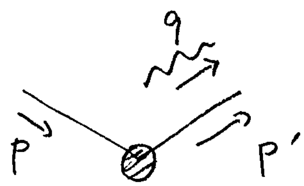
last time:

$$d\sigma(i \to f + \gamma) = d\sigma(i \to f) \int \frac{d^3q}{(2\pi)^3 2E_q} e^2 \left| \frac{p' \cdot \epsilon}{p' \cdot q} - \frac{p \cdot \epsilon}{p \cdot q} \right|^2$$



note: $p' = p \Rightarrow$ ISR + FSR cancel.

focus on 1 term:

$$\frac{dP(\text{extra}\,\gamma)}{dx} \approx \frac{\alpha}{\pi} \frac{1}{x} \log E^2/m_e^2 \qquad \text{soft } \gamma \text{ approx.}$$

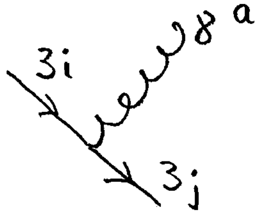
Q^2 or s (E^2)

momentum fraction of γ (x)

full answer:
coll. limit

$$\frac{dP}{dx} = \frac{\alpha}{\pi} \log \frac{E^2}{m_e^2} \; \frac{1+(1-x)^2}{2x}$$

gluon?



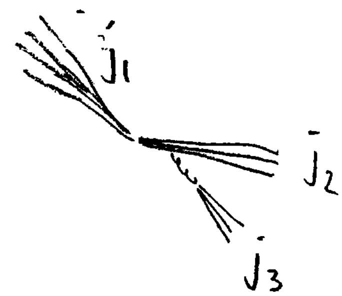
average over i

$$\frac{1}{3} \sum_{a,j,i} T^a_{ij} T^{a*}_{ji} =$$

$$= \frac{1}{3} \sum_a \mathrm{tr}(T^a T^a) = \frac{1}{3}\frac{1}{2}\delta^{aa} = \frac{4}{3} \equiv C_F$$

$$\Rightarrow \alpha_{em} \to \alpha_{str.} \cdot \frac{4}{3}$$

three jet events: $e^+e^- \to \underbrace{q\bar{q}}_{\text{indistinguishable}} g$



