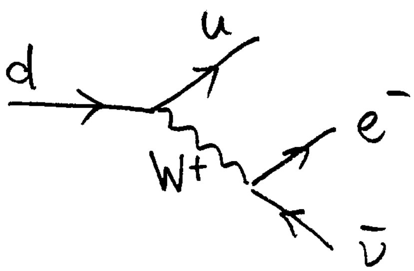
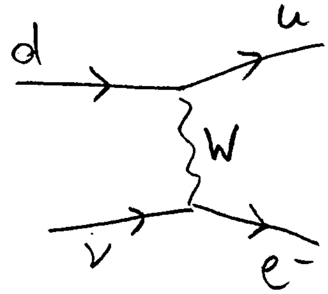
Weak interactions (Peskin 5)

famous example: $n \to p^+ e^- \bar{\nu}_e$



crossing $\Longleftrightarrow$



flavor changing:

$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix} \begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix} \qquad W^\pm$$

parity violating (unlike QED, QCD)

parity: $q_L \leftrightarrow q_R$ (both helicity and chirality)

$$\Rightarrow \quad \mathcal{M}^{QED}(e^-_L e^+_R \to q_L \bar{q}_R) = \mathcal{M}^{QED}(e^-_R e^+_L \to q_R \bar{q}_L)$$

weak interactions: only left chirality particles couple to $W^\pm$

corresponding spinors $\quad u_L = P_L u = \begin{pmatrix} 1 & \\ & 0 \end{pmatrix} u = \begin{pmatrix} u_L \\ 0 \end{pmatrix} \xrightarrow[p \parallel \hat{z}]{m \to 0} \sqrt{2E} \begin{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ 0 \\ 0 \end{pmatrix}$

abuse of notation

$$V_L = P_L V = \begin{pmatrix} V_L \\ 0 \end{pmatrix} \longrightarrow \sqrt{2E} \begin{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \end{pmatrix} \end{pmatrix}$$

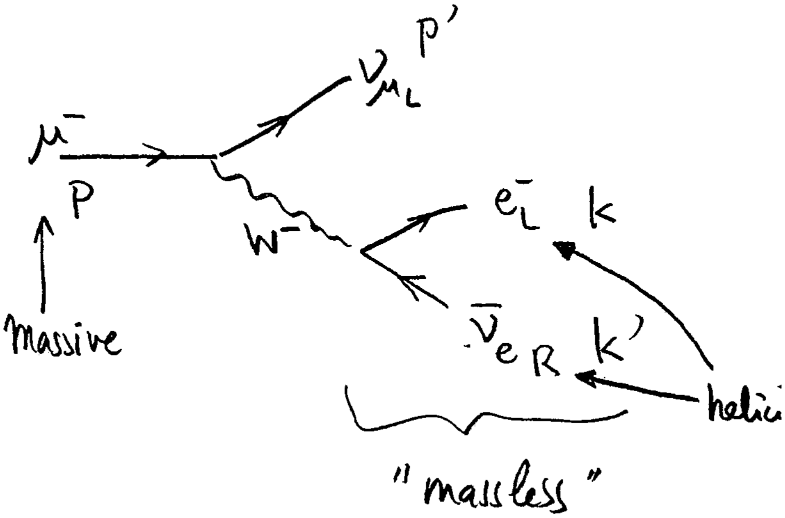
$\leftarrow$ opposite spin of anti-parti

"weak" ?
$$\frac{g_w^2}{p^2 - M_w^2} \quad \text{vs.} \quad \frac{e^2}{p^2}$$

$$\Rightarrow \frac{\text{weak}}{\text{QED}} \sim \frac{g_w^2}{e^2}\,\frac{p^2}{p^2-m_w^2} \sim \begin{cases} 10^{-6} & \text{in } \mu\text{-decay} \\ 1 & \text{at LHC} \end{cases}$$

$$g_s \sim 1\,,\; g_w \sim 2/3\,,\; e \sim 1/3 \quad @\; M_Z$$

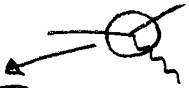
famous calculation : μ-decay



in rest frame:

