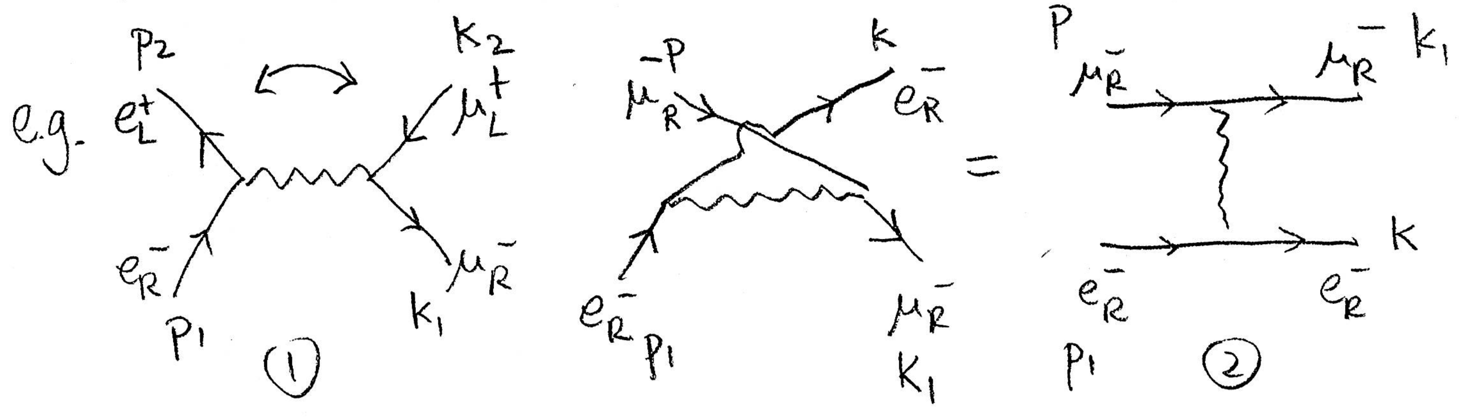
Crossing symmetry: relating matrix elements for processes with similar Feynman diagrams.



note: - charge flip $e^+ \to e^+$

- momentum flip $p_2 \to -k$

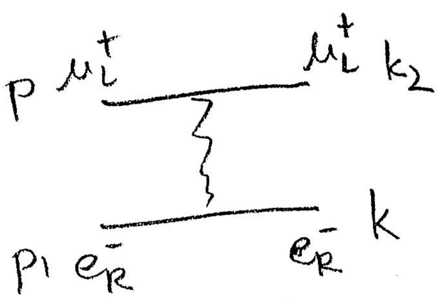
  $k_2 \to -p$

- helicity flip $L \to R$

$$\mathcal{M}_2 = \mathcal{M}_1(p_2 \to -k, k_2 \to -p)$$

$$= 4e^2 \frac{-k\cdot k_1}{(p_1-k)^2} = -2e^2 \frac{s}{t}$$

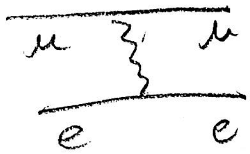
③ flip $e_L^+ \leftrightarrow \mu_R^-$ $\Rightarrow$ $e_R^- \mu_L^+ \to e_R^- \mu_L^+$



$$\mathcal{M}_3 = \mathcal{M}_1(p_2 \to -k, k_1 \to -p)$$

$$= 4e^2 \frac{+p\cdot k}{(p_1-k)^2} = -2e^2 \frac{u}{t}$$

$e^-\mu^- \to e^-\mu^-$



$$\mathcal{M}(e_L^-\mu_L^- \to e_L^-\mu_L^-) = \mathcal{M}(e_R^-\mu_R^- \to e_R^-\mu_R^-) = -2e^2\,\frac{s}{t}$$

$$\mathcal{M}(e_L^-\mu_R^- \to e_L^-\mu_R^-) = \mathcal{M}(e_R^-\mu_L^- \to e_R^-\mu_L^-) = 2e^2\,\frac{u}{t}$$

$$\frac{d\sigma}{d\cos\theta}(e_L^-\mu_L^- \to e_L^-\mu_L^-) = \frac{1}{32\pi}\,\frac{1}{E_{CM}^2}\,|\mathcal{M}|^2$$

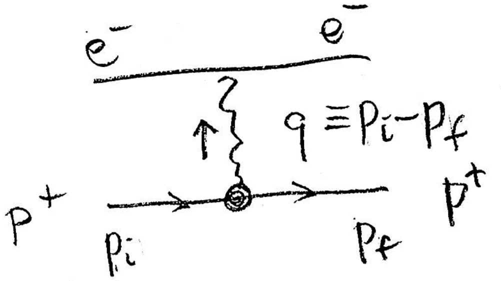
$(s/t)^2$

$$= \frac{\pi}{2}\,\frac{\alpha^2}{E_{CM}^2}\left(\frac{4}{1-\cos\theta}\right)^2 = \frac{8\pi\alpha^2}{E_{CM}^2}\,\frac{1}{(1-\cos\theta)^2}$$

$(\sin\theta/2)^4$

Coulomb scattering.