define: $x_i = \dfrac{E_i}{\frac{1}{2}E_{CM}} \Rightarrow \sum x_i = 2$

$x_1 > x_2 > x_3$

in soft-collinear limit,

expect: $x_1 \sim 1$

$\dfrac{dP}{dx_3} \sim \dfrac{1}{x_3}$

$x_2 = 1 - x_3$

$\boxed{\text{PLOT 38}}$ x_i - distributions

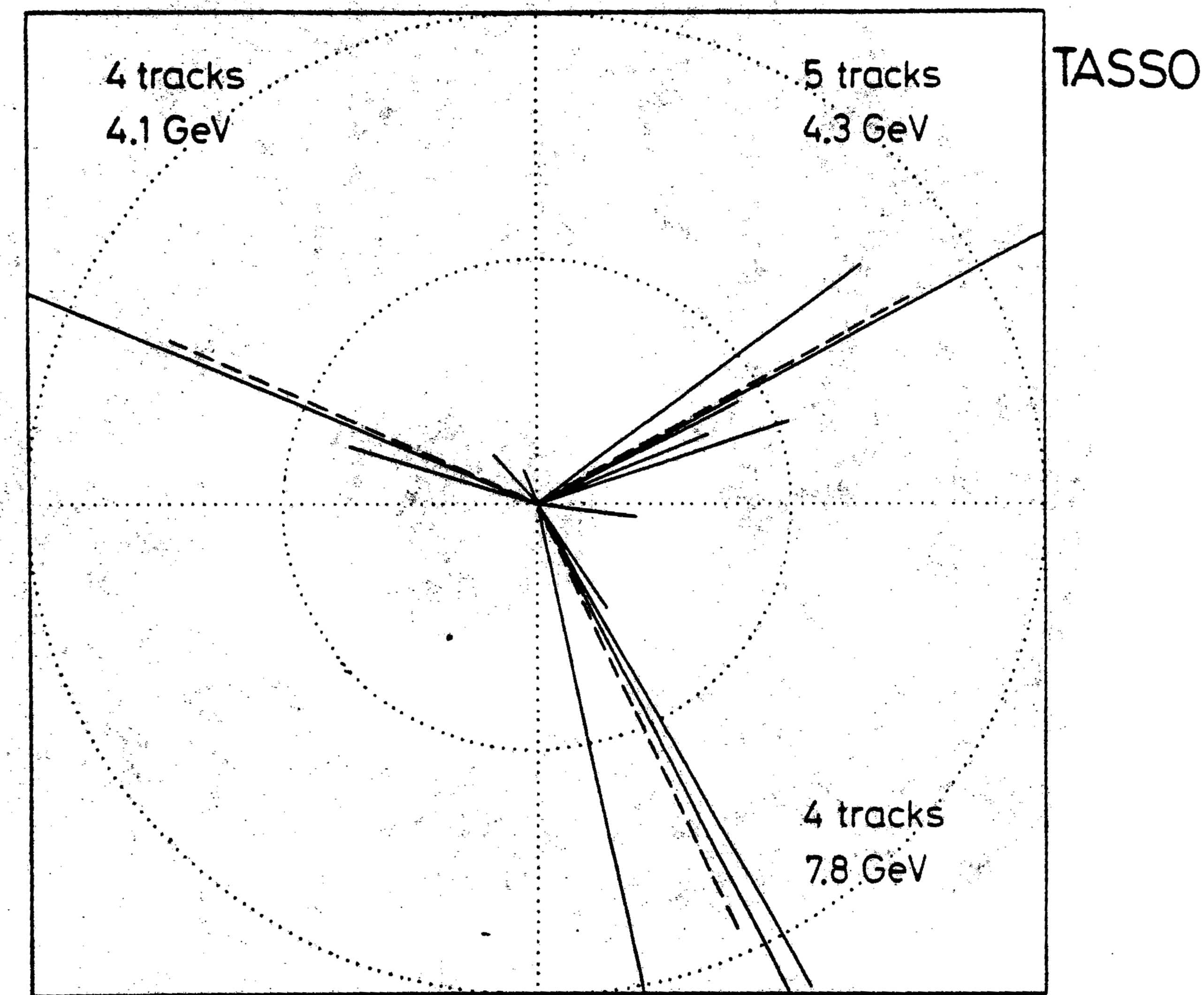
$\boxed{\text{PLOT 39}}$ " (comparing - vector " gluon, - scalar, - tensor)

($\mathcal{M}$ depended on spin)

when is a gluon its own jet?