$$\Gamma = \underbrace{\frac{1}{2m_\mu}}_{\substack{\text{instead of} \\ \frac{1}{2E_1}\frac{1}{2E_2}\frac{1}{|v_1-v_2|}}} \int\frac{d^3p'}{(2\pi)^3 2E_{p'}} \int\frac{d^3k}{(2\pi)^3 2E_k}\int\frac{d^3k'}{(2\pi)^3 2E_{k'}} (2\pi)^4\delta^4(p-k-k'-p')\;\frac{1}{2}\sum_{\text{spins}}|M|^2$$

average over μ-spin



$$iM = \bar{u}_{p'} \frac{i g_w}{\sqrt{2}} \gamma^\mu P_L u_p \frac{-i g_{\mu\nu}}{(p-p')^2 - M_W^2} \bar{u}_k \, i \frac{g_w}{\sqrt{2}} \gamma^\nu P_L V_{k'}$$

$$\Rightarrow \; M \approx -\frac{g^2}{2M_W^2} \bar{u}_{p'} \gamma^\mu P_L u_p \;\; \bar{u}_k \gamma_\mu P_L V_{k'}$$

$$\frac{1}{2}\Sigma |M|^2 = \frac{g^4}{8M_W^4} \,\mathrm{tr}\Big[\underbrace{u_{p'}\bar{u}_{p'}}_{\not{p}+m} \gamma^\mu P_L u_p \bar{u}_p \gamma^\nu P_L\Big] \,\mathrm{tr}\Big[u_k \bar{u}_k \gamma_\mu P_L V_{k'} \bar{V}_{k'} \gamma_\nu P_L\Big]$$

but $\quad P_L(\not{p}+m)\gamma^\mu P_L = P_L(\not{p}+\cancel{m}) P_R \gamma^\mu \quad$ mass drop out

$$\mathrm{tr}\left[\not{p}' \gamma^\mu \not{p} \gamma^\nu P_L\right] \mathrm{tr}\left[\not{k} \gamma_\mu \not{k}' \gamma_\nu P_L\right]$$

$$= \frac{2g^4}{M_W^4} \, p\cdot k' \; p'\cdot k$$

final ν's not observable $\Rightarrow$ integrate over

$$J^{\mu\nu} \equiv \int \frac{d^3p'}{(2\pi)^3 2E_{p'}} \int \frac{d^3k'}{(2\pi)^3 2E_{k'}} (2\pi)^4 \delta^4(X - p' - k') \, k'^\mu p'^\nu$$

$$= A\, X^\mu X^\nu + B\, X^2 g^{\mu\nu}$$

correct Lorentz indices, X^2 by dim'l analysis, A,B = #!

determine A, B: pick $X = (m\ 0\ 0\ 0)$

$$\Rightarrow p' = \frac{m}{2}(1, \hat{n}) \quad k' = \frac{m}{2}(1, -\hat{n})$$

$$J^{00}_{RHS} = (A+B)\, m^2 = J^{00}_{LHS} = \int \frac{d^3p'}{(2\pi)^2\, 4E_{p'}^2}\, \delta(m - 2E_{p'})\, p'^0 k'^0$$

$$= \frac{1}{16\pi^2} \int d\Omega\, \frac{1}{2}\, \frac{m^2}{4} = m^2/32\pi$$

$$J^{ij}_{RHS} = -B\, m^2\, \delta^{ij} = J^{ij}_{LHS} = -\int \frac{d^3p'}{(2\pi)^2\, 4}\, \delta(m - 2E_{p'})\, n^i n^j$$

$$= -\frac{1}{16\pi^2} \underbrace{\int d\Omega\, n^i n^j}_{\equiv a\delta^{ij}}\, \frac{m^2}{8} = -\frac{1}{3}\, \frac{m^2}{32\pi}\, \delta^{ij}$$

take trace: $\int d\Omega\, 1 = 4\pi = 3a \Rightarrow a = \frac{4}{3}\pi$

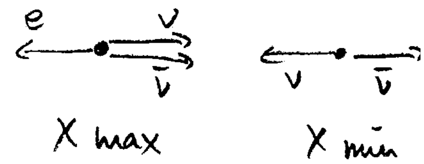
$$\Rightarrow \quad B = \frac{1}{3}\, \frac{1}{32\pi}, \quad A = \frac{2}{3}\, \frac{1}{32\pi}$$

$$J^{\mu\nu} = \frac{1}{32\pi}\left(\frac{2}{3} X^\mu X^\nu + \frac{1}{3} X^2 g^{\mu\nu}\right)$$

