computation with Feynman rules:

$$\mathcal{M} = e^2(1+c_\theta) \; ! \quad \text{only off by factor of 2}$$

$$\Rightarrow \frac{d\sigma}{d\cos\theta}(e_R^- e_L^+ \to \mu_R^- \mu_L^+) = \frac{1}{32\pi E_{CM}^2} e^4 (1+c_\theta)^2 = \frac{\alpha_{em}^2}{E_{CM}^2} \frac{\pi}{2} (1+c_\theta)^2$$

couplings

forward scattering enhanced!

dim'l analysis $\sigma \sim \text{Area} \sim \frac{1}{\text{Energy}^2}$

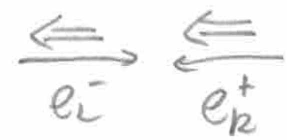
spin

$$\sigma = \int_{-1}^{1} d\cos\theta \, \frac{d\sigma}{dc_\theta} = \frac{\alpha^2}{E_{CM}^2} \frac{\pi}{2} \underbrace{\int_{-1}^{1} dc_\theta (1+c_\theta)^2}_{8/3}$$

$$= \frac{\alpha^2}{E_{CM}^2} \frac{4}{3}\pi \quad \blacksquare$$

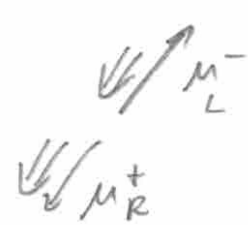
What about other polarizations?

- $e_L^- e_R^+ \to \mu_R^- \mu_L^+$ initial spins flipped



$$\Rightarrow \vec{\epsilon}_-^{\,in} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-i\\0\end{pmatrix} \Rightarrow \epsilon_+^{out\,\dagger}\cdot\epsilon_-^{in} = -1+\cos\theta$$

- $e_R^- e_L^+ \to \mu_L^- \mu_R^+$ final spins flipped



$$\Rightarrow \vec{\epsilon}_-^{\,out} = \begin{pmatrix}c_\theta\\-i\\-s_\theta\end{pmatrix}\frac{1}{\sqrt{2}} \Rightarrow -1+\cos\theta$$

- $e_L^- e_R^+ \to \mu_L^- \mu_R^+ \quad (-1)^2 \quad \Rightarrow \quad 1+\cos\theta$

all other combinations vanish for massless fermions because the coupling to the photon vanishes. For nonzero mass one gets $\frac{m}{E}$ suppressed contributions to $\mathcal{M}$.

massless case:

$$\frac{d\sigma}{d\cos\theta}(e_R^- e_L^+ \to \mu_R^- \mu_L^+) = \frac{d\sigma}{d\cos\theta}(e_L^- e_R^+ \to \mu_L^- \mu_R^+) = \frac{\alpha^2}{E_{CM}^2}\frac{\pi}{2}(1+c_\theta)^2$$

$$\frac{d\sigma}{d\cos\theta}(e_R^- e_L^+ \to \mu_L^- \mu_R^+) = \frac{d\sigma}{d\cos\theta}(e_R^- e_L^+ \to \mu_L^- \mu_R^+) = \frac{\alpha^2}{E_{CM}^2}\frac{\pi}{2}(1-c_\theta)^2$$

all others vanish e.g $(e_L^- e_L^+ \to \dots) = 0$

- in most experiments we cannot measure final spin.

  $\Rightarrow$ add different possibilities for final state helicities $\sum\limits_{\substack{\mu^- = L,R \\ \mu^+ = L,R}}$

- Most beams are unpolarized (exception SLAC SLC)

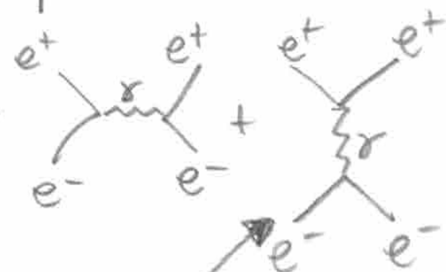
  $\Rightarrow$ average over initial helicities $\frac{1}{2\cdot 2}\sum\limits_{\substack{e^- = L,R \\ e^+ = L,R}}$

$$\frac{d\sigma}{d\cos\theta} = \frac{1}{4}\frac{\alpha^2}{E_{CM}^2}\frac{\pi}{2}\left[2(1+c_\theta)^2 + 2(1-c_\theta)^2\right]$$