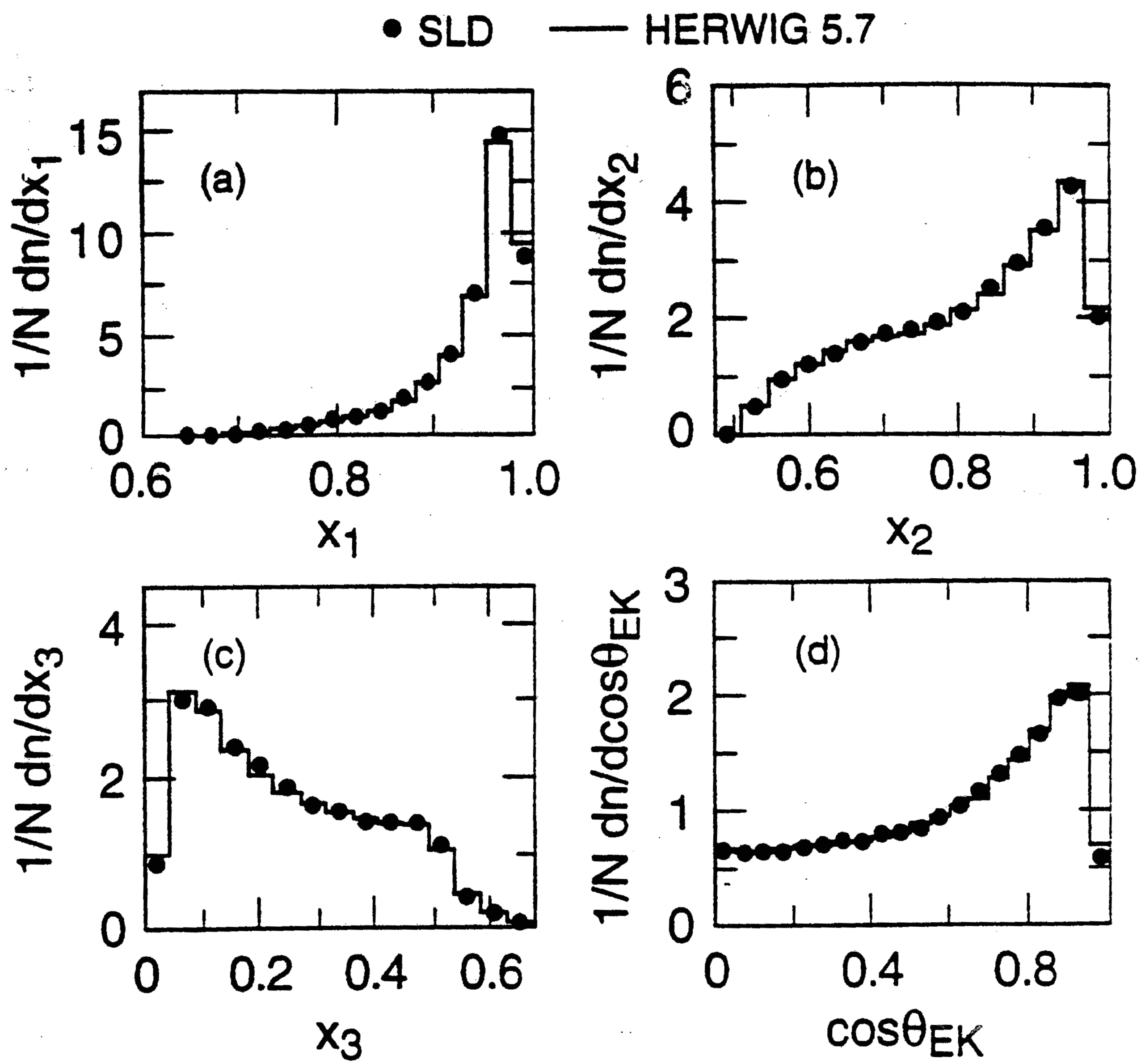
jet clustering: e.g. $m_{jet}^2 \equiv \left(\sum_i^{jet} p_i\right)^2 < \left(\sum^{all} p_i\right)^2 = E_{CM}^2 = s$

Three jet event



from G. Wolf, in
proceedings of the 1979 Lepton-Photon conference.