Similarly: $e^-p^+ \to e^-p^+$



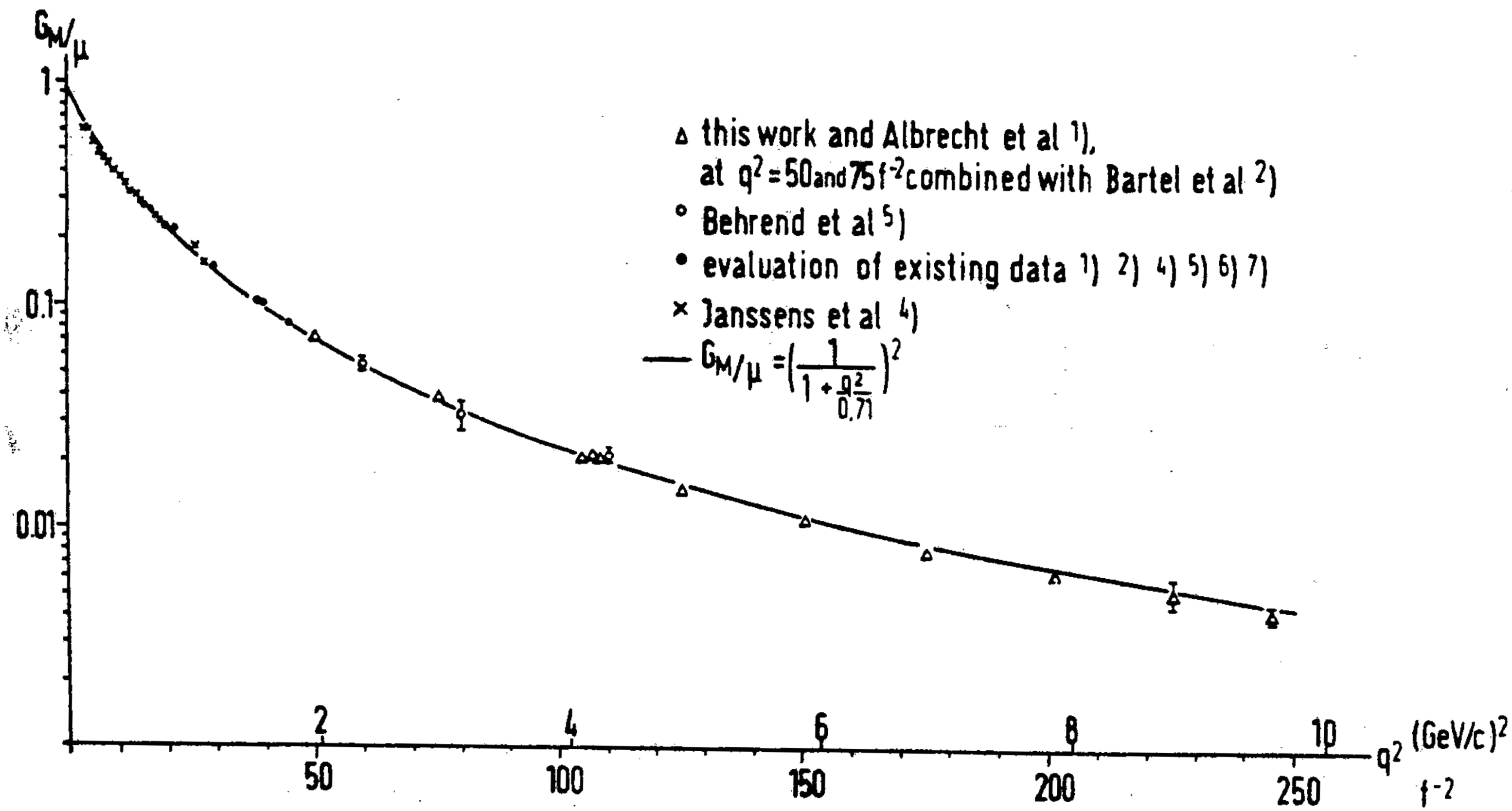
as long as proton point-like, i.e. $q^2 \ll m_p^2$ expect same answer. In general, parameterize

$$\frac{d\sigma}{d\cos\theta}(e_L^-p_L^+ \to e_L^-p_L^+) = \frac{8\pi\alpha^2}{E_{CM}^2}\,\frac{1}{(1-\cos\theta)^2}\,|G(q^2)|$$