$$= \frac{\alpha^2}{E_{CM}^2}\frac{\pi}{2}(1+c_\theta^2)$$

$$\sigma_{total} = \frac{\alpha^2}{E_{CM}^2}\frac{4}{3}\pi$$

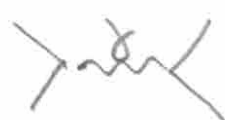
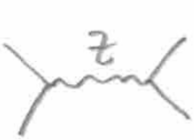
PLOTS :
- Peskin notes pg 22 lower plot — $e^+e^- \to \mu^+\mu^-$ expect $(1+c_\theta^2)$ data confirms (note detector acceptance near $\theta = 0, \pi$)
- pg 22 upper plot — $e^+e^- \to e^+e^-$ forward peaked!

  there is another diagram 

  coulomb scattering (HW 2)

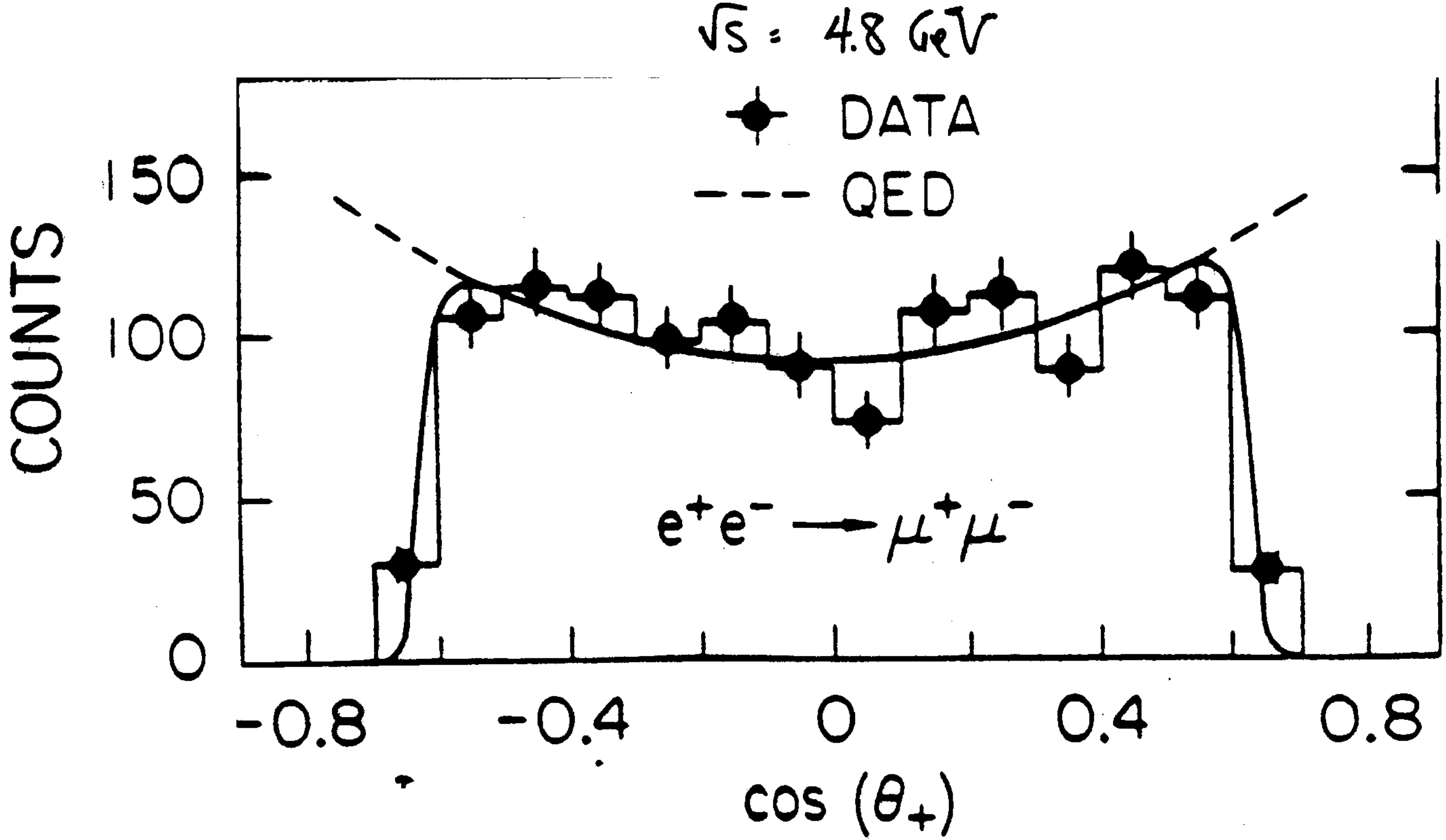
- pg23 asymmetry : L, R couple differently to Z^0