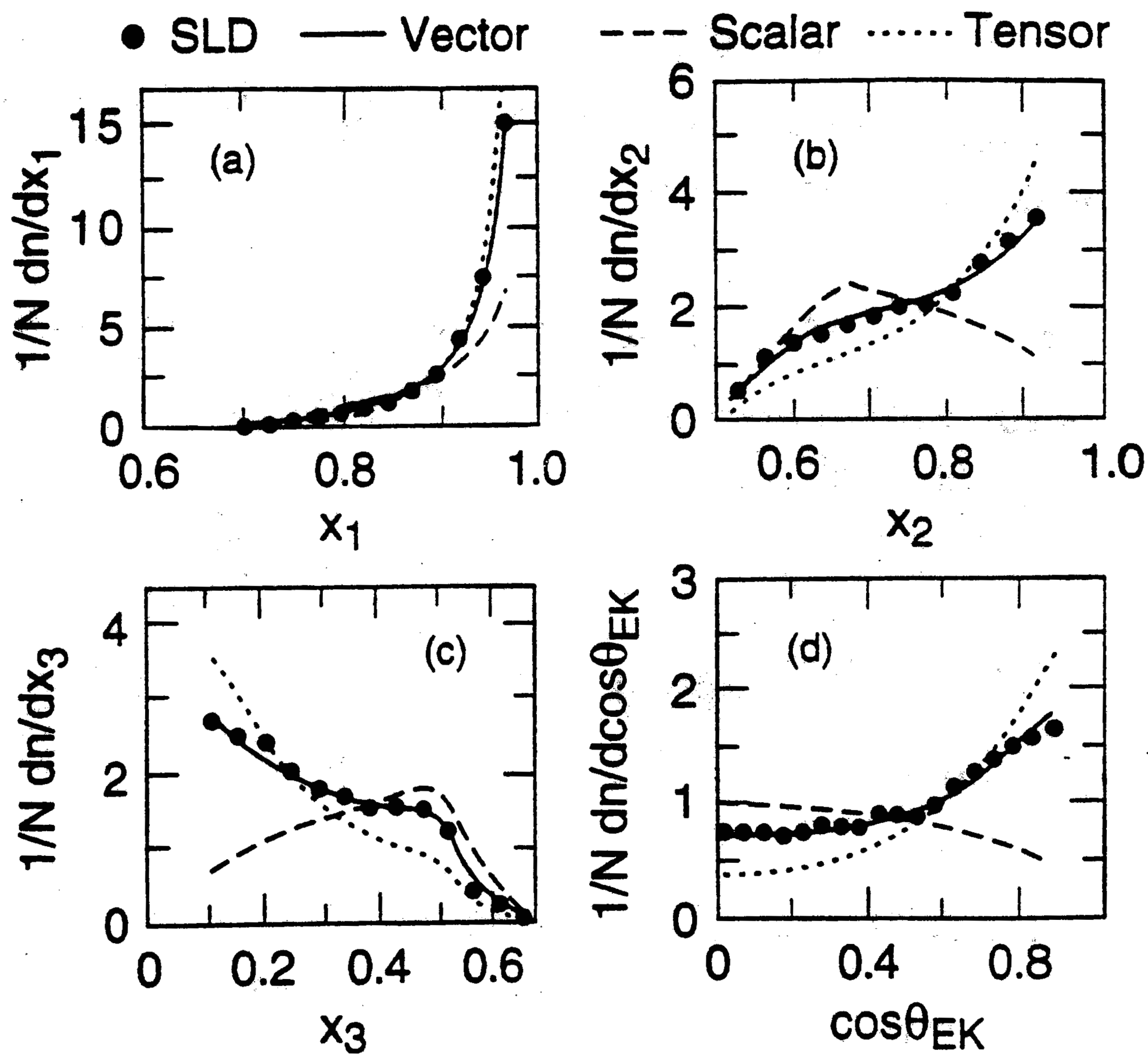
x_i distributions



K. Abe et al, Phys. Rev. D 55, 2533 (1997)
(SLD)

Is the gluon ...

- scalar
- vector
- tensor



K Abe et al. (SLD),
Phys. Rev. D 55, 2533 (1997)

def. $m^2_{cut} \equiv y_{cut}\, s \qquad y \in [0,1]$

cluster only momenta up to $m^2_{jet} \leq m^2_{cut}$

3j small y_{cut}

2j large y_{cut}

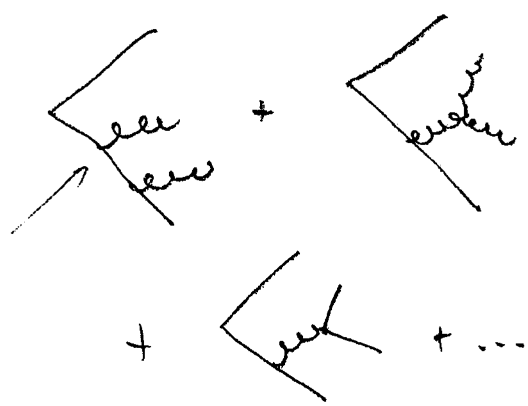
$\Rightarrow$ Prob for gluon to be jet depends on y_{cut}

$$\frac{dP}{dx}\left(q \to j_q + j_g\right) \approx \frac{\alpha_s}{\pi}\,\frac{4}{3}\,\frac{1-(1-x)^2}{2x}\,\log\left(1/y_{cut}\right)$$

