$\Rightarrow (D^\mu\phi)^* D_\mu \phi$ gauge invariant kinetic term

D_μ = "covariant derivative"

Aside: $(D^\mu\phi)^* = (\partial^\mu + igA^\mu)\phi^* = (\partial^\mu - ig(-1)A^\mu)\phi^*$

charge of ϕ^*

generalization to field of charge Q:

$$D_\mu \phi^Q = (\partial_\mu - igQA_\mu)\phi^Q$$

A_μ new field, needs kinetic term to make its energy density dynamical

$\partial_\nu A_\mu \partial^\nu A^\mu$ not gauge-invariant

define $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu \longrightarrow F_{\mu\nu} + \cancel{\frac{1}{g}\partial_\mu\partial_\nu \alpha} - \cancel{\frac{1}{g}\partial_\nu\partial_\mu \alpha}$

$$\Rightarrow \mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + D_\mu\phi^* D^\mu\phi - V(\phi^*\phi)$$

gauge invariant Lagrangian "scalar QED"

vary $A_\mu + \delta A_\mu$ to get EOM

$$\Rightarrow \partial_\mu F^{\mu\nu} + \underbrace{gi\left(\phi^* D^\nu \phi - D^\nu(\phi^*)\phi\right)}_{\equiv j^\nu} = 0$$

multiple scalars, charges Q_i $\phi_i \to e^{i\alpha Q_i}\phi_i$

$$\Rightarrow j^\nu = \sum_i iQ_i\left(\phi^* D^\nu \phi - D^\nu(\phi^*)\phi\right)$$

charged fermion? $\psi^\dagger i\bar{\sigma}^\mu(\partial_\mu - igQA_\mu)\psi$

$$j^\mu = Q\psi^\dagger \bar{\sigma}^\mu \psi$$

Dirac $\bar{\Psi}(i\not{D} - m)\Psi$, $j^\mu = Q\bar{\Psi}\gamma^\mu\Psi$

gauged scalar EOM: $(D_\mu D^\mu + m^2)\phi = 0$

Dirac EOM: $(i\not{D} - m)\Psi = 0$

$\vec{E}$ and $\vec{B}$? $A^\mu = (\phi, \vec{A})$

$F^{00} = 0$

$F^{0i} = \dot{A}^i + \nabla^i\phi \equiv -E^i$

$$F^{ij} = -\left(\nabla^i A^j - \nabla^j A^i\right) = -\epsilon^{ijk} B^k \qquad (\vec{B} = \vec{\nabla} \times \vec{A})$$

$$F = \left(\begin{array}{c|ccc} 0 & & -\vec{E}^T & \\ \hline & 0 & -B_3 & B_2 \\ +\vec{E} & B_3 & 0 & -B_1 \\ & -B_2 & B_1 & 0 \end{array}\right)$$

Maxwell? $\quad \vec{\nabla}\cdot\vec{B} = 0 \quad \dot{\vec{B}} + \vec{\nabla}\times\vec{E} = 0$ follow from $\epsilon^{\mu\nu\alpha\beta}\partial_\nu F_{\alpha\beta} = 0$

(def of F)

$\vec{\nabla}\cdot\vec{E} = -g\rho$

$-\dot{\vec{E}} + \vec{\nabla}\times\vec{B} = -g\vec{j}$ $\quad$ " " $\partial_\mu F^{\mu\nu} = -g j^\nu$

(EOM)

Non-Abelian gauge fields

$\bar{\psi}_i \, i\not{\partial} \psi_i$ preserves flavor, $W_\mu^\pm$ couplings don't

$\Rightarrow$ generalization needed.

SU(N) transformations $\quad \psi = \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_N \end{pmatrix} \longrightarrow U\psi \equiv e^{i\alpha}\psi$

unitary $U^\dagger U = \mathbb{1}_N$ $\qquad$ hermitian $\alpha^\dagger = \alpha$

$$\alpha = \sum_{a=1}^{N^2-1} \alpha^a T^a + \alpha^0 \mathbb{1}$$

real parameters $\quad$ hermitian, traceless basis = "generators" of SU(N) ...

normalization: $\mathrm{tr}[T^a T^b] = \frac{1}{2}\delta^{ab}$

$-i[T^a, T^b]$ is traceless, hermitian $N\times N$ $\Rightarrow$ can expand in basis

$= f^{abc} T^c$

$\nwarrow$ structure constants, define the transformation group

$\{T^a\}$ = Lie algebra

$\{U\}$ = Lie group

$f^{abc} = -i2\,\mathrm{tr}\left([T^a, T^b]T^c\right)$ totally antisymmetric.

Example: $SU(2)$: $T^a = \frac{\sigma^a}{2}$ $\quad \mathrm{tr}\frac{(\sigma^a\sigma^b)}{4} = \frac{1}{4}\mathrm{tr}\begin{pmatrix}1&0\\0&1\end{pmatrix}\delta^{ab} = \frac{1}{2}\delta^{ab}$

$-i\frac{1}{4}[\sigma^a, \sigma^b] = \epsilon^{abc}\frac{\sigma^c}{2} \Rightarrow f^{abc}_{SU(2)} = \epsilon^{abc}$

$SU(3)$ $\quad T^a$ = Gell-Mann matrices $\quad a = 1..8 \quad T^{1,2,3} = \frac{1}{2}\left(\begin{array}{c|c}\sigma^a & \begin{matrix}0\\0\end{matrix}\\ \hline \begin{matrix}0&0\end{matrix} & 0\end{array}\right)$ ..

$T^{4,5} = \frac{1}{2}\begin{pmatrix}0 & & 1,-i\\ & 0 & \\ 1,i & & 0\end{pmatrix} \quad T^{6,7} = \begin{pmatrix} & 0 & 0\\ & & 1,-i\\ & & \end{pmatrix} \quad T^8 = N\begin{pmatrix}1 & & \\ & 1 & \\ & & -2\end{pmatrix}$

$\mathrm{Tr}\, T^8 T^8 = N^2 6 \Rightarrow N = 1/\sqrt{12}$