Peskin 4 : gluon

- $e^+e^- \to$ hadrons : thrust, jets, angular distribution
  $\Longrightarrow$ quarks

- $R_h = \dfrac{\sigma(\text{hadrons})}{\sigma(\mu^+\mu^-)}(E_{cm}) \Rightarrow$ quark masses, 3 colors
  bound state resonances
  $\Rightarrow$ strong interaction potential (color field)

- DIS : proton structure
  quark pdf's, 50% of momentum missing (gluons)

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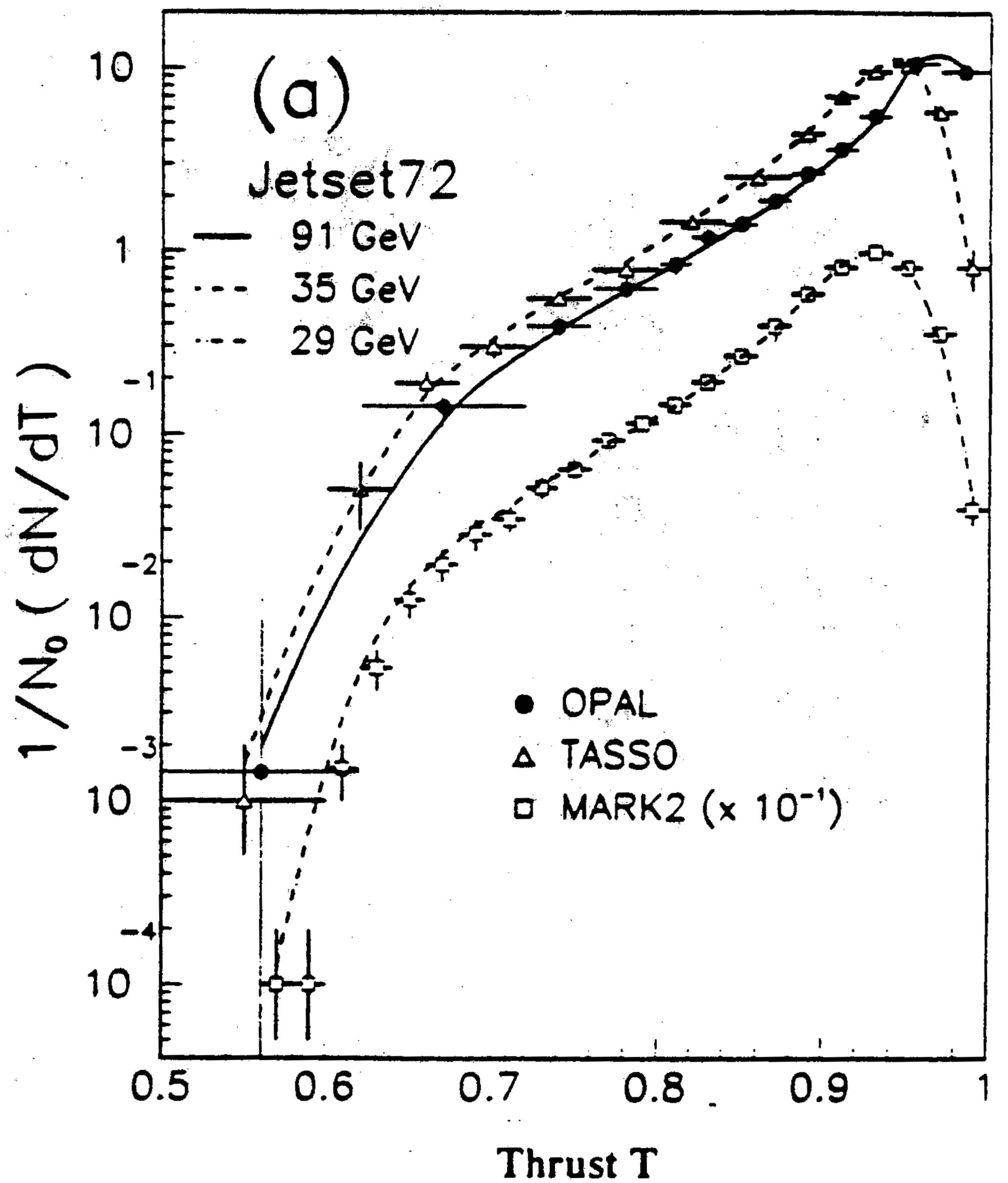
<u>gluon jets</u> : Thrust $T = \max\limits_{\hat{n}} \dfrac{\sum_i |\vec{p}_i \cdot \hat{n}|}{\sum_i |\vec{p}_i|}$ measures "dijetness"