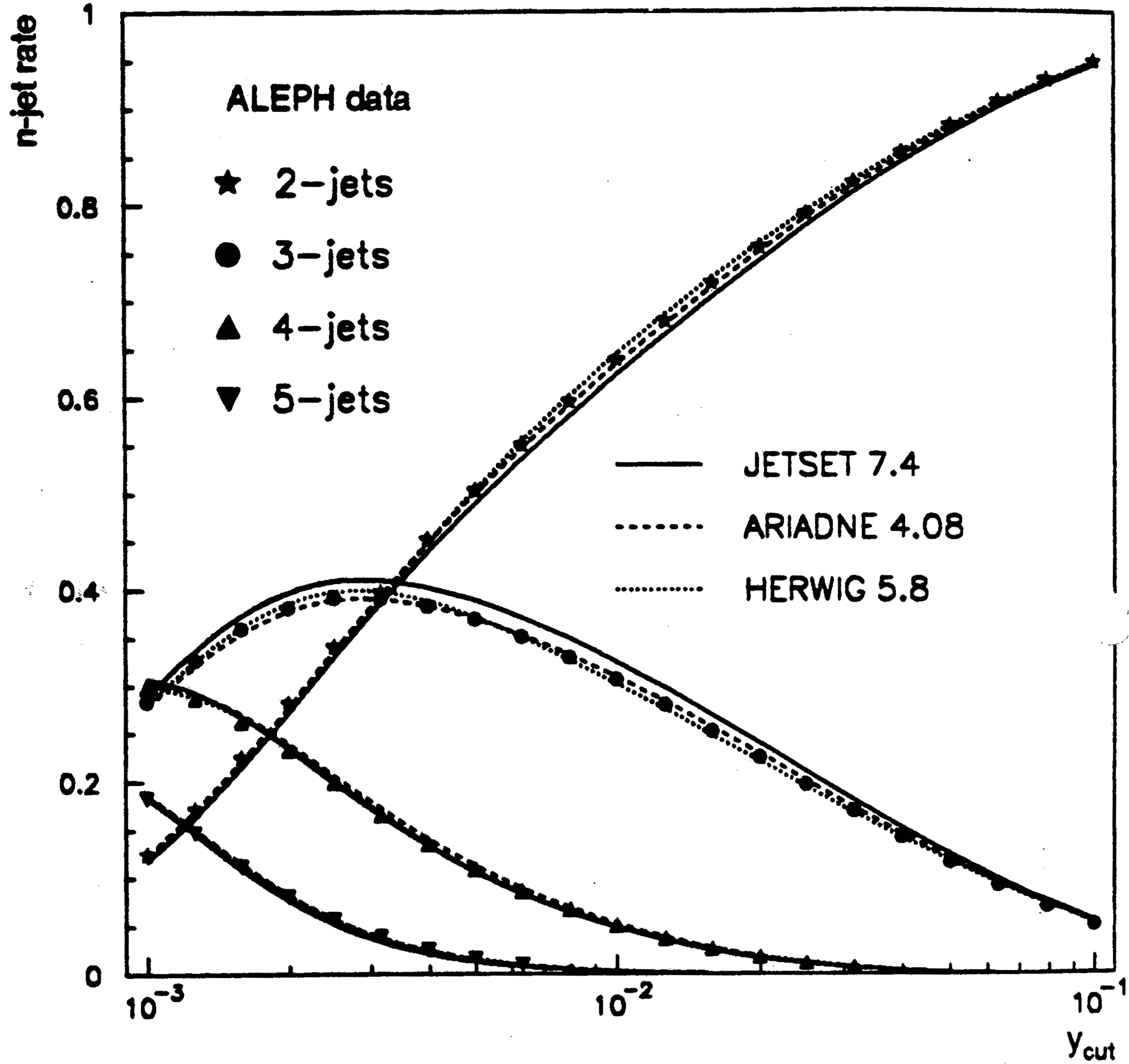
$\boxed{\text{PLOT 40}}$ 3j: linear rise with $-\log y_{cut}$