μ^- rest frame: $p = (m\ 0)$ $\quad k^\mu = \frac{m}{2} x (1, \hat{n})$

$\qquad\qquad 0 < x < 1$

$$X^2 = (p-k)^2 = m^2 - 2p\cdot k$$
$$= m^2(1-x)$$



$$p\cdot X = p^2 - p\cdot k = m^2\left(1 - \tfrac{x}{2}\right)$$

$$k\cdot X = k\cdot p = \tfrac{1}{2} x m^2$$

$$\Rightarrow \Gamma = \frac{1}{2m_\mu}\frac{2g^4}{m_W^4}\int\frac{d^3k}{(2\pi)^3 2E_k}\frac{1}{32\pi}\frac{1}{3}\left(2X\cdot p\, X\cdot k + X^2 k\cdot p\right)$$

$$= \frac{g^4}{m\, m_W^4}\frac{1}{32\pi}\frac{1}{3}\frac{4\pi}{16\pi^3}\left(\frac{m}{2}\right)^2\int_0^1 x\,dx\, m^4\left[x\left(1-\frac{x}{2}\right) + (1-x)\frac{x}{2}\right]$$

$$= \frac{m^5}{M_W^4}\frac{1}{96\pi^3}\frac{1}{16}\underbrace{\int_0^1 dx\, x^2\left(\frac{3}{2}-x\right)}_{\frac{1}{2}-\frac{1}{4}=1/4} = \boxed{\frac{m_\mu^5}{192\pi^3}\underbrace{\frac{g^4}{32M_W^4}}_{\equiv G_F^2} = \Gamma_\mu}$$

$\equiv G_F^2$ Fermi constant

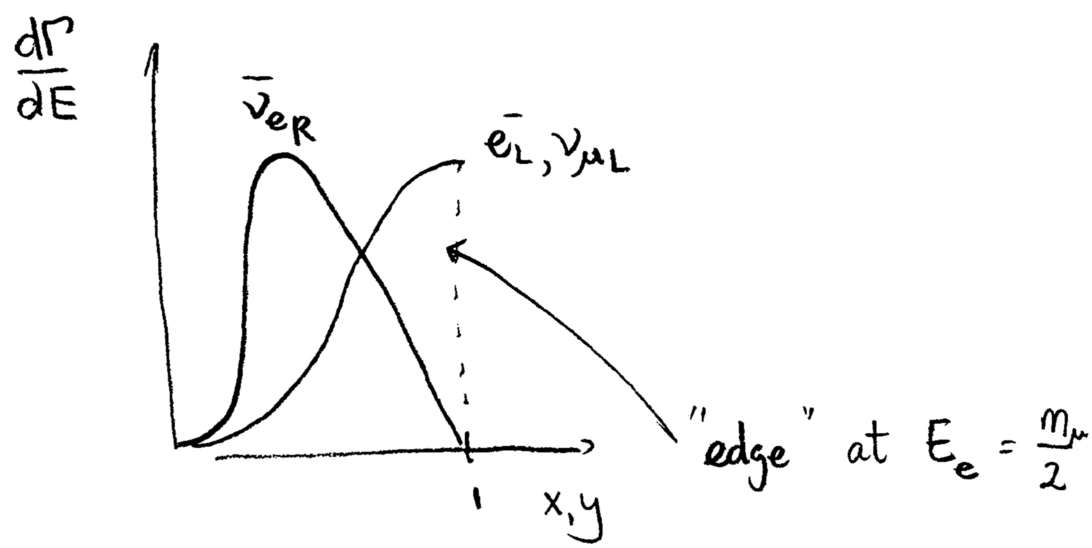
$$G_F = \frac{g^2}{\sqrt{2}\, 4m_W^2} \sim 10^{-5}\ \text{GeV}^{-2}$$

$$\frac{d\Gamma}{dx} \propto x^2\left(\frac{3}{2}-x\right) \qquad \text{max at } x=1$$