form factor

$\boxed{\text{PLOT 25}}$

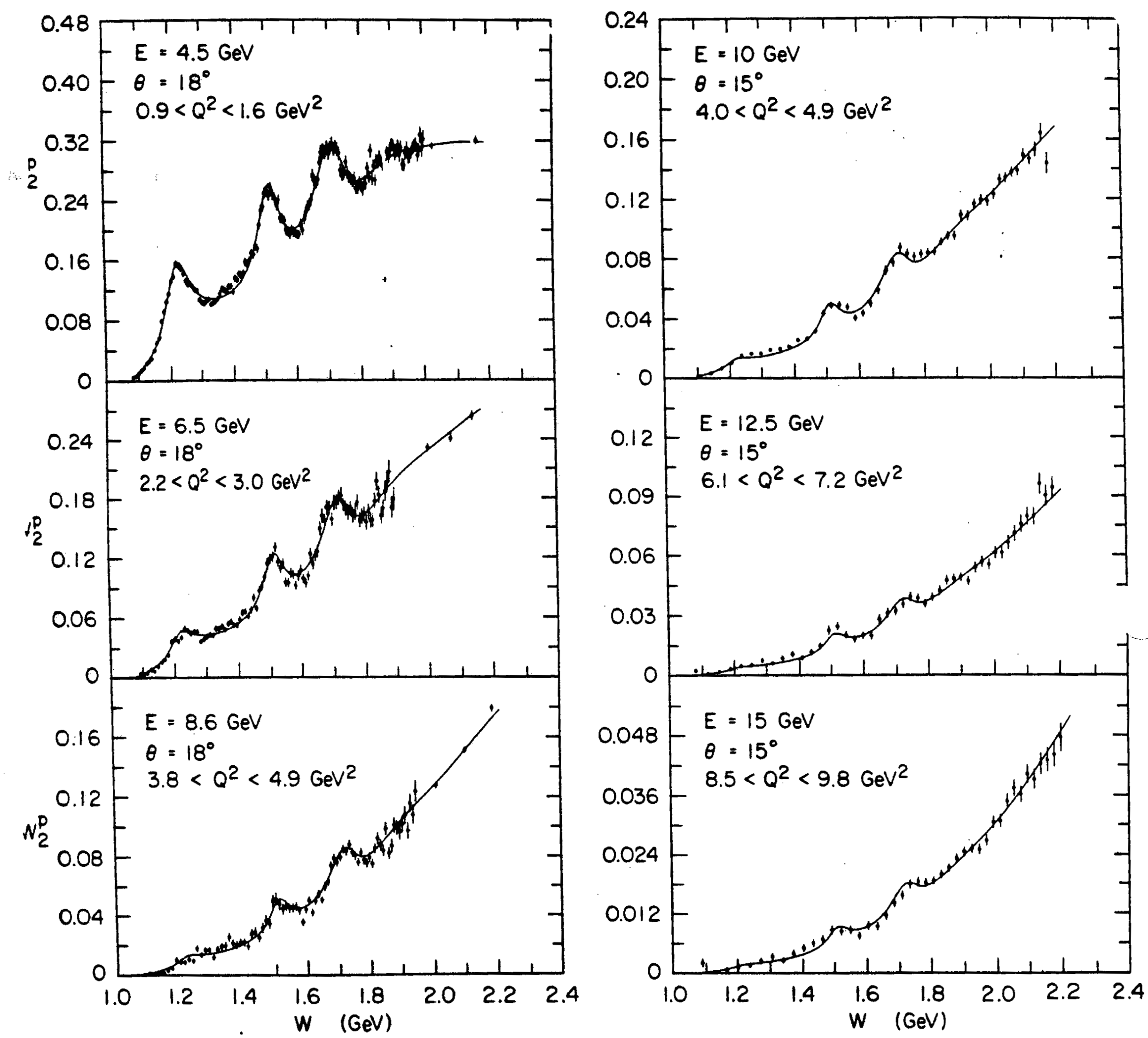
e-p+ —> e-p+ form factor G



W. Albrecht et al.

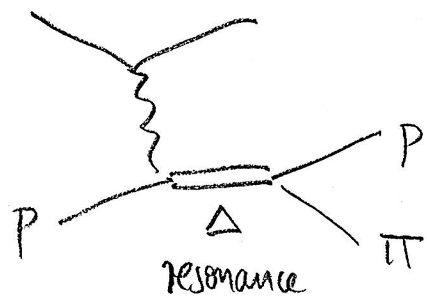
Phys. Rev. Lett. 18, 1014 (1967)

e-p+ —> e-p+ hadronic resonances



A. Bodek et al Phys. Rev. D 20, 1471 (1979)
(SLAC-MIT)

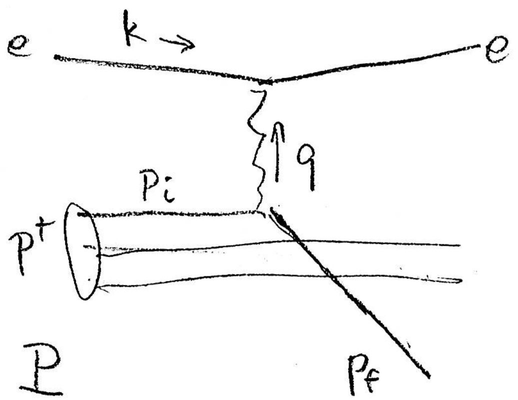
$q^2 \gtrsim m_p^2$ $\boxed{\text{PLOT 26}}$



inelastic

$q^2 \gg m_p^2$ photon "sees" free quarks, p^+ binding energy irrelevant

"<u>deep inelastic scattering</u>"



define $p_i \equiv \xi \underline{P}$ $\quad 0 \leq \xi \leq 1$

momentum fraction carried by quark

(note: $p_{i_\perp} \lesssim m_p$ negligible)

simple QED times probability to find quark with momentum fraction ξ in proton.

kinematics $\xrightarrow[k]{e^-}$ $\xleftarrow[P]{p^+}$ $s = E_{CM}^2 = (k+P)^2$

$$\hat{s} = (k+p)^2 = \hat{E}_{CM}^2$$

$$= (k+\xi P)^2 \cong 2\xi\, k\cdot P = \xi s$$

$$t = (k-k')^2 = -\frac{\hat{s}}{2}(1-\cos\hat{\theta})$$

partonic CM

t is Lorentz-invariant, measurable from e^- only,

$\cos\theta_{CM}$ is not " , $\hat{s}$ not L.I.

change variables $dt = \frac{\hat{s}}{2}\, d\cos\hat{\theta}$

$$\frac{d\hat{\sigma}}{dt}(e_L^- q_L^- \to e_L^- q_L^-) = \frac{2}{\hat{s}}\frac{d\hat{\sigma}}{d\cos\hat{\theta}} = \frac{\pi\alpha^2}{\hat{s}^2}\left(\frac{2\hat{s}}{t}\right)^2 Q_q^2 = \frac{4\pi\alpha^2}{t^2} Q_q^2$$

$$\frac{d\hat{\sigma}}{dt}(e_L^- q_R^- \to e_L^- q_R^-) = \frac{4\pi\alpha^2}{\hat{s}^2} Q_q^2 \left(\frac{\hat{u}}{t}\right)^2 = \frac{4\pi\alpha^2}{t^2} Q_q^2 \left(1+\frac{t}{\hat{s}}\right)^2$$ using $\hat{u}+\hat{s}+t=$