  $\Rightarrow$ linear c_θ term in $|\mathcal{M}|^2$ does not cancel

  $\mathcal{M} =$  $+$ 

- pg 25 $e^+e^- \to \tau^+\tau^-$ threshold at $E_{CM} = 2m_\tau$

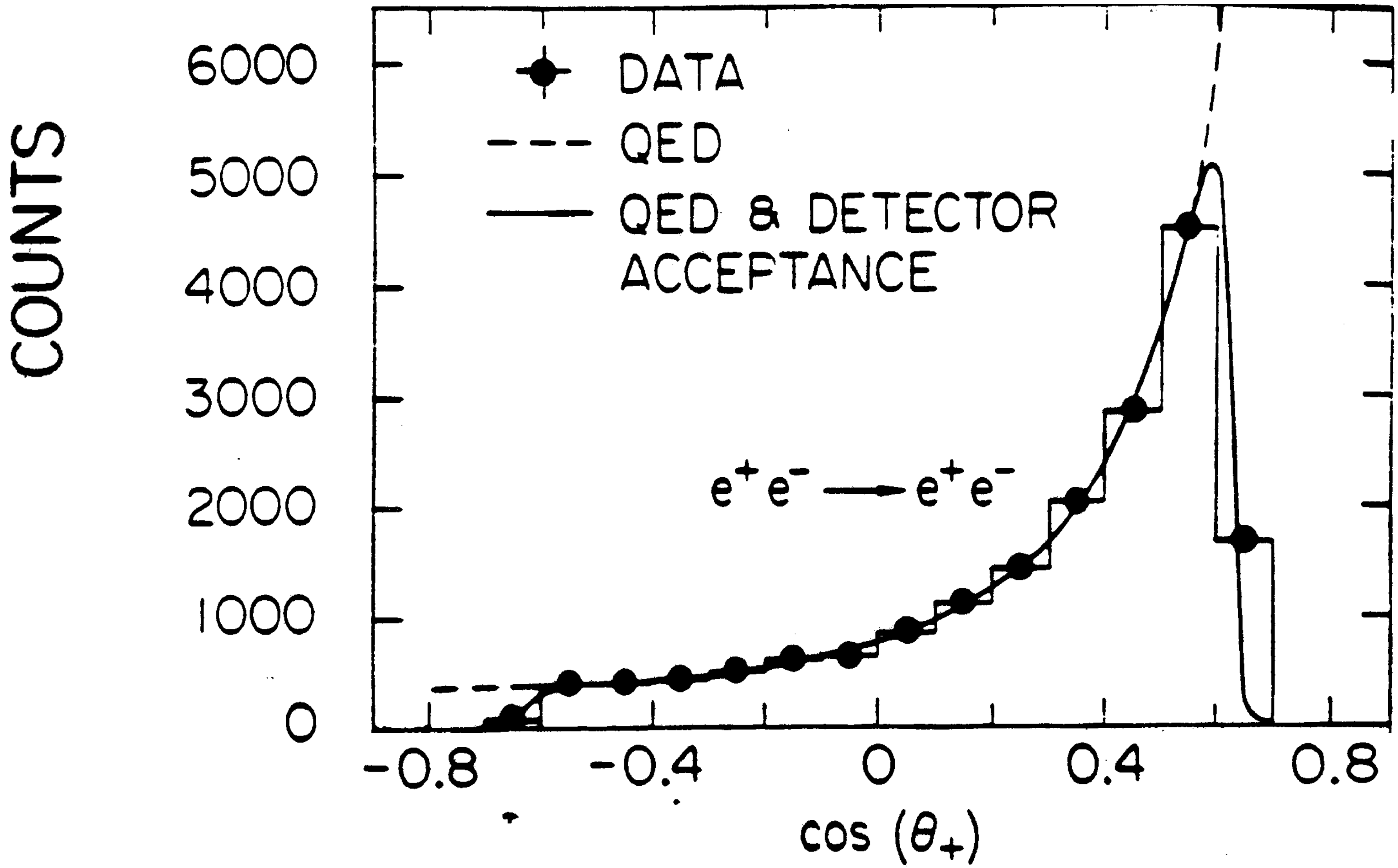
  $\Rightarrow$ can measure τ-mass by tuning E_{CM}.