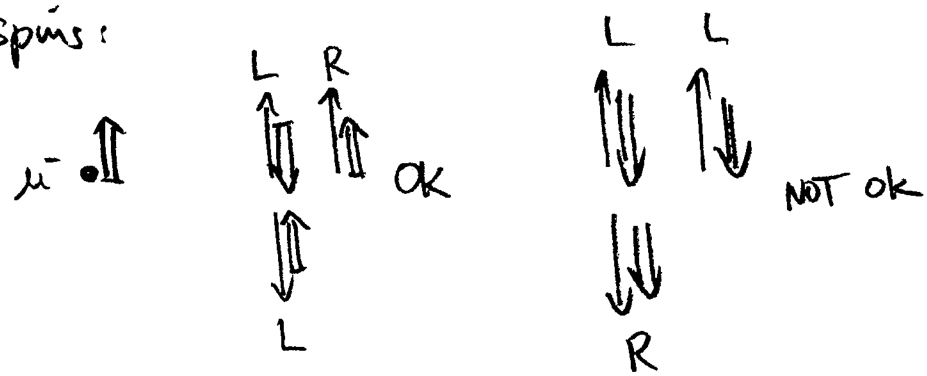
call $\bar{\nu}_e$ energy $\frac{m}{2}y$

$$\frac{d\Gamma}{dy} \sim y^2(1-y) \qquad 0 \text{ at } y=1$$

$\nu_{\mu L}$? $|M|^2$ symmetric under $p' \leftrightarrow k$ $\Rightarrow$ same as e^-



Spins:



$\Rightarrow$ only L particles can get max E, highest energy opposite to μ^- spin

$\boxed{\text{PLOT 44}}$ e^- shape

$\boxed{\text{PLOT 45, 46}}$ 45: highest X e^- backward (rel to μ-spin)

46: muons in B-field precess, select e^- above X-threshold $\Rightarrow$ oscillation. μ-spin towards detector $\Rightarrow$ no high-E e^-.

---

deep inelastic $\bar{\nu}$-scattering

$$\nu_\mu + (p,n) \xrightarrow{W} \mu^- + X \qquad \text{charged current CC}$$

$$\nu_\mu + (p,n) \xrightarrow{Z^0} \nu_\mu + X \qquad \text{neutral current NC}$$

$\boxed{\text{PLOT 47}}$ no muon track in NC event

Currents: (a convenient notation)

$Q e A_\mu \bar{\psi} \gamma^\mu \psi$ photon coupling to fermion ψ in $\mathcal{L}$

$\sum_i Q_i e A_\mu \bar{\psi}_i \gamma^\mu \psi_i$ multiple fermions.