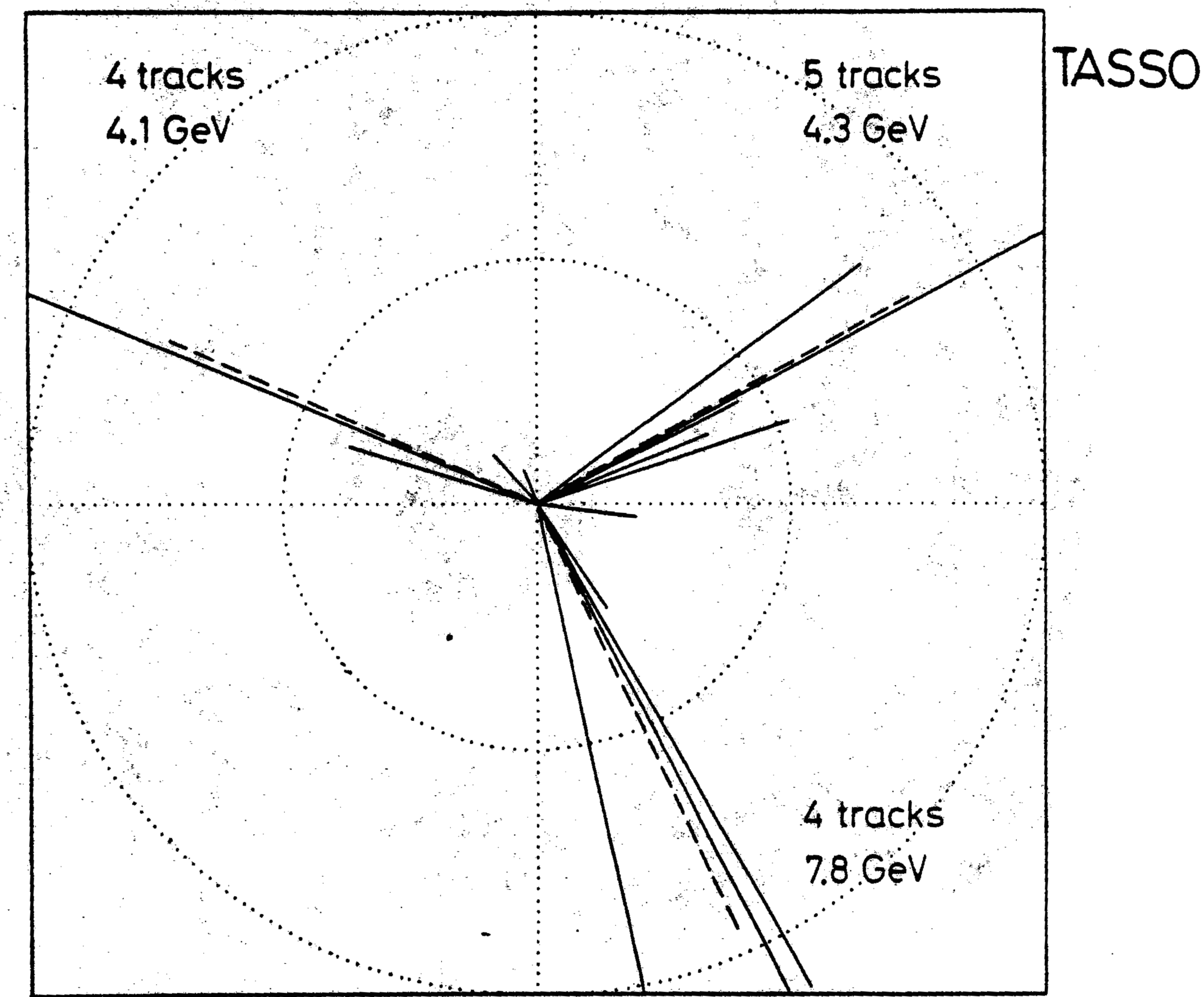
$\boxed{\text{PLOT 33}}$ peaks near $T \approx 1$, "shoulder" at $T \sim 2/3$