4j:

$\sim \left(\log y_{cut}\right)^2$



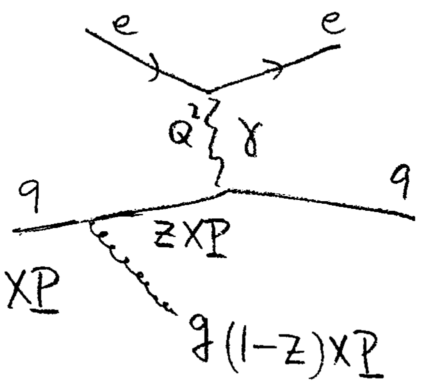
N-jet rate as function of y-cut



R. Barate et al (ALEPH)

Physics Reports (to appear)

Scaling violations → Altarelli-Parisi-Eqs



$$\frac{d\,\mathrm{Prob}^{ISR}}{dz} \cong \frac{\alpha_s}{2\pi}\,\frac{4}{3}\,\frac{1+z^2}{1-z}\,\log\frac{Q^2}{(\mathrm{GeV})^2}$$

⇒ effective x reduced to zx

$$P \sim \log Q^2$$

⇒ P to remain at x $(f_q(x)) \propto (1-\log Q^2)$ scaling violation

---

quantitatively:

$$\frac{d}{d\ln Q^2} f_q(x,Q^2) = \underset{\substack{\uparrow\\ \text{all initial}\\ \text{quark}}}{\int_0^1 dx'}\ \underset{\substack{\uparrow\\ \text{all}\\ \text{possible}\\ \text{gluons}}}{\int_0^1 dz}\ \underbrace{\frac{\alpha_s}{2\pi}\,\frac{4}{3}\,\frac{1+z^2}{1-z}}_{\frac{dP}{d\ln Q^2}}\, f_q(x',Q^2)\ \underbrace{\delta(x-zx')}_{\substack{\text{selects correct}\\ \text{remaining } q\\ \text{energy}}}$$

$$-\frac{\alpha_s}{2\pi}\,\frac{4}{3}\, a\, f_q(x,Q^2)$$

choose s.th. $\frac{d}{dQ^2}\left(\int_0^1 dx\, f_q(x,Q^2)\right) = 0$

$$= \int_x^1 \frac{dz}{z}\,\frac{\alpha_s}{2\pi}\,\frac{4}{3}\left(\frac{1+z^2}{1-z} - a\,\delta(z-1)\right) f_q\!\left(\frac{x}{z},Q^2\right)$$

Appendix: determining a

$$0=\int_0^1 dx\,\frac{d}{d\ln Q^2}f(x,Q^2)=\int_0^1 dx\int_x^1\frac{dz}{z}\left(\frac{1+z^2}{1-z_+}-a\,\delta(1-z)\right)f\left(\frac{x}{z}\right)\frac{\alpha_s}{2\pi}\frac{4}{3}$$

$$\Rightarrow a\int_0^1 dx\,f(x)=\int_0^1 dx\int_x^1\frac{dz}{z}\,\frac{1+z^2}{1-z_+}\,f\left(\frac{x}{z}\right)$$

$$\frac{x}{z}=x' \Rightarrow dx'=\frac{-x}{z^2}dz=-x'\frac{dz}{z}$$

$$=\int_0^1 dx\int_x^1\frac{dx'}{x'}\,\frac{1+(x/x')^2}{1-x/x'}\,f(x')$$

$$=\int_0^1\frac{dx'}{x'}f(x')\int_0^1\frac{dx}{x'}\,\frac{x'^2+x^2}{x'-x}\,\theta(x'-x)$$