Sum final, average initial

$$\frac{d\hat{\sigma}}{dQ^2} = \frac{2\pi\alpha^2}{Q^4} Q_q^2 \left[1+\left(1-\frac{Q^2}{\xi s}\right)^2\right] \qquad Q^2 \equiv -t = |(k-k')^2|$$

partonic cross section.

for $\frac{d\sigma}{dQ^2}(ep \to eX)$ need

$f_q(\xi)\,d\xi$ = prob for quark q of mom. fraction $\in [\xi, \xi+d\xi]$

"parton distribution function" pdf

obvious sum rules:

$$\int_0^1 d\xi\; f_u(\xi) - f_{\bar{u}}(\xi) = 2$$ net # of u in proton

$$\int_0^1 d\xi\; f_d(\xi) - f_{\bar{d}}(\xi) = 1$$

" s " $\bar{s}$ $= 0$

$$\sum_i \int_0^1 \underbrace{d\xi\;\; \xi \underline{P}\, f_i(\xi)}_{\substack{\text{momentum carried}\\ \text{by parton } i \text{ in interval}\\ [\xi,\, \xi+d\xi]}} = \underline{P}$$

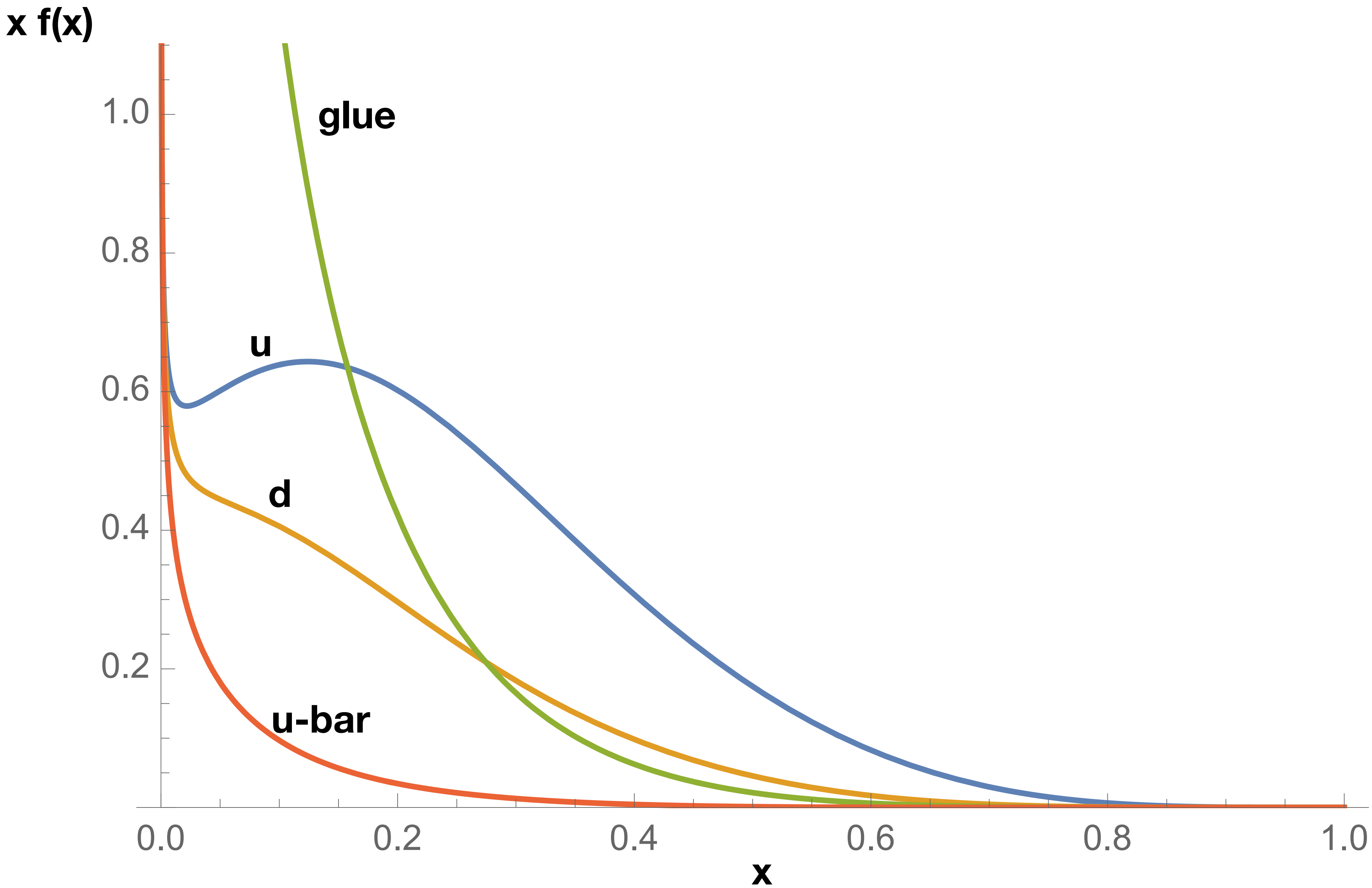
$\boxed{\text{PLOT Mathematica pdfs}}$

$$\boxed{\begin{aligned}\frac{d\sigma}{dQ^2}(ep \to eX) &= \sum_{f \in q, \bar{q}} \int_0^1 d\xi\; f_f(\xi)\, \frac{d\hat{\sigma}}{dQ^2} \\ &= \sum_f \int_0^1 d\xi\; f_f(\xi)\, Q_f^2\, \frac{2\pi\alpha^2}{Q^4}\left[1 + \left(1 - \frac{Q^2}{\xi s}\right)^2\right]\end{aligned}}$$

deep inelastic scattering

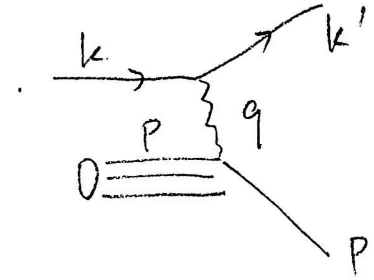
10/2/1[illegible]

pdf's - momentum fraction carried



in experiment, know k, $\underline{P}$, measure k'.