Thrust

M Z Akrawy et al (OPAL)
Z Phys. C 47, 505 (1990)

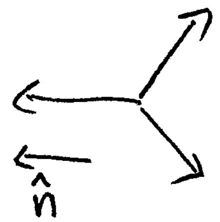


Three jet event

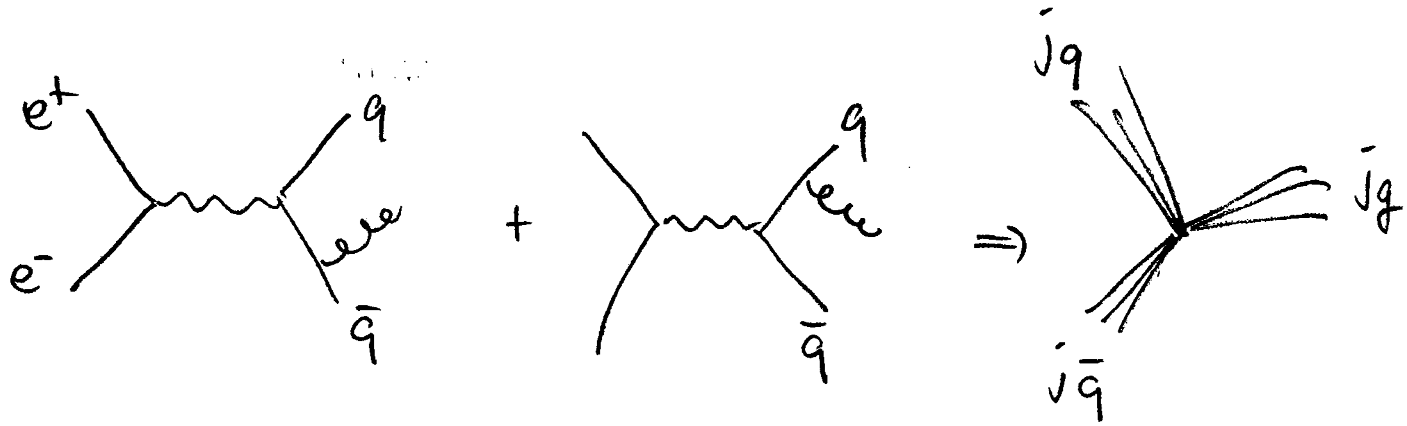


from G. Wolf, in
proceeding of the 1979 Lepton-Photon conference.

3 jet event, symmetric case minimises T

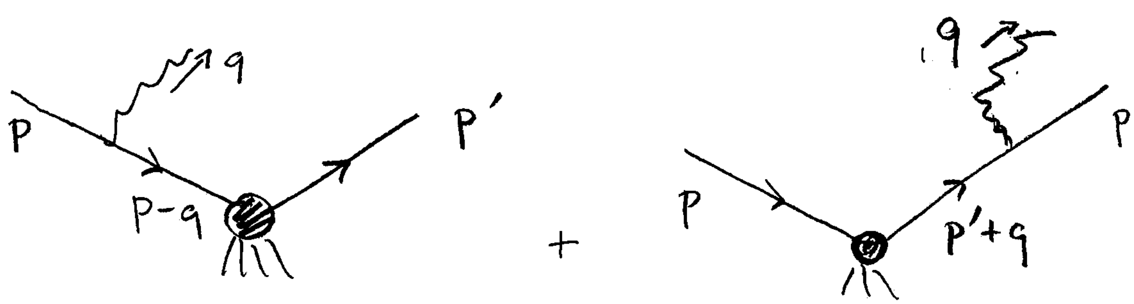


$$T = \frac{p + 2p\cos 60}{3p} = 2/3$$

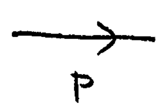


$$\sigma_{3j} \sim \frac{1}{10}\,\sigma_{2j}$$

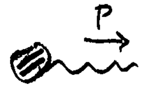
<u>Soft photon emission from electron</u> (apply to gluon later)