$$=\int_0^1 dx'\,f(x')\,\underbrace{\frac{1}{x'^2}\int_0^{x'}dx\,\frac{x^2+x'^2}{x-x'_+}}$$

$$=\int_0^1 dy\,\frac{1+y^2}{1-y_+}=2\underbrace{\int_0^1 dy\,\frac{1}{1-y_+}}_{=0}-\int_0^1(y+1)\,dy=-\frac{3}{2}$$

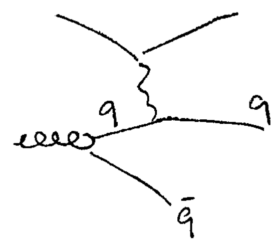
$$\Rightarrow a=-3/2.$$

$$\frac{d}{d\ln Q^2} f_q(x,Q^2) \equiv \int_x^1 \frac{dz}{z} \frac{\alpha}{2\pi} \underbrace{P_{q\leftarrow q}(z)}_{\text{splitting function}} f_q\left(\frac{x}{z}, Q^2\right)$$

$$P_{q\leftarrow q}(z) = \frac{4}{3}\left(\frac{1+z^2}{1-z_+} + \frac{3}{2}\delta(1-z)\right)$$

Principal value, $z=1$ is $\omega_{gluon}=0$

another term,
gluons can create quarks:



$$\frac{d}{d\ln Q^2} f_q(x,Q^2) = \int_x^1 \frac{dz}{z}\frac{\alpha}{2\pi}\left[P_{q\leftarrow q}(z) f_q\left(\frac{x}{z},Q^2\right) + P_{q\leftarrow g}(z) f_g\left(\frac{x}{z},Q^2\right)\right]$$

$$P_{q\leftarrow g}(z) = \frac{1}{2}\left(z^2+(1-z)^2\right)$$

$$\frac{d}{d\ln Q^2} f_{\bar q}(x,Q^2) = \text{''} \left[P_{q\leftarrow q} f_{\bar q} + P_{q\leftarrow g} f_g\right]$$

$$\frac{d}{d\ln Q^2} f_g(x,Q^2) = \text{''} \left[P_{g\leftarrow q}\sum_i (f_{q_i}+f_{\bar q_i}) + P_{g\leftarrow g} f_g\right]$$

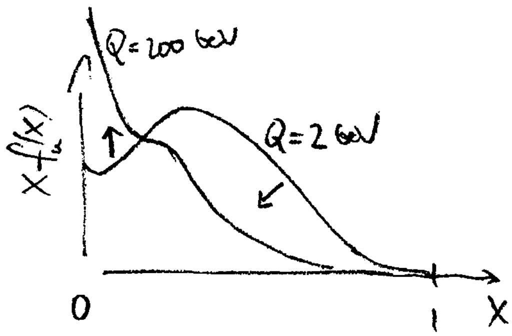
$$P_{g\leftarrow q} = \frac{4}{3}\frac{1+(1-z)^2}{z} \qquad P_{g\leftarrow g} = 6\left[\frac{1-z}{z} + \frac{z}{1-z} + z(1-z) + \left(\frac{11}{12} - \frac{n_f}{18}\right)\delta(1-z)\right]$$

coupled set of integral equations for $f_i(x, Q^2)$.