$\Rightarrow$ Lorentz invariants $Q^2 = -(k-k')^2$



$$S = (\underline{P}+k)^2 = 2\underline{P}\cdot k$$

$$2\underline{P}\cdot q = 2\underline{P}\cdot(k-k')$$

dimensionless ratios: $x \equiv \dfrac{Q^2}{2\underline{P}\cdot q}$

$$y = \frac{2\underline{P}\cdot q}{2\underline{P}\cdot k} = \frac{q^0}{k^0}$$

in proton rest frame (Lab frame)

$\Rightarrow$ y = fraction of e^- energy transferred to p^+.

x? $\quad 0 = p'^2 = (p+q)^2 = 2p\cdot q + q^2 = \xi\, 2\underline{P}\cdot q - Q^2$

$$\Rightarrow \xi = \frac{Q^2}{2\underline{P}\cdot q} = x$$

$\Rightarrow$ fixed x $\Leftrightarrow$ fixed momentum fraction of initial parton.

domain: $0 \leq x, y \leq 1$

$$\frac{d\sigma}{dQ^2dx}(ep \to eX) = \underbrace{\sum_f f_f(x)\, Q_f^2}_{\text{proton structure}}\ \underbrace{\frac{2\pi\alpha^2}{Q^4}\left(1+\underbrace{\left(1-\frac{Q^2}{xS}\right)^2}_{=(1-y)^2}\right)}_{\text{QED } \equiv \Sigma(x,Q^2)}$$

$$\frac{d\sigma}{dQ^2dx} \Big/ \Sigma(x,Q^2) = \sum_f f_f(x)\, Q_f^2$$

"Bjorken scaling"
Q^2-independent
measures pdfs

$\boxed{\text{FIG 27}}$: experiment

$\boxed{\text{PLOT 28}}$: scaling plot

ep measures : $\frac{4}{9}\left(u(x)+\bar{u}(x)\right) + \frac{1}{9}\left(d+\bar{d}\right)$

ed : $\frac{5}{9}\left(u+\bar{u}+d+\bar{d}\right)$ (neutron pdfs are "opposite" of proton pdfs $n \leftrightarrow p \Leftrightarrow u \leftrightarrow d$ "isospin"

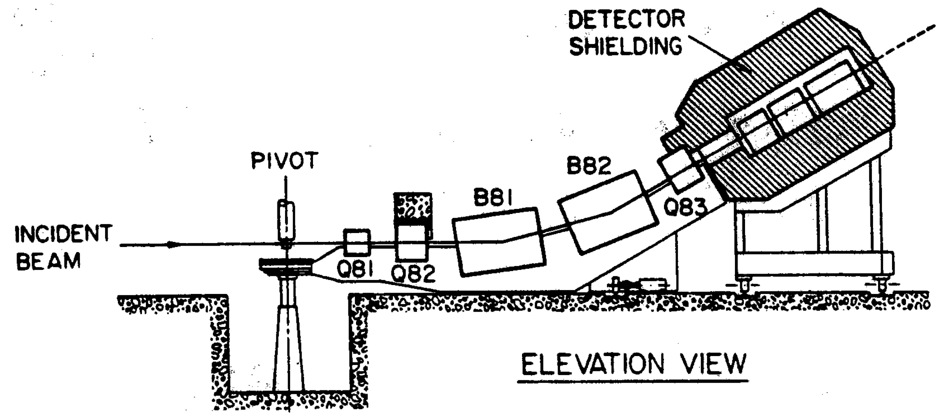
↑
deuterium
p+n

$\Rightarrow$ get $u+\bar{u}$ and $d+\bar{d}$ independently.