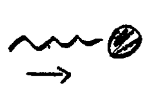
need Feynman rules :



$\dfrac{i(\not{p}+m)}{p^2-m^2}$ fermion propagator



$\epsilon^*_\mu(p)$ out γ



$\epsilon_\mu(p)$ in γ

recall massive spin 1 propagator

$$(\partial_t^2 - \vec{\nabla}^2 + m^2) A^\mu = J^\mu$$

$$\Rightarrow \quad \tilde{A}^\mu = \frac{-g^{\mu\nu}}{p^2 - m^2} \tilde{J}_\nu$$

$\times i =$ 〰〰

fermion: $(i\partial_\mu \gamma^\mu - m)\psi = \eta$

$$\tilde{\psi} = \text{"}\frac{1}{\not{p} - m}\text{"} \tilde{\eta} = \frac{\not{p}+m}{(\not{p}-m)(\not{p}+m)} \tilde{\eta} = \frac{\not{p}+m}{p^2 - m^2} \tilde{\eta}$$

$\times i$ propagator

polarization vectors: $\quad p^\mu = p(1, 0\,0\,1)$

$$\epsilon^\mu_\pm = \frac{1}{\sqrt{2}}(0 \;\, 1 \;\, \pm i \;\, 0) \quad \text{or} \quad \begin{cases} \epsilon^\mu_1 = (0\;1\;0\;0) \\ \epsilon^\mu_2 = (0\;0\;1\;0) \end{cases}$$

$$iM = \bar{u}_{p'}\, iM_0(p', p-q)\, i\,\frac{\not{p} - \not{q} + m}{(p-q)^2 - m^2}\, (-ie\gamma^\mu)\, u_p\, \epsilon^*_\mu(q)$$

$$+ \bar{u}_{p'}\, (-ie\gamma^\mu)\, \frac{i(\not{p}' + \not{q} + m)}{(p'+q)^2 - m^2}\, iM_0(p'+q, p)\, u_p\, \epsilon^*_\mu(q)$$

denominators: $(p-q)^2 - m^2 = \cancel{p^2} - \cancel{m^2} - 2p\cdot q + \cancel{q^2}$

$(p'+q)^2 - m^2 = 2p'\cdot q$

numerators: $q \ll p, p' \Rightarrow M_0(p', p)$, drop $\not{q}$

$$\mathcal{M} = e\,\epsilon^*_\mu(q)\;\bar{u}_{p'}\left[M_0 \frac{\not{p}+m}{-2p\cdot q}\gamma^\mu + \gamma^\mu \frac{\not{p}'+m}{2p'\cdot q} M_0 \right] u_p$$

$$(\not{p}+m)\gamma^\mu u_p = \Big(\underbrace{\{\not{p},\gamma^\mu\}} + \gamma^\mu \overset{=0}{\cancel{(m-\not{p})}}\Big) u_p$$

$$p_\nu \underbrace{\{\gamma^\nu, \gamma^\mu\}}_{2g^{\nu\mu}} = 2p^\mu$$

$$\mathcal{M} = e\,\epsilon^*_\mu(q)\;\bar{u}_{p'}\left(\frac{p^\mu}{-p\cdot q} M_0 + \frac{p'^\mu}{p'\cdot q} M_0 \right) u_p$$

$$= \underbrace{\bar{u}_{p'} M_0 u_p}\; e\left(\frac{\epsilon^*\cdot p'}{p'\cdot q} - \frac{\epsilon^*\cdot p}{p\cdot q} \right)$$