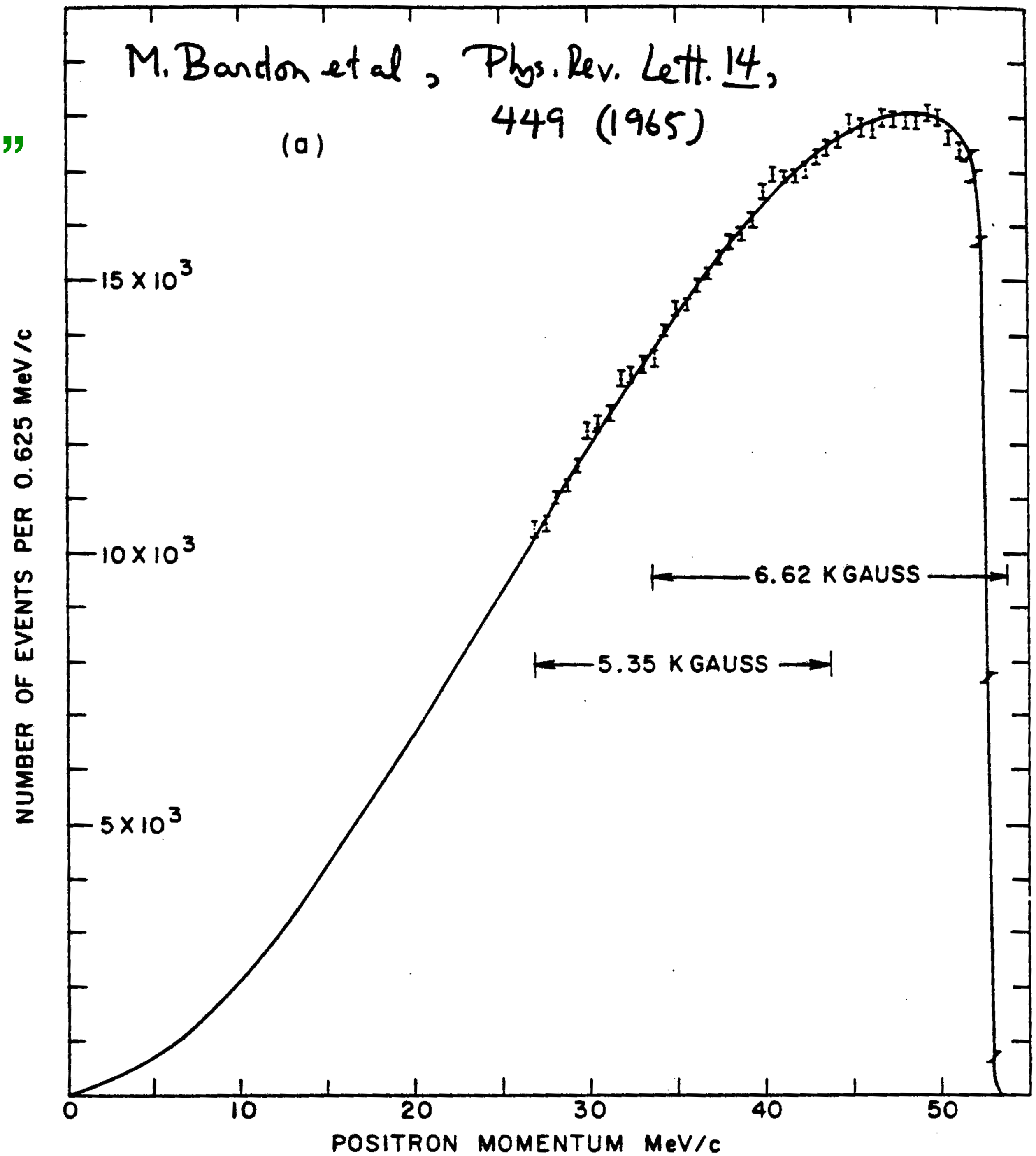
$j^\mu = \sum_i Q_i \bar{\psi}_i \gamma^\mu \psi_i$ fermion current, hermitean

Q_i ↑ charge of particle

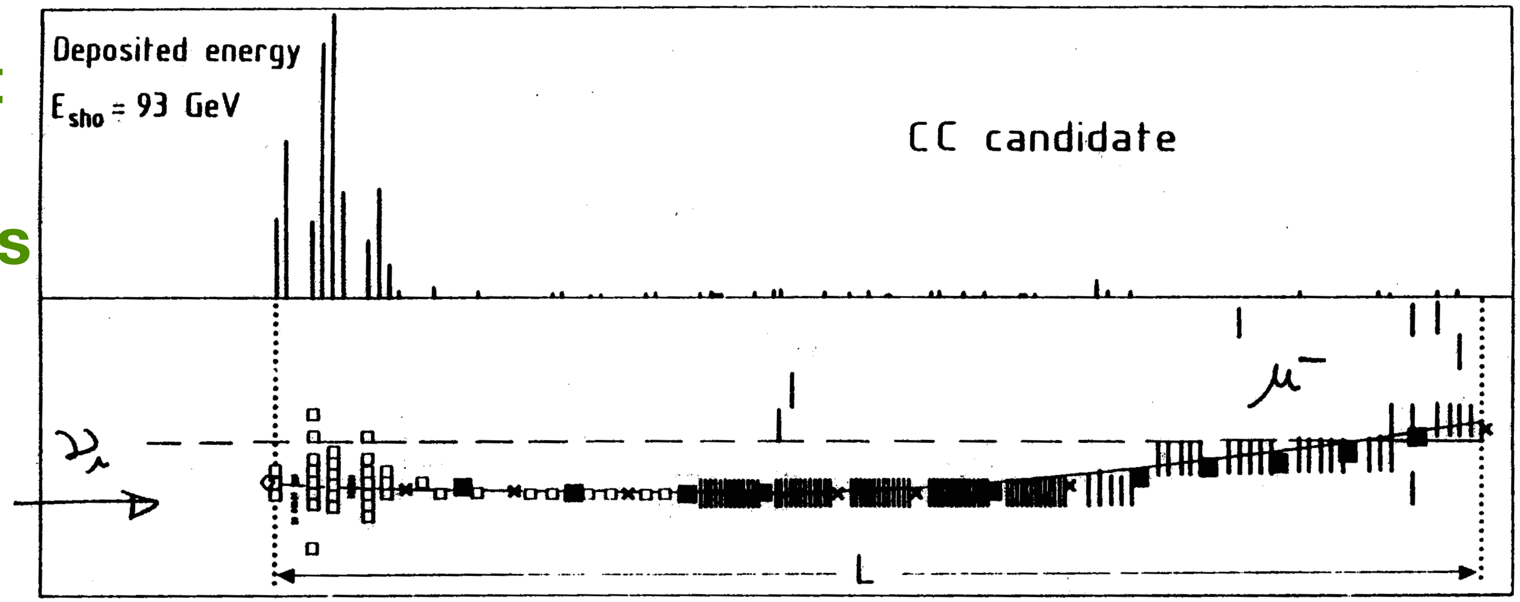
ψ can $\begin{cases} \text{destroy particle} \longrightarrow u \\ \text{create anti-p.} \longrightarrow v \end{cases}$