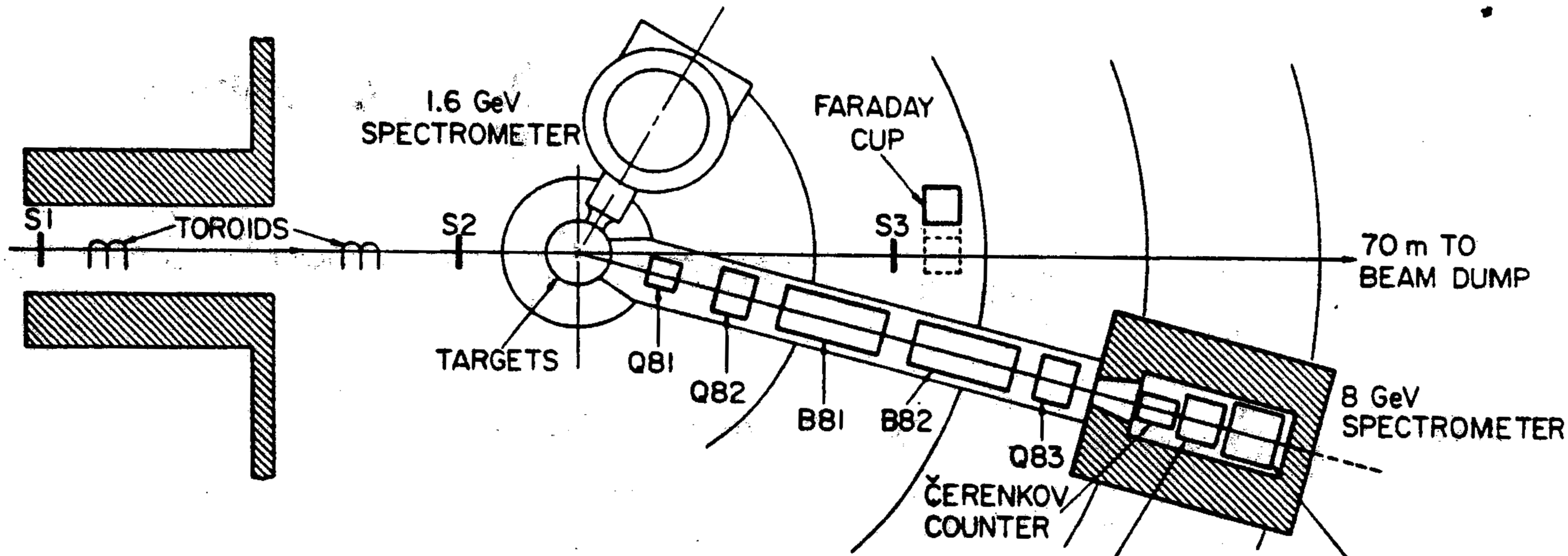
DIS - experiment

A. Bodek et al (SLAC-MIT)

Phys. Rev. D 20 1471 (1979)