$=$ 

⇒ amplitude for additional soft γ:

$$e\left(\frac{p'\cdot\epsilon^*}{p'\cdot q} - \frac{p\cdot\epsilon}{p\cdot q}\right)$$

$$d\sigma(i\to f+\gamma) = d\sigma(i\to f)\cdot\int^{\text{soft}}\frac{d^3q}{(2\pi)^3 2E_q}\, e^2\sum_i\left|\frac{p'\cdot\epsilon_i^*}{p'\cdot q} - \frac{p\cdot\epsilon_i^*}{p\cdot q}\right|^2$$

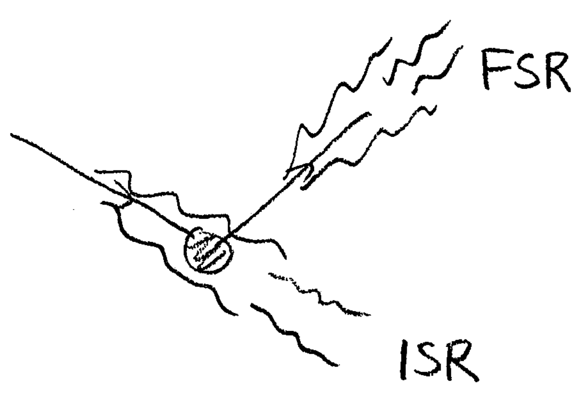
when is this big?

- $q\to 0$? $\quad \int\frac{q^2 dq}{q}\,\frac{1}{q^2}\sim\int\frac{dq}{q}\sim\log 0$

  "soft"

- $q\,/\!/\,p$ or $q\,/\!/\,p'$ $\Rightarrow$ $p\cdot q = \alpha p^2 \simeq 0$

  "collinear"



E&M field of incoming charge ISR

E&M " " outgoing " FSR

focus on ISR (similar for FSR) $q \propto p$

$$p^\mu = E(1\ 0\ 0\ 1)$$

$$q^\mu = \omega(1\ \sin\theta\ 0\ \cos\theta) \approx \omega(1\ \theta\ 0\ 1-\frac{\theta^2}{2})$$

$$\Rightarrow p\cdot q = E\omega\frac{\theta^2}{2}$$

$$\epsilon_1^\mu = (0\ 0\ 1\ 0) \qquad \epsilon_2^\mu = (0\ 1\ 0\ -\theta)$$

$$p\cdot\epsilon_1^* = 0$$

$$p\cdot\epsilon_2^* = \theta E$$

$$\left|\frac{p\cdot\epsilon_2^*}{p\cdot q}\right|^2 = \left|\frac{\theta \cancel{E}}{\cancel{E}\omega\frac{\theta^2}{2}}\right|^2 = \frac{4}{(\omega\theta)^2}$$

Prob of ISR γ: $\displaystyle\int\frac{\omega^2 d\omega}{(2\pi)^3 2\omega}\, 2\pi\int\theta\, d\theta\, \frac{4e^2}{\omega^2\theta^2}$

$$= \frac{\alpha}{\pi}\int\frac{d\omega}{\omega}\int\frac{d\theta^2}{\theta^2}$$

set $x = \frac{\omega}{E}$ fraction of energy carried away by γ

$$\approx \frac{\alpha}{\pi}\int\frac{dx}{x}(-\log\theta^2_{min})$$

θ^2_{min} ? how small $\theta^2 = 2\frac{p\cdot q}{E\omega}$ get ?

$$q = (\omega \;\; 0 \;\; 0 \;\; \omega)$$

$$p = (E \;\; 0 \;\; 0 \;\; p)$$

$$\theta^2 = 2\,\frac{E-P}{E} = 2\,\frac{\sqrt{p^2+m^2} - P}{E} = 2\,\frac{P}{E}\left(\sqrt{1+m^2/p^2} - 1\right)$$

$$\cong \frac{m^2}{E^2}$$

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$$\frac{d(\text{Prob } e^- \to e\gamma)}{dx} \approx \frac{\alpha}{\pi}\log\frac{E^2}{m_e^2}\,\frac{1}{x}$$ (soft γ approximation)

Including hard γ's: $$\frac{dP(e^- \to e^-\gamma)}{dx} = \frac{\alpha}{\pi}\log\frac{E^2}{m_e^2}\,\frac{1}{x}\,\frac{1+(1-x)^2}{2}$$