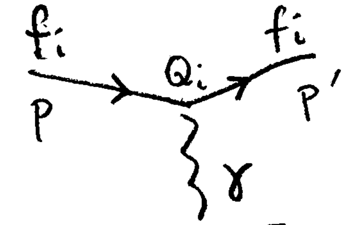
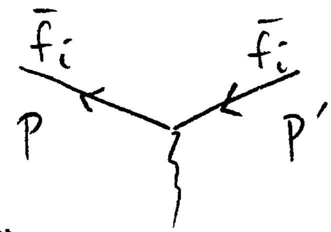
Feynman rule:



$\bar{\psi}\gamma^\mu\psi$

matrix element $\langle f_i | j^\mu | f_i \rangle = Q_i \bar{u}_{p'} \gamma^\mu u_p$

↑ ↗ particles



anti-particles

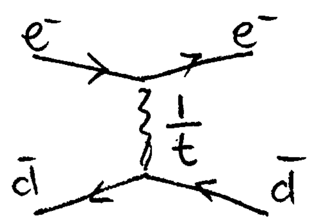
$\langle \bar{f}_i | j^\mu | \bar{f}_i \rangle = Q_i \bar{v}_p \gamma^\mu v_{p'}$

QED photon exchange ⟷ current-current interaction

$$\frac{e^2}{2} j^\mu_{EM} \frac{1}{q^2} j_{\mu EM} \Rightarrow M = \frac{e^2}{2q^2} \langle f | j^\mu_{EM} j_{\mu EM} | i \rangle$$

10/21/13

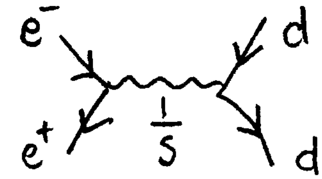
example 1. $e^-\bar{d} \to e^-\bar{d}$



$$M = \frac{e^2}{2t} \left(\langle e^- | j^\mu | e^- \rangle \langle \bar{d} | j_\mu | \bar{d} \rangle + \underbrace{\langle e^- | j_\mu | e^- \rangle \langle j^\mu \rangle}_{\text{same at 1st term, cancels } 1/2} \right.$$

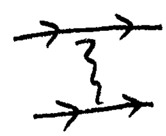
$$\mathcal{M} = \frac{e^2}{t}\ \bar{u}\gamma^\mu u\ \bar{v}\gamma_\mu v\ \underset{\uparrow\ Q_{e^-}}{(-1)}\underset{\uparrow\ Q_d}{(-\tfrac{1}{3})}$$

example 2: $e^+e^- \to d\bar{d}$



$$\mathcal{M} = \frac{e^2}{s}\frac{1}{3}\ \bar{v}\gamma^\mu u\ \bar{u}\gamma_\mu v$$

example 3: $e^-e^- \to e^-e^-$


$-$


$$\mathcal{M} = \frac{e^2}{2}(-1)^2\left[\frac{1}{t}\ \bar{u}\gamma^\mu u\,\bar{u}\gamma_\mu u - \frac{1}{u}\ \bar{u}\gamma^\mu u\,\bar{u}\gamma_\mu u\right]\cdot 2$$

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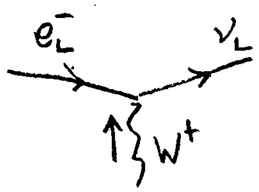
charged current:



$$-i\frac{g}{\sqrt{2}}\ \overset{\nu_e}{\bar{u}}\ \gamma^\mu P_L \overset{e}{u}$$

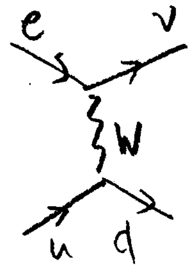
$$j_L^{\mu +} = \bar{\nu}_e\gamma^\mu P_L e$$ ← electron field operator destroys e^-, creates e^+

$$+\bar{u}\gamma^\mu P_L d$$

+ other 2 generations

$$j_L^{\mu -} = \left(j_L^{\mu +}\right)^\dagger \sim e^\dagger P_L \gamma^{\mu\dagger}\gamma^0 \nu = \bar{e}\gamma^\mu P_L \nu$$

example: $eu \to \nu d$



$$\mathcal{M} = \frac{g^2}{2} \frac{1}{q^2 - M_W^2} \langle f | j^{\mu +} j_\mu^- | i \rangle$$

$$= \frac{g^2}{2} \frac{1}{t - M_W^2} \bar{u}_\nu \gamma^\mu P_L u_e \, \bar{u}_d \gamma_\mu P_L u_u$$

$SU(2)_{weak}$ notation: $\begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \begin{pmatrix} u_L \\ d_L \end{pmatrix}$ $SU(2)_{weak}$ doublets "isospin"

def $\sigma^\pm = \frac{1}{2}(\sigma_1 \pm i \sigma_2) = \begin{cases} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \end{cases}$

$$\Rightarrow \quad j_L^{\mu \pm} = \overline{\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}} \sigma^\pm \gamma^\mu \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} = \overline{\begin{pmatrix} \nu \\ e \end{pmatrix}} \sigma^\pm \gamma^\mu P_L \begin{pmatrix} \nu \\ e \end{pmatrix}$$

$$+ \overline{\begin{pmatrix} u \\ d \end{pmatrix}} \sigma^\pm \gamma^\mu \begin{pmatrix} u \\ d \end{pmatrix} + \text{other generations}$$

$\Rightarrow$ current-current interaction:

$$\frac{g^2}{2} \sum_{\text{doublets } i} \overline{(\,)}_i \sigma^+ \gamma^\mu P_L (\,)_i \frac{1}{q^2 - M_W^2} \sum_j \overline{(\,)}_j \sigma^- \gamma_\mu P_L (\,)_j \xrightarrow{q^2 \ll M_W^2} -\frac{4G_F}{\sqrt{2}} j^+_{\mu_L} j^{\mu -}_L$$

this is not SU(2) invariant. SU(2) invariant would be

$$\frac{1}{4}\vec{\sigma}\cdot\vec{\sigma} = \frac{1}{4}\sigma_a\,\sigma_a = \frac{1}{4}\Big((\sigma_1 + i\sigma_2)(\sigma_1 - i\sigma_2) + \sigma_3\sigma_3\Big)$$

$$= \sigma^+\sigma^- + \frac{\sigma_3}{2}\,\frac{\sigma_3}{2}$$

would be

$\Rightarrow$ SU(2) invariant current-current theory

$$-\frac{4G_F}{\sqrt{2}}\left(j^{\mu +}_L\, j^-_{\mu L} + j^\mu_{3L}\, j_{3L\mu}\right)$$

with $j^\mu_{3L} \equiv \sum_i \overline{(\ )}_i\, \frac{\sigma_3}{2}\,\gamma^\mu P_L\, (\ )_i$ "neutral current"

Experiment shows :

$$\boxed{-\frac{4G_F}{\sqrt{2}}\left(\overset{W}{j^{\mu +}_L\, j^-_{\mu L}} + \overbrace{\left(j^{\mu 3}_L - \sin^2\theta_W\, j^\mu_{EM}\right)\left(j^3_{\mu L} - \sin^2\theta_W\, j_{\mu EM}\right)}^{Z}\right) + \overset{\gamma}{\frac{e^2}{2}\frac{1}{q^2}\, j^\mu_{EM}\, j_{\mu EM}}}$$

weak mixing angle

$\sin^2\theta_W \approx 0.23$

electroweak interactions