J E Augustin et al (Mark 1)

Phys. Rev. Lett. 34, 233 (1975)

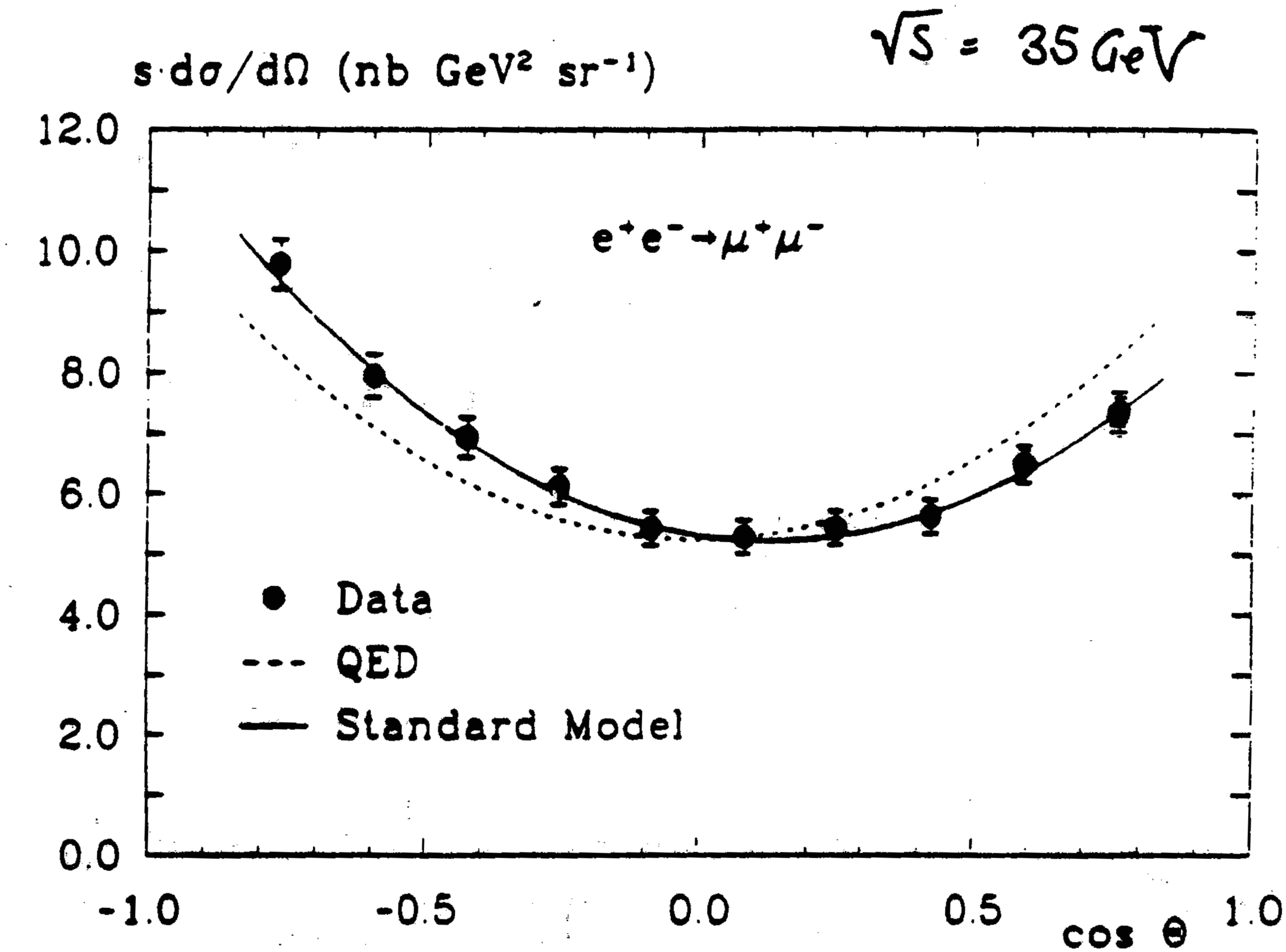
$\sqrt{s} = 4.8$ GeV



J.E. Augustin et al (Mark 1)