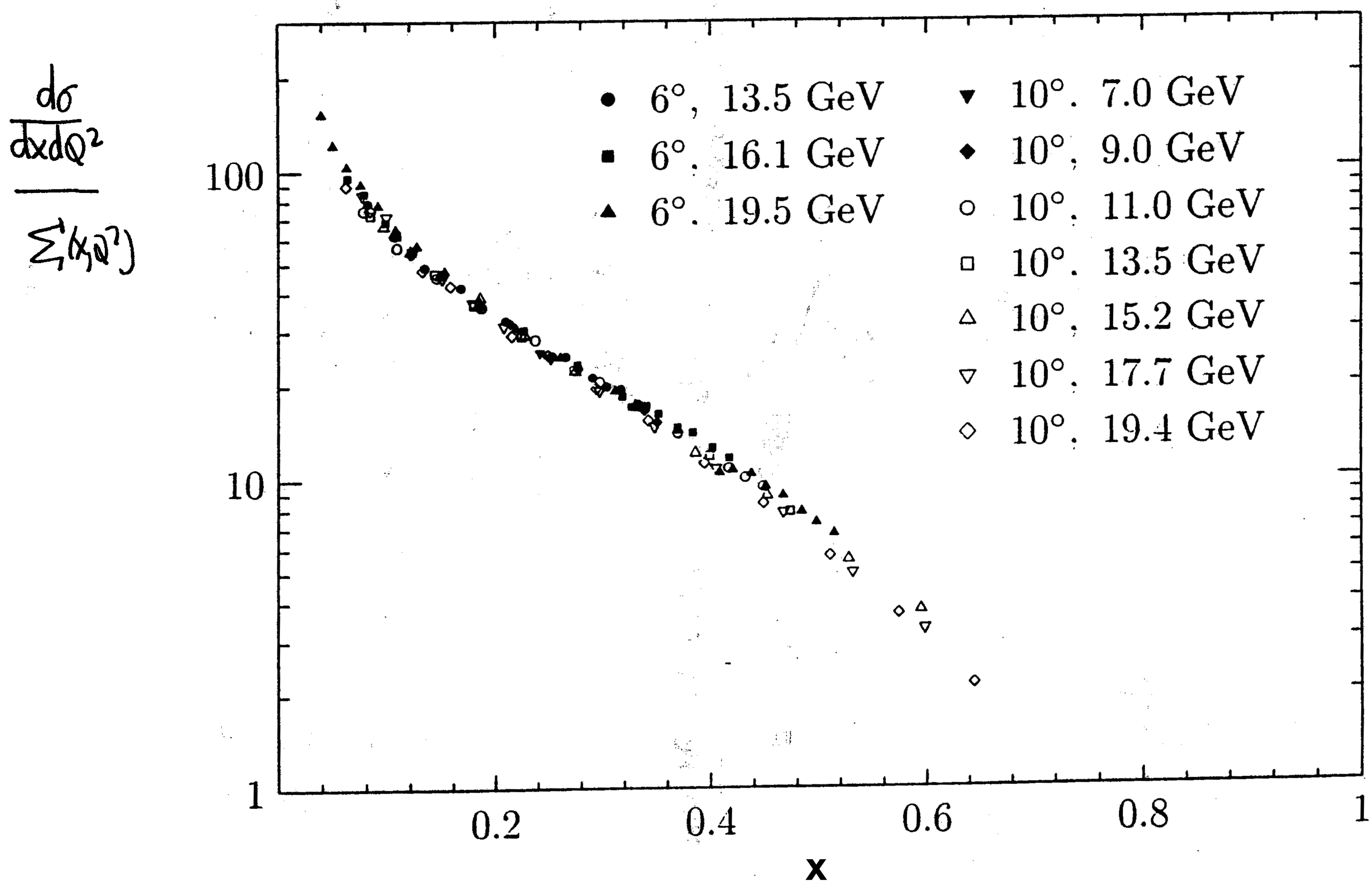
ELEVATION VIEW



PLAN VIEW

Bjorken scaling

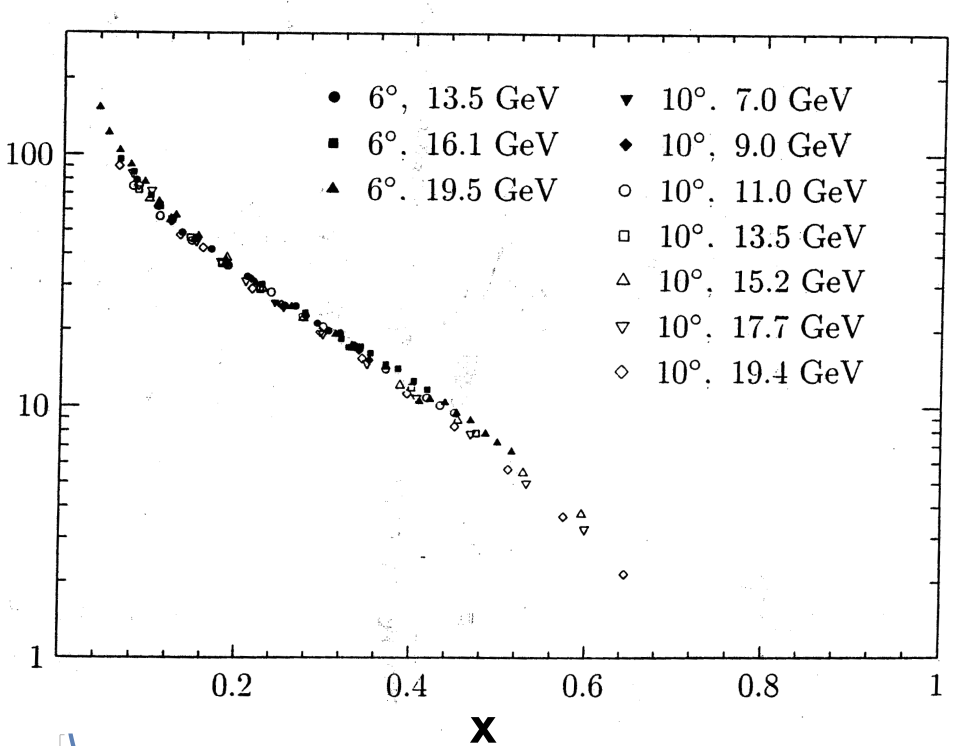
data from J.S. Poucher et al (SLAC-MIT)
Phys. Rev. Lett 32, 118 (1974)

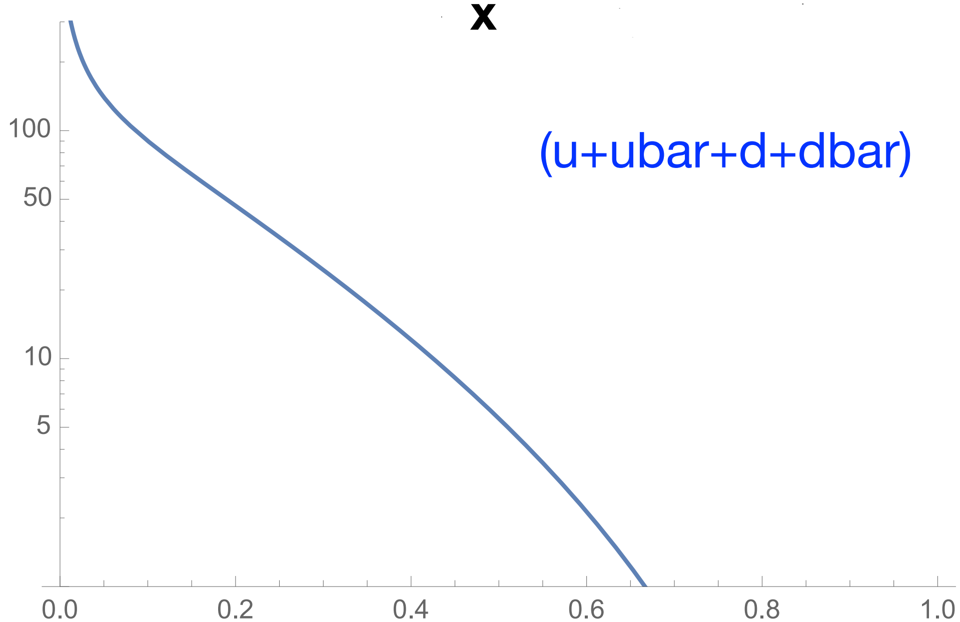


Bjorken scaling

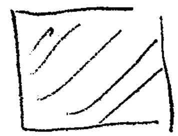
data from J.S. Poucher et al (SLAC-MIT)
Phys. Rev. Lett 32, 118 (1974)

$$\frac{\frac{d\sigma}{dx\,dQ^2}}{\Sigma(x,Q^2)}$$



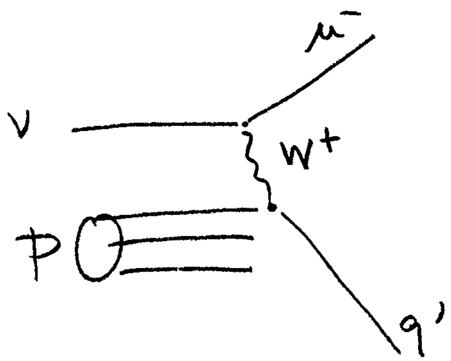


q vs. $\bar{q}$? deep inelastic ν-scattering

ν beam: $\pi^\pm \longrightarrow \begin{matrix}\mu^+\nu \\ \mu^-\bar{\nu}\end{matrix}$  $\longrightarrow \begin{matrix}\nu \\ \bar{\nu}\end{matrix}$