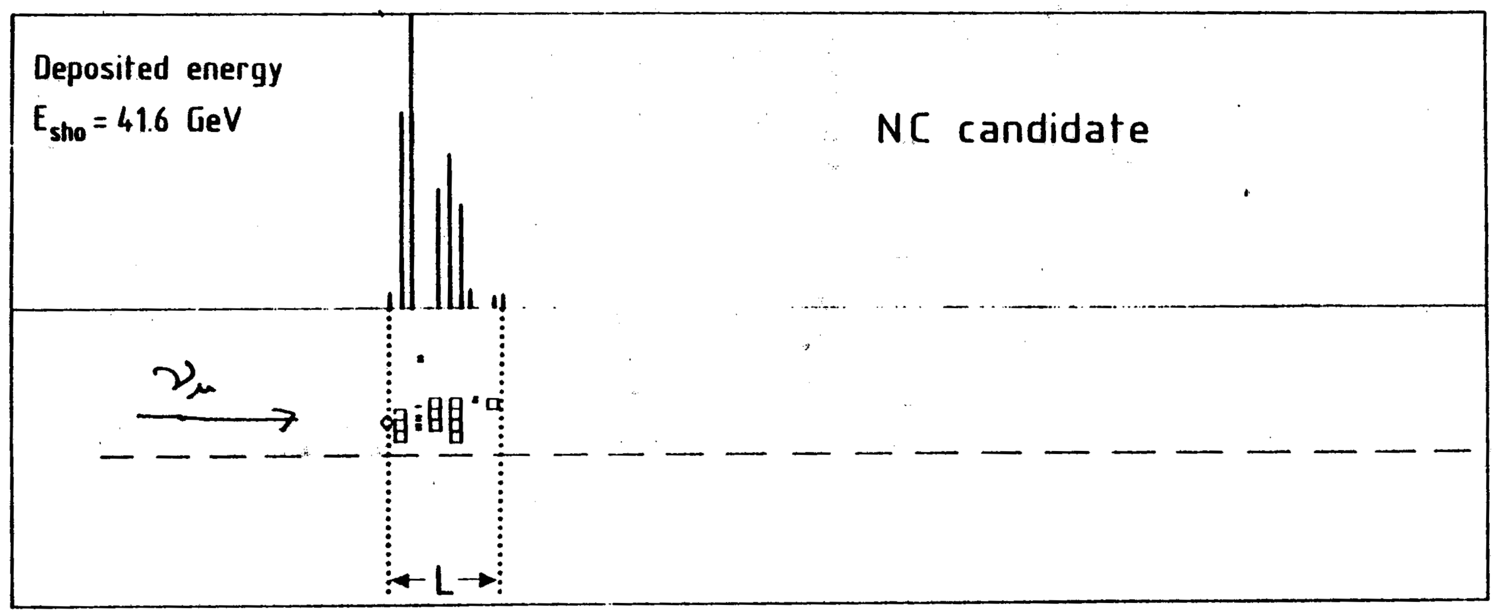
$\bar{\psi}$ opposite

Spectrum of “Michel electrons”



Charged and neutral current interactions of muon neutrinos





A. Blondel et al (CDHS) Z.Phys. C45, 361 (1990)