Phys. Rev. Lett. 34, 233 (1975)

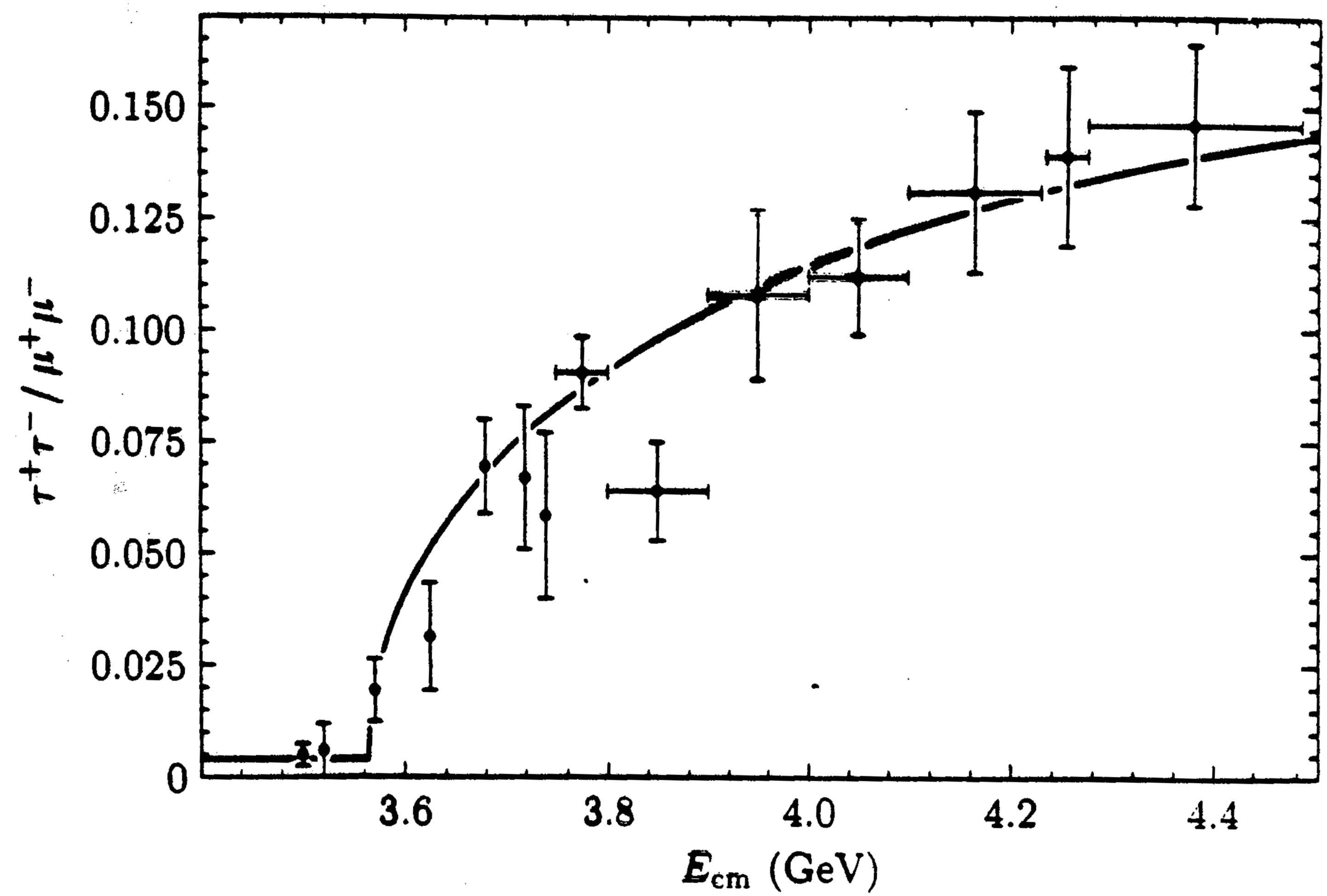
Forward-backward Asymmetry



S. Hegner et al (JADE)
Z Phys. C 46, 547 (1990)

formula for massive final state particle (see HW3)

$$\sigma = \frac{4}{3}\pi \frac{\alpha^2}{E_{CM}^2} \underbrace{\sqrt{1-\left(\frac{2m}{E_{CM}}\right)^2}}_{\text{phase space}} \underbrace{\left(1+\frac{2m^2}{E_{CM}^2}\right)}_{|\mathcal{M}|^2}$$

