

SU(2) - invariant weak interactions

$$L \equiv \begin{pmatrix}\nu \\ e^-\end{pmatrix}_L \qquad L \to UL = e^{i\alpha^a \frac{\sigma^a}{2}} L$$

$$e_R \qquad D_\mu L = \left(\partial_\mu - ig W_\mu^a \frac{\sigma^a}{2}\right) L$$

acts in SU(2) weak space

$$\mathcal{L} \sim L^+ i\bar{\sigma}^\mu D_\mu L + e_R^+ i\bar{\sigma}^\mu \partial_\mu e_R$$

acts in spinor space

$$\sigma^{\pm} = \frac{1}{2}(\sigma^1 \pm i\sigma^2) = \begin{cases} \begin{pmatrix} 0 & 1 \\ & \end{pmatrix} \\ \begin{pmatrix} & \\ 1 & 0 \end{pmatrix} \end{cases}$$

$$W^{\pm}_\mu = \frac{1}{\sqrt{2}}\left(W^1_\mu \mp i W^2_\mu\right)$$

$$\Rightarrow \quad D_\mu L = \left(\partial_\mu - i\,\frac{g}{\sqrt{2}}\left(W^+_\mu \sigma^+ + W^-_\mu \sigma^-\right) - i g W^3 \frac{\sigma^3}{2}\right) L$$

$$\Rightarrow \mathcal{L} \sim \frac{g}{\sqrt{2}}\left(W^+_\mu\, \nu_L^\dagger \bar{\sigma}^\mu e_L + W^-_\mu\, e_L^\dagger \bar{\sigma}^\mu \nu_L\right)$$

$$+ \frac{g}{2} W^3_\mu \left(\nu_L^\dagger \bar{\sigma}^\mu \nu_L - e_L^\dagger \bar{\sigma}^\mu e_L\right)$$

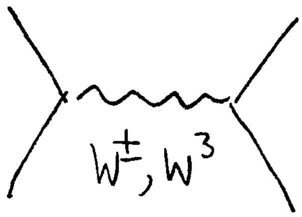
$$= g\left(W^{\pm}_\mu\, j_L^{\mu\pm} + W^3_\mu\, j_L^{\mu 3}\right)$$

$$j_L^{\mu +} = \frac{1}{\sqrt{2}}\left(\nu_L^\dagger \bar{\sigma}^\mu e_L + u_L^\dagger \bar{\sigma}^\mu d_L\right)$$

$$j_L^{\mu -} = \text{h.c.}$$

$$j_L^{\mu 3} = \frac{1}{2}\left(\nu_L^\dagger \bar{\sigma}^\mu \nu_L - e_L^\dagger \bar{\sigma}^\mu e_L + u_L^\dagger \bar{\sigma}^\mu u_L - d_L^\dagger \bar{\sigma}^\mu d_L\right)$$

weak interaction current-current:



$$\frac{1}{p^2 - m_W^2} \approx \frac{-1}{m_W^2}$$

$$-\frac{g^2}{m_W^2}\left(j_L^{\mu +} j_{\mu L}^{-} + \frac{1}{2} j_L^{\mu 3} j_{\mu L}^{3} \right)$$

cancels against 2 in matrix element

same as few lectures ago.

Standard Model: $SU(2) \times U(1)$ Glashow, Weinberg, Salam

$$L \to \underbrace{e^{i\alpha^a T^a}}_{SU(2)} \underbrace{e^{i\beta Y}}_{U(1)} L$$

(Y ← hypercharge)

$$e_R \to e^{i\beta Y} e_R$$

	L_L	E_R	Q_L	U_R	D_R
$SU(2)$	2	–	2	–	–
$U(1)_Y$	$-1/2$	-1	$1/6$	$2/3$	$-1/3$
Q	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$	-1	$\begin{pmatrix} 2/3 \\ -1/3 \end{pmatrix}$	$2/3$	$-1/3$

← 3 copies each