Fe-beam dump

$\uparrow$ can select charge with magnets



$\nu p \to \mu^- X$

W only couples to left chirality fermions: ν_L, q_L, $\bar{\nu}_R, \bar{q}_R$ ← helicity

$$\frac{d\hat{\sigma}}{dQ^2}(\nu_L d_L \to \mu^-_L u_L) = \# \frac{\alpha_W^2}{(Q^2+m_W^2)^2} \cdot 1 \quad \leftarrow \text{HW}$$

$$\nu_L \bar{u}_R \to \mu^-_L \bar{d}_R = \quad '' \quad '' \quad \cdot (1-y)^2$$

$$\bar{\nu}_R u_L \to \mu^+_R d_L \qquad (1-y)^2$$

$$\bar{\nu}_R \bar{d}_R \to \mu^+_R \bar{u}_R \qquad 1$$

$$\frac{d\sigma}{dQ^2dx}(\nu_L p^+) \sim d(x) + (1-y)^2\,\bar{u}(x)$$

$$\frac{d\sigma}{dQ^2dx}(\nu_L n^0) \sim u(x) + (1-y)^2\,\bar{d}(x)$$

get $u, \bar{u}, d, \bar{d}$ separately.

experiment w. solid targets e.g. $Fe \Rightarrow n_p\, p + n_n\, n$

similar

$\boxed{\text{PLOTS 29}}$ $\nu_L H \sim 1$

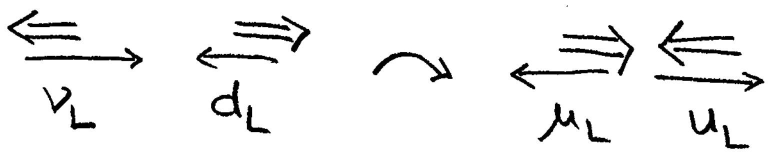
$\bar{\nu}_R H \sim (1-y)^2$

$\boxed{\text{PLOT 30}}$ $\nu\, Fe \to u, d, (\bar{u}, \bar{d})$

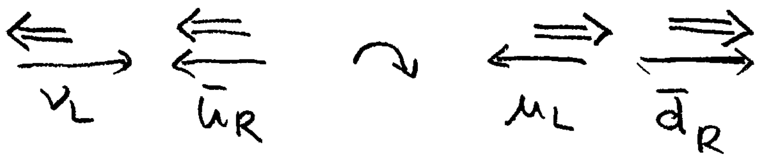
$e\, Fe \to u, d, (\bar{u}, \bar{d})$

angular dependence $\Leftrightarrow$ y-dependence

$y = 1$ maximum momentum transfer $\Rightarrow$ back-scatter



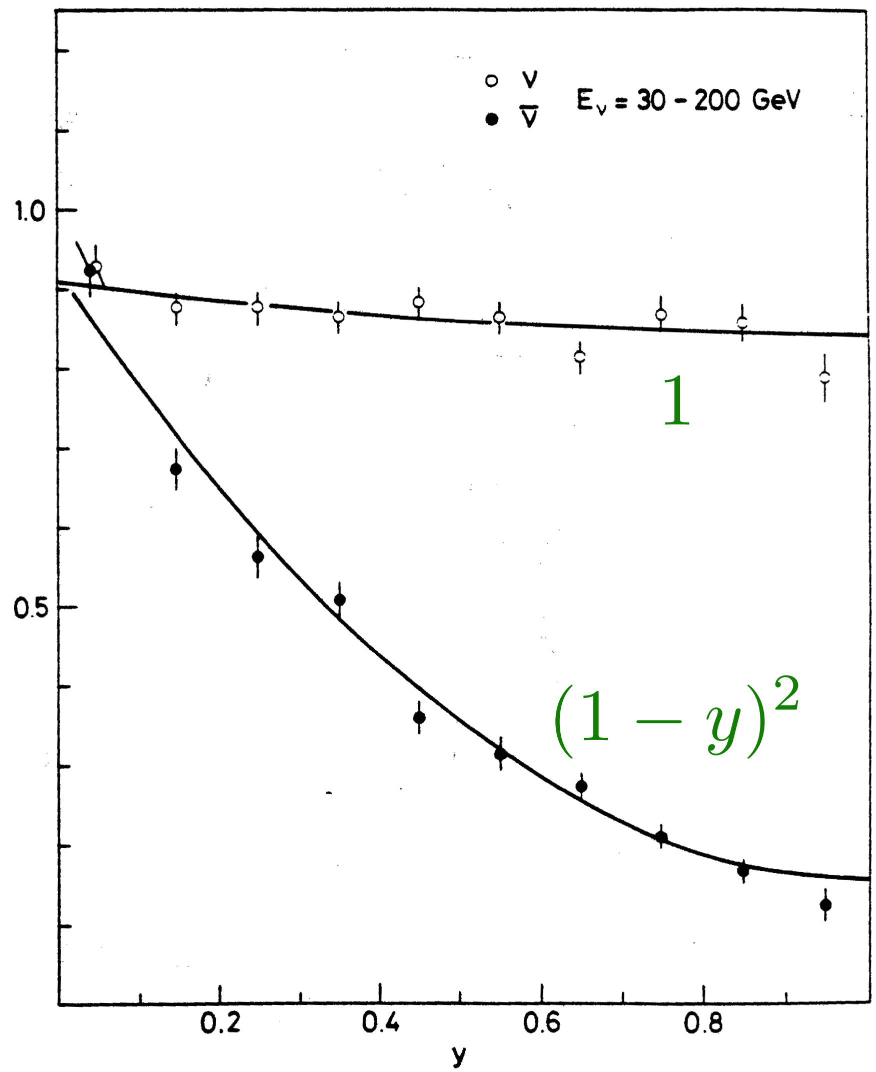
OK



$(1-y)^2$ from spin flip

DIS:
neutrinos
on Hydrogen gas

J G H de Groot et al (CDHS)
Z Phys C1, 143 (1979)



DIS:
electron-deuterium

$$\sim (f_u + f_d + f_{\bar{u}} + f_{\bar{d}})\,(1 + (1-y)^2)$$

neutrino-iron

$$\sim f_u + f_d + (f_{\bar{u}} + f_{\bar{d}})(1-y)^2$$