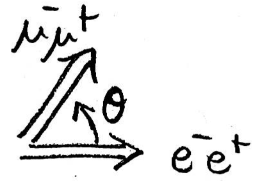


W. Bacino et al (DELCO)

Phys. Rev. Lett. 41, 13 (1978)

$m_\tau = 1782^{+2}_{-7}$ MeV

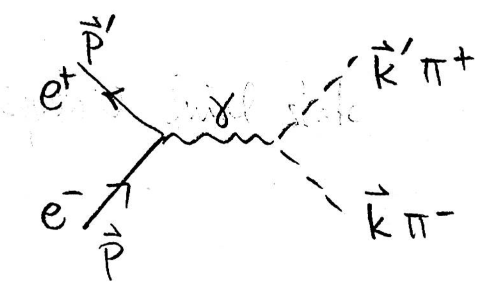
<u>Peskin 2</u> $e_R^- e_L^+ \to \mu_R^- \mu_L^+$



$$\frac{d\sigma}{d\cos\theta} \sim (1+\cos\theta)^2 \quad \text{from spins} \quad \epsilon_+^{\theta *} \cdot \epsilon_+^{0} \sim 1+\cos\theta$$

hadron production?

$e_R^- e_L^+ \to \pi^- \pi^+$



CM: $\vec{p}+\vec{p}\,'=0=\vec{k}+\vec{k}'$

initial state: $\epsilon_+^0 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ i\\ 0\end{pmatrix}$

final state, no spin. $\mathcal{M}$ scalar $\Rightarrow$ must "$\cdot$" into final state momentum

$$\vec{k}-\vec{k}' = 2\vec{k} \qquad (\vec{k}+\vec{k}'=0)$$

$$\mathcal{M} \sim \vec{k}\cdot\epsilon_+^0$$

$$\Rightarrow |\mathcal{M}|^2 \sim \vec{k}^T \epsilon^* \epsilon^T \vec{k} = \frac{1}{2}\vec{k}^T \underbrace{\begin{pmatrix}1\\ -i\\ 0\end{pmatrix}(1\ i\ 0)}_{\begin{pmatrix}1 & & \\ & 1 & \\ & & 0\end{pmatrix}} \vec{k} = \frac{1}{2}(\vec{k}^2 - k_z^2) = \frac{1}{2}k^2 \sin^2\theta$$

$e_L^- e_R^+ \to \pi^- \pi^+$ same $(\epsilon_+ \to \epsilon_-, \; i \to -i)$

$e_L^- e_L^+, \; e_R^- e_R^+$ vanish (for $m_e = 0$)

$$\Rightarrow \frac{d\sigma}{d\cos\theta}(e^- e^+ \to \pi^- \pi^+) = \frac{1}{4} \pi \alpha^2 \frac{\sin^2\theta}{E_{CM}^2}$$

$\leftarrow$ spin

$\leftarrow$ dim'l analysis

calculation

e^4

$\int d\varphi$

$$e^4 \frac{(2\pi)^4 \delta^4}{(2\pi)^6} \, 2\pi \sim \pi \alpha^2$$

E_{CM}-dependence? for $E_{CM} \gg m_e, m_\pi$

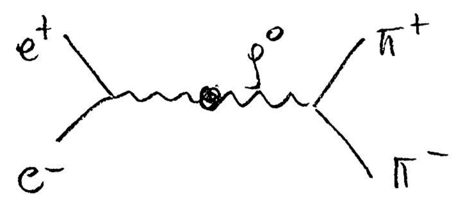
$E_{CM} = 2p = 2k$ only scale in process.

$$\sigma = \frac{\pi}{4} \frac{\alpha^2}{E_{CM}^2} \int_{-1}^{1} d\cos\theta \, (1 - \cos^2\theta) = \frac{\pi}{4} \frac{\alpha^2}{E_{CM}^2} \frac{4}{3}$$

$$\Rightarrow R_{\pi^+\pi^-} \equiv \frac{\sigma(e^+e^- \to \pi^+\pi^-)}{\sigma(e^+e^- \to \mu^+\mu^-)} = \frac{1}{4} = \text{const}$$

$\boxed{\text{PLOT pg 7}}$

R wrong!



resonance $m_\rho \sim 0.8\,\text{GeV}$