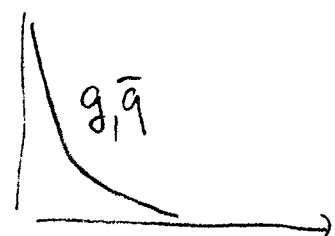
Measure pdfs at $Q^2 = Q_0^2$, integrate AP eqns to all Q^2.

qualitatively:

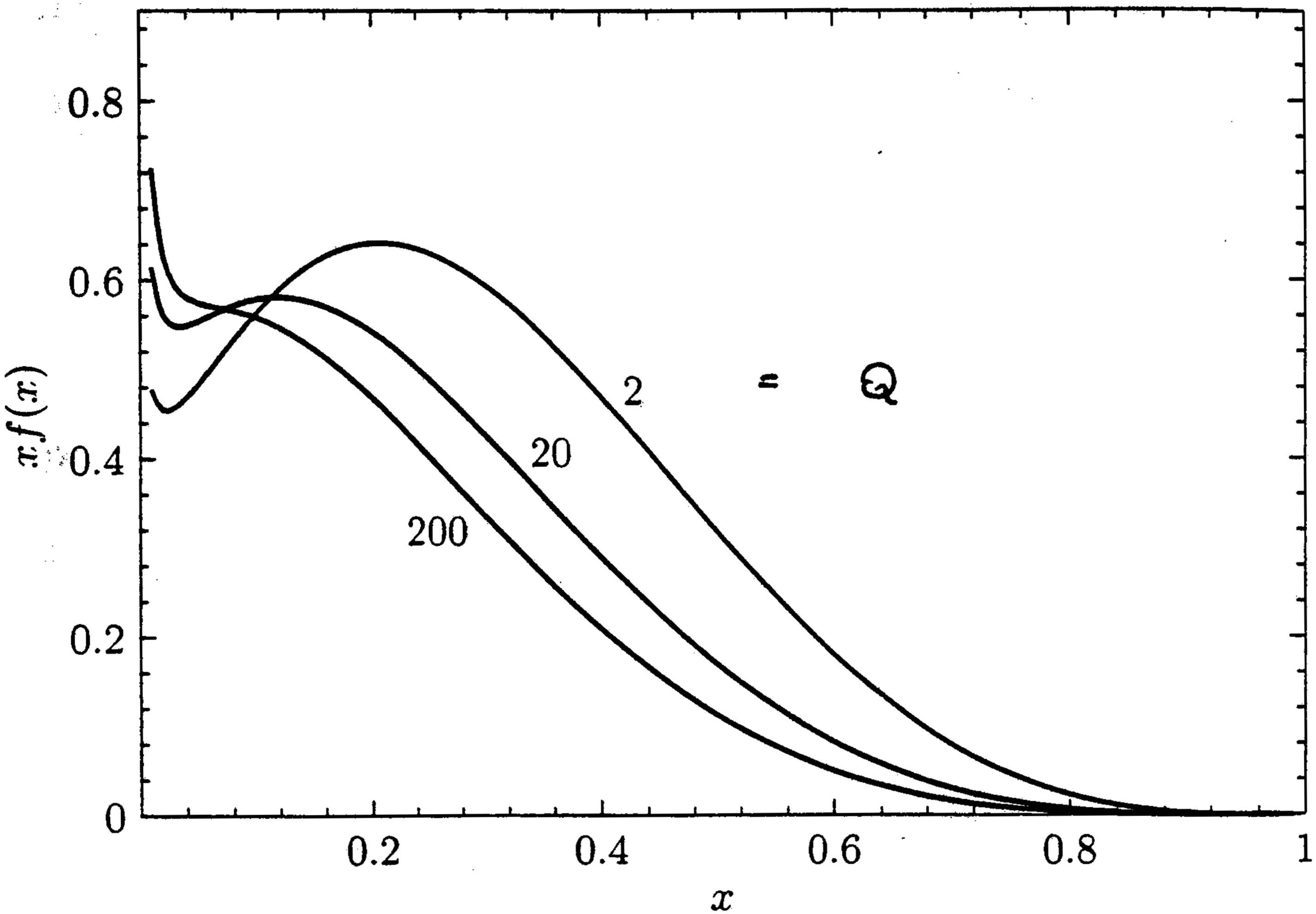
$Q^2 \uparrow \Rightarrow$ more radiation from hard quarks



$\boxed{\text{PLOT 41}}$



scaling violation
Q-dependence of pdfs



J. Botts et al (CTEQ)

Phys Lett. B304, 159